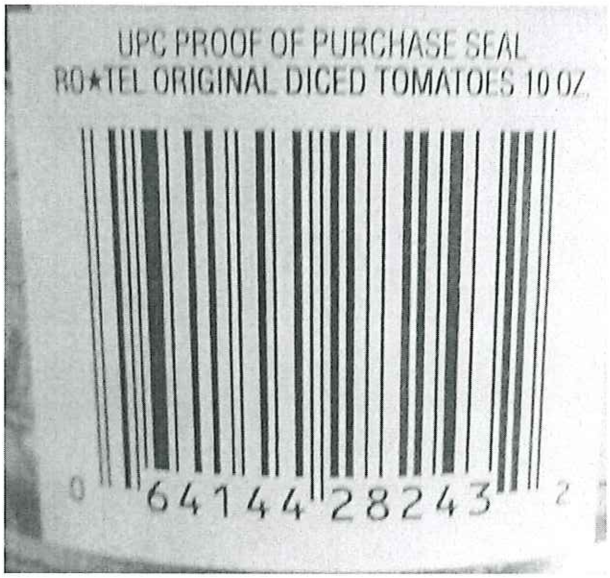
CHAPTER 16 – IDENTIFICATION NUMBERS

Consider the UPC code on a can of RO★TEL tomatoes



The scanner is not working so the clerk enters the numbers by hand as

0 64144 28263 2 ↙ check digit

and this is invalid even though the product code for the mild version of this is 28263. What happened?

The UPC codes use a ***check digit*** to minimize scanning errors. A check digit is a digit included in a code to help detect errors.

For the UPC code $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}$, the check digit, a_{12}, is chosen so that S a multiple of 10 where

$$S = 3a_1 + a_2 + 3a_3 + a_4 + 3a_5 + a_6 + 3a_7 + a_8 + 3a_9 + a_{10} + 3a_{11} + a_{12}.$$

What is the check digit for the mild RO★TEL if the first eleven digits are
0 6 4 1 4 4 2 8 2 6 3? a_{12} ↙ check digit

check digit

$$S = 3(0)+(6)+3(4)+(1)+3(4)+(4)+3(2)+(8)+3(2)+(6)+3(3)+(a_{12})$$
$$= 70 + a_{12}$$

S needs to be a multiple of 10, so $a_{12} = 0$
so the check digit is zero.

When talking about check digits, modular arithmetic will be helpful.

Definition: Congruence Modulo m sec ✓17.1 of book

Let a, b, and m be integers with $m \geq 2$. Then a is congruent to b modulo m, written

$$a \equiv b \bmod m$$

if $(a-b) \div m$ has a remainder of 0. This means $a-b$ is a multiple of m. One way to find a value for ~~b~~ a is to find the remainder when ~~a~~ b is divided by m.

Determine if the congruences below are true or false:

25 mod 6 ≡ 1 } mean same thing

$25 \equiv 1 \bmod 6$

True

$\frac{25-1}{6} = \frac{24}{6}$ $\begin{array}{r} 4 \\ 6\overline{)24} \\ -24 \\ \hline 0 \end{array}$ remainder → 0

$100 \equiv 20 \bmod 10$

True

$\frac{100-20}{10} > \frac{80}{10}$ $\begin{array}{r} 8 \\ 10\overline{)80} \\ -80 \\ \hline 0 \end{array}$ ← remainder

$52 \equiv 0 \bmod 13$ True

$\frac{52-0}{13} = \frac{52}{13}$ $\begin{array}{r} 4 \\ 13\overline{)52} \\ -52 \\ \hline 0 \end{array}$ ← remainder

$75 \equiv 7 \bmod 5$ False

$\frac{75-7}{5} = \frac{68}{5}$ $\begin{array}{r} 13 \\ 5\overline{)68} \\ -5 \\ \hline 18 \\ -15 \\ \hline 3 \end{array}$ ← remainder

Find the following values:

(a) 34 mod 5 ≡ 4 $\begin{array}{r} 6 \\ 5\overline{)34} \\ -30 \\ \hline 4 \end{array}$

(b) 78 mod 11 ≡ 1 $\begin{array}{r} 7 \\ 11\overline{)78} \\ -77 \\ \hline 1 \end{array}$

