CHAPTER 14: Apportionment

14.1 The Apportionment Problem

An ***apportionment problem*** is to round a set of fractions so their sum is maintained at its original value.

The rounding procedure used in an apportionment problem is called an ***apportionment method***.

The total population, p, divided by the house size, h, is called the ***standard divisor, s***.

$$s = \frac{p}{h}$$

A group's ***quota*** q_i is the group's population, p_i, divided by the standard divisor, s.

$$q_i = \frac{p_i}{s}$$

Different apportionment methods will use different rounding rules.

When q is not already an integer, there are multiple ways to round.

- Round q up to the next integer, $\lceil q \rceil$.
- Round q down to the previous integer, $\lfloor q \rfloor$.
- Round to the nearest integer, $[q]$. If q is halfway to the next integer or larger, round up to the next integer. Otherwise, round down to the previous integer.
- Round according to the geometric mean. The geometric mean of $\lfloor q \rfloor$ and $\lceil q \rceil$ is $q^* = \sqrt{\lfloor q \rfloor \lceil q \rceil}$. If q is equal to or larger than q^*, round up to the next integer. Otherwise, round down to the previous integer.

Example

Complete the following chart.

q	$\lceil q \rceil$	$\lfloor q \rfloor$	$[q]$	q^*	Round according to q^*
5	5	5	5	$\sqrt{5 \cdot 5} = 5$	5
3.6	4	3	4	$\sqrt{3 \cdot 4} \approx 3.4641$	4
3.5	4	3	4	3.4641	4
3.465	4	3	3	3.4641	4
3.464	4	3	3	3.4641	3
0.02	1	0	0	$\sqrt{0 \cdot 1} = 0$	1

14.2 Hamilton Method

Step 1 Compute the standard divisor.
Step 2 Compute the quota for each "state" (group).
Step 3 Round each quota *down*.
Step 4 Calculate the number of seats left to be assigned.
Step 5 Assign the remaining seats to the states with the *largest fractional part* of q.

Example

Use the Hamilton method to apportion 36 silver coins to Doris, Mildred, and Henrietta if Doris paid \$5900, Mildred paid \$7600, and Henrietta paid \$1400.

$s = \frac{14900}{36} = 413.\overline{8}$

Lq

Person	Contribution	q	Rounded quota	Hamilton Apportionment
Doris	\$5900	$\frac{5900}{s} = 14.2550$	14	14
Mildred	\$7600	$\frac{7600}{s} = 18.3624$	18	18
Henrietta	\$1400	$\frac{1400}{s} = 3.3826$	3	+1 4
TOTAL	14,900		35	36

36-35=1 coin left to apportion

Example

A county has four districts, North, South, East, and West. They will apportion for a 100 member advisory council using the Hamilton method. Determine the number of council members from each district.

$s =$ $\frac{64920}{100} = 649.20$

Lq

District	Population	q	Rounded quota	Hamilton Apportionment
North	27,460	$\frac{27460}{s} = 42.2982$	42	42
South	17,250	26.5712	26	+1 27
East	19,210	29.5903	29	+1 30
West	1000	1.5404	1	1
TOTAL	64920		98	100

100 - 98 = 2 seats left to apportion