(c) 13 mod 15 ≡ 13 $\begin{array}{r} 0 \\ 15\overline{)13} \\ -0 \\ \hline 13 \end{array}$

(d) 12 mod 2 ≡ 0 $\begin{array}{r} 6 \\ 2\overline{)12} \\ -12 \\ \hline 0 \end{array}$

Some types of errors when dealing with identification numbers are

- Replacing one digit with a different digit (single digit error)
- Transposing two adjacent digits (adjacent transposition error)
- Transposing two digits that are separated by another digit (jump transposition error)

Assume that the correct code was 5678 and provide an example of these errors:

Single digit error: 5778 6678 5674 and lots more

Adjacent Transposition Error: 6578 5687 5768

Jump Transposition Error: 7658 5876

Note that some of the digits in the UPC code are multiplied by 3. Those digits had a ***weight*** of 3. Other codes use different weights.

A code $a_1a_2a_3a_4a_5$ (check digit) uses the last digit as a check digit. The check digit is found using the formula

$$a_5 = (a_1 + 7a_2 + a_3 + 7a_4) \bmod 10$$

(a) What is the check digit for the code 2374? check digit is $\underline{8}$

$a_5 = (2) + 7(3) + (7) + 7(4) \bmod 10$

$a_5 = 58 \bmod 10$

$a_5 = 8$

$$\begin{array}{r} 5 \\ 10\overline{)58} \\ -50 \\ \hline 8 \end{array}$$

(b) Find the value of the missing digit x in the code $468x3$

$3 = (4) + 7(6) + (8) + 7(X) \bmod 10$

$3 = 54 + 7X \bmod 10$

$$\begin{array}{r} 6 \\ 10\overline{)61} \\ -60 \\ \hline 1 \end{array}$$ ← remainder

X =	0	1	2	3	4	5	6	7	8	9
Sum =	54	61	68	75	82	89	96	103	110	117
Sum Mod 10 =	4	1	8	5	2	9	6	3	0	7

$\boxed{\text{So } X = 7}$

(c) Will this code find an error if a single digit is entered incorrectly?

Let's look at an error in the first digit, a_1.

Correct Code: $a_1a_2a_3a_4$

Incorrect Code: $e_1a_2a_3a_4$

So the correct check digit is

$(a_1 + 7a_2 + a_3 + 7a_4)$ mod 10

and the incorrect check digit is

$(e_1 + 7a_2 + a_3 + 7a_4)$ mod 10

✱ The error will NOT be caught if

$(a_1 + 7a_2 + a_3 + 7a_4) - (e_1 + 7a_2 + a_3 + 7a_4)$ is a multiple of 10.

This simplifies to

$a_1 - e_1$ is a multiple of 10,

Multiples of 10 are $\pm\{0,\ 10, 20, 30,\ \ldots\}$.

$$\begin{array}{r} a_1 - e_1 = 0 \\ +e_1 \qquad +e_1 \\ \hline a_1 = e_1 \end{array}$$

If $a_1 - e_1 = 0$, then $a_1 = e_1$ so we did not really make an error.

If $a_1 - e_1 = \pm 10$ (which can also be written as $|a_1 - e_1| = 10$), then a_1 and e_1 are digits that are separated by 10 units. The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are never separated by more than 9 units, so this and all higher multiples of 10 are impossible.

Therefore, all single-digit errors in the first digit would be caught.

The same logic also applies to the third digit.

Therefore, we need to check the even-numbered positions.

Let's look at an error in the second digit, a_2.

Correct Code: $a_1a_2a_3a_4$

Incorrect Code: $a_1e_2a_3a_4$

So the correct check digit is

$(a_1 + 7a_2 + a_3 + 7a_4)$ mod 10

and the incorrect check digit is

$(a_1 + 7e_2 + a_3 + 7a_4)$ mod 10

The error will NOT be caught if

$(a_1 + 7a_2 + a_3 + 7a_4) - (a_1 + 7e_2 + a_3 + 7a_4)$ is a multiple of 10.

This simplifies to

$7a_2 - 7e_2$ is a multiple of 10,

which means $7(a_2 - e_2)$ is a multiple of 10,

which means, $a_2 - e_2$ is a multiple of $\frac{10}{7}$.

Multiples of $\frac{10}{7}$ are $\pm\left\{0, \frac{10}{7}, \frac{20}{7}, \frac{30}{7}, \frac{40}{7}, \frac{50}{7}, \frac{60}{7}, 10, \frac{80}{7}, \dots\right\}$.

If $a_2 - e_2 = 0$, then $a_2 = e_2$ so we did not really make an error.

If $a_2 - e_2 = \pm\frac{10}{7}$ (which can also be written as $|a_1 - e_1| = \frac{10}{7}$), then a_1 and e_1 are digits that are separated by $\frac{10}{7}$. The difference between digits is always an integer, so this and all other non-integer multiples of $\frac{10}{7}$ are impossible.

If $|a_2 - e_2| = 10$, then then a_1 and e_1 are digits that are separated by 10 units. The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are never separated by more than 9 units, so this and all higher multiples of $\frac{10}{7}$ are impossible.

Therefore, all single-digit errors in the second digit would be caught.

The same logic also applies to the fourth digit.

We have now checked all four digits for single-digit errors and have not found any that would not be detected. Therefore, this scheme detects all single-digits errors.

(d) Will this code find all adjacent transposition errors?

Let's look at an adjacent transposition of the first two digits, a_1 and a_2.

Correct Code: $a_1 a_2 a_3 a_4$

Incorrect Code: $a_2 a_1 a_3 a_4$

So the correct check digit is

$(a_1 + 7a_2 + a_3 + 7a_4) \bmod 10$

and the incorrect check digit is

$(\underline{a_2} + 7\underline{a_1} + a_3 + 7a_4) \bmod 10$

The error will NOT be caught if

$(a_1 + 7a_2 + a_3 + 7a_4) - (a_2 + 7a_1 + a_3 + 7a_4)$ is a multiple of 10.

This simplifies to

$a_1 + 7a_2 - a_2 - 7a_1$ is a multiple of 10,

which means $6a_2 - 6a_1 = 6(a_2 - a_1)$ is a multiple of 10,

which means, $a_2 - a_1$ is a multiple of $\frac{10}{6} = \frac{5}{3}$.

Multiples of $\frac{5}{3}$ are $\pm\left\{0, \frac{5}{3}, \frac{10}{3}, 5, \frac{20}{3}, \frac{25}{3}, 10, \frac{35}{3}, \ldots\right\}$.

If $a_2 - a_1 = 0$, then $a_2 = a_1$ so we did not really make an error.

If $a_2 - a_1 = \pm\frac{5}{3}$ (which can also be written as $|\, a_2 - a_1| = \frac{5}{3}$), then a_2 and a_1 are digits that are separated by $\frac{5}{3}$. The difference between digits is always an integer, so this and all other non-integer multiples of $\frac{5}{3}$ are impossible.

If $|\, a_2 - a_1| = 5$, then then a_2 and a_1 are digits that are separated by 5 units. The digits that are separated by 5 units are 0 and 5, 1 and 6, 2 and 7, 3 and 8, and 4 and 9. These errors will NOT be caught.

If $| a_2 - a_1| = 10$, then then a_2 and a_1 are digits that are separated by 10 units. The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are never separated by more than 9 units, so this and all higher multiples of $\frac{5}{3}$ are impossible.

Therefore, all adjacent transposition errors of the first two digits will be caught unless the digits are separated by 5 units. (This is the same as saying $| a_2 - a_1| = 5$).

The same logic also applies to the remaining adjacent digits.

Therefore, this scheme detects all adjacent transposition errors other than the interchange of 0 and 5, 1 and 6, 2 and 7, 3 and 8, or 4 and 9.

(e) Will this code find all jump transposition errors?

Let's look at 1st & 3rd digit transposition

NO jump transpositions are caught ☹

b/c order of addition will not change the check digit and 1st & 3rd digits have the same weight as each other and 2nd & 4th digits have the same weight as each other

Data can be encoded in identification numbers.

An Illinois driver's license contains a portion (Y-YDDD) that represents the last two digits of the year of birth of the driver and codes the birthday according to the following formulas where m represents birth month and d represents the birth date.

$$\text{Male} = 31(m-1)+d$$
$$\text{Female} = 31(m-1)+d+600$$

(a) What would the Y-YDDD digits of an of Illinois driver's license number look like for a man born on February 12, 1967?

Y-Y is 6-7

$m=2 \quad d=12$

DDD for Male $= 31(m-1)+d$

$= 31(2-1)+12$

$= 31(1)+12 = 43$

$\boxed{\begin{array}{l} \text{Y-Y DDD} \\ 6\text{-}7043 \end{array}}$

(b) What do you know about a person whose Y-YDDD digits are 1-0642?

Born in "10" either 1910 or 2010

Female b/c DDD > 600

$642 = 31(m-1)+d+600$

$-600 \qquad\qquad -600$

$42 = 31(m-1)+d$

$$\begin{array}{r} 1 \\ 31\overline{)42} \\ -31 \\ \hline 11 \end{array} \quad \begin{array}{l} \leftarrow m-1=1 \\ \\ \\ \leftarrow d \end{array}$$

$m-1=1$

$+1 \quad +1$

$m=2$

(She was born Feb 11, 1910)

(c) What do you know about a person whose Y-YDDD digits are 9-0373?

Male b/c DDD < 600

Born in 1990

$373 = 31(m-1)+d$

$$\begin{array}{r} 12 \\ 31\overline{)373} \\ -31 \\ \hline 63 \\ -62 \\ \hline 1 \end{array} \quad \begin{array}{l} \leftarrow m-1 \\ \\ \\ \\ \\ \leftarrow d \end{array}$$

Can't be born in 13th month, so this # is impossible

SAMPLE EXAM QUESTIONS FROM CHAPTER 16

1. Determine the check digit that should be appended to the identification number 634498, if the check digit is the number needed to bring the total of all the digits to a multiple of 10. if X is check digit

(A) The code is invalid [✓ (B) 6] (C) 8 (D) 4

(E) None of these

(6+3+4+4+9+8+X) needs to be a multiple of 10

34 + X needs to be a multiple of 10

2. Which, if any, of the statements below are true? Mark all correct answers.

[✓ (A)] $101 \equiv 1 \bmod 2$ True $\frac{101-1}{2} = \frac{100}{2}$ 2)100 = 50, −100, 0 ←

[✓ (B)] $77 \equiv 0 \bmod 11$ True $\frac{77-0}{11} = \frac{77}{11}$ 11)77 = 7, −77, 0 ←

[✓ (C)] $49 \equiv 1 \bmod 12$ True $\frac{49-1}{12} = \frac{48}{12}$ 12)48 = 4, −48, 0 ←

(D) $39 \equiv 5 \bmod 5$ False $\frac{39-5}{5} = \frac{34}{5}$ 5)34 = 6, −30, 4 ← not 0, so False

(E) None of these are true.

3. The number 4320 is accidentally entered as 4321.

What type of error is this?

(A) A transposition error

(B) A jump transposition error

[✓ (C)] A single digit error

(D) A baseball error

(E) None of these

4. The last three digits of a person's ID are calculated based on their birthday where m represents birth month and d represents the birth date.

$$\text{Male} = 35(m-1)+d$$
$$\text{Female} = 35(m-1)+d+500$$

(a) What are the last three digits of a man's ID number if he was born on October 8th? $m=10$ $d=8$

$$\begin{aligned}\text{Male} &= 35(10-1)+8\\ &= 35(9)+8\\ &= 315+8\\ &= 323\end{aligned}$$

$$\boxed{323}$$

(b) What do you know about a person if the last three digits of the person's ID number are 603?

Female b/c > 500

$$\begin{array}{rcl} 35(m-1)+d+500 &=& 603\\ -500 && -500\\ \hline 35(m-1)+d &=& 103\end{array}$$

$$\begin{array}{r} 2 \leftarrow m-1\\ 35\overline{)103}\\ -70\\ \hline 33 \leftarrow d\end{array}$$

Born March 33_3???

There is not a 33^{rd} day in March, so incorrect

March 31 $35(3-1)+31+500=601$

April 1 $35(4-1)+1+500=606$

(c) What do you know about a person if the last three digits of the person's ID number is 320? $35(m-1)+d=320$

Male b/c < 500

$$\begin{array}{r} 9 \leftarrow m-1\\ 35\overline{)320}\\ -315\\ \hline 5 \leftarrow d\end{array}$$

He was born October 5th

check digit

5. A code is given by $a_1a_2a_3a_4$ where a_4 is the check digit. The check digit is $a_4 = 7a_1 + 2a_2 + 5a_3$ mod 9.

check digit

(a) Determine the value of x in the code $2x45$, given that the check digit is valid.

$5 \equiv 7(2) + 2(x) + 5(4)$ mod 9

$5 \equiv (14 + 2x + 20)$ mod 9

$5 \equiv (34 + 2x)$ mod 9

$x =$	0	1	2	3	4	5	6	7	8	9
Sum	34	36	38	40	42	44	46	48	50	52
Sum mod 9	7	0	2	4	6	8	1	3	5	7

$$\begin{array}{r} 3 \\ 9\overline{)34} \\ -27 \\ \hline 7 \end{array}$$

$x = 8$

(b) Determine if the check digit will find all single digit errors in the second position.

Correct code $a_1\ a_2\ a_3$

Incorrect code $a_1\ e_2\ a_3$

Correct check digit $7a_1 + 2a_2 + 5a_3$ mod 9

Incorrect check digit $7a_1 + 2e_2 + 5a_3$ mod 9

The error will NOT be caught if $(7a_1 + 2a_2 + 5a_3) - (7a_1 + 2e_2 + 5a_3)$ is a multiple of 9

This simplifies to $2a_2 - 2e_2$ is a multiple of 9

$2(a_2 - e_2)$ is a multiple of 9

$a_2 - e_2$ is a multiple of $\frac{9}{2}$

Multiples of $\frac{9}{2}$ are $\pm\{0, \frac{9}{2}, 9, \frac{27}{2}, \ldots\}$

If $a_2 - e_2 = 0$, there is no error b/c $a_2 = e_2$

If $a_2 - e_2 = \pm\frac{9}{2}$, this error is impossible b/c digits are always separated by an integer.

If $a_2 - e_2 = \pm 9$, then error will not be caught if the digits are separated by 9. This can happen when 0 and 9 are interchanged.

p11
continued

If $a_2 - e_2 =$ Larger multiples, this error is impossible b/c digits are never separated by more than 9.

All single-digit errors in the second position for this scheme will be caught EXCEPT $|a_2 - e_2| = 9$ (in other words when 0 and 9 are interchanged.

Look at part a and notice that all X produced different check digits except $X=0$ and $X=9$

(c) Determine if the check digit will find all transposition errors in the second and third positions.

Correct code $a_1\ a_2\ a_3$

Incorrect code $a_1\ a_3\ a_2$

Correct check digit $(7a_1 + 2a_2 + 5a_3) \bmod 9$

Incorrect check digit $(7a_1 + 2a_3 + 5a_2) \bmod 9$

Error will NOT be caught if

$(7a_1 + 2a_2 + 5a_3) - (7a_1 + 2a_3 + 5a_2)$ is a multiple of 9

This simplifies to

$2a_2 + 5a_3 - 2a_3 - 5a_2$ is a multiple of 9

$3a_3 - 3a_2$ is a multiple of 9

$3(a_3 - a_2)$ is a multiple of 9

$a_3 - a_2$ is a multiple of $\frac{9}{3} = 3$

Multiples of 3 are $\pm \{0, 3, 6, 9, 12, \ldots\}$

If $a_3 - a_2 = 0$, there is no error b/c $a_3 = a_2$

If $a_3 - a_2 = \pm 3$, digits that are separated by 3 are NOT caught

If $a_3 - a_2 = \pm 6$, digits that are separated by 6 are NOT caught

If $a_3 - a_2 = \pm 9$, digits that are separated by 9 are NOT caught

If $a_3 - a_2 = \pm 12$, this and all higher multiples are impossible b/c digits are never separated by more than 9.

This scheme will NOT catch transposition errors at 2nd and 3rd digits when $|a_3 - a_2| = 3$

$|a_3 - a_2| = 6$

or $|a_3 - a_2| = 9$