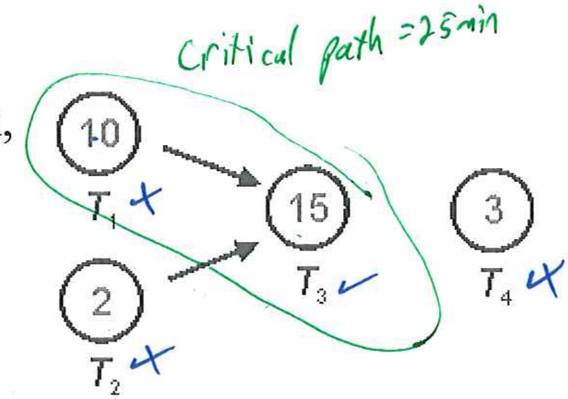
CHAPTER 3 – PLANNING AND SCHEDULING

Pizza Dinner

To make a pizza, you need to pre-heat the oven, unwrap the frozen pizza, and bake it for 15 minutes. You also need to find plates. The priority list for these tasks is T_3, T_2, T_4, T_1.

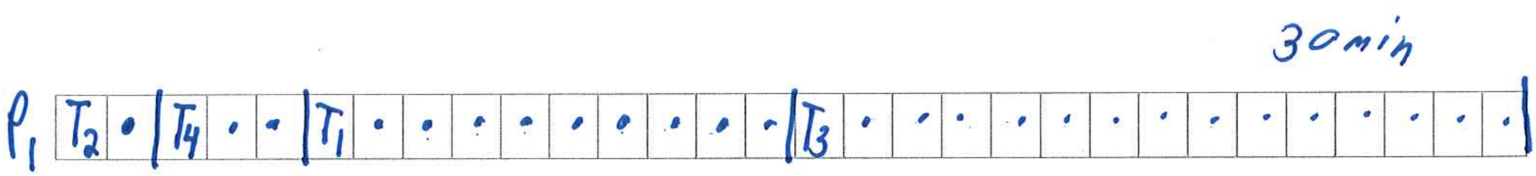


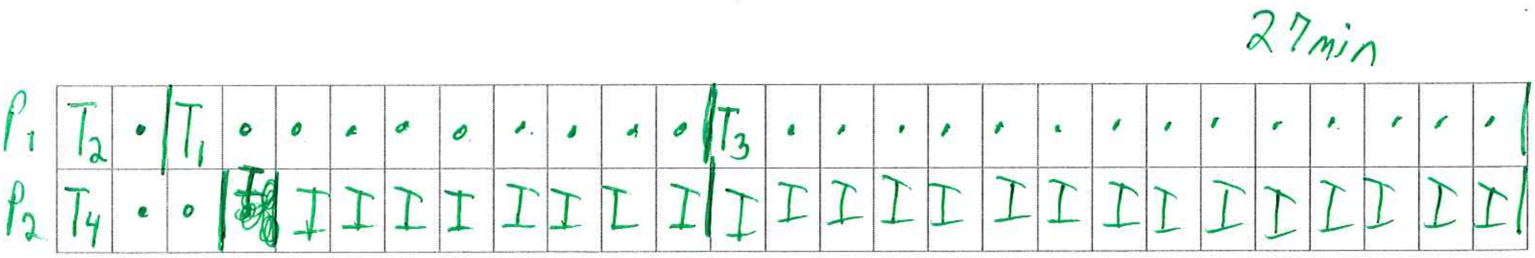
Schedule these tasks with one person working. Will dinner get ready faster with two people? Yes

Assumptions and Rules

- If a processor starts working on a task, the work will continue until that task is complete. No multitasking
- No processor stays idle if there is a task to be done.
- The scheduling problem has an associated order-requirement weighted digraph.
- The tasks are arranged in a ***priority list*** that is independent of the digraph.

A task is considered ***ready*** if all its predecessors in the digraph have been completed. We will show the schedules on a ***Gantt chart***.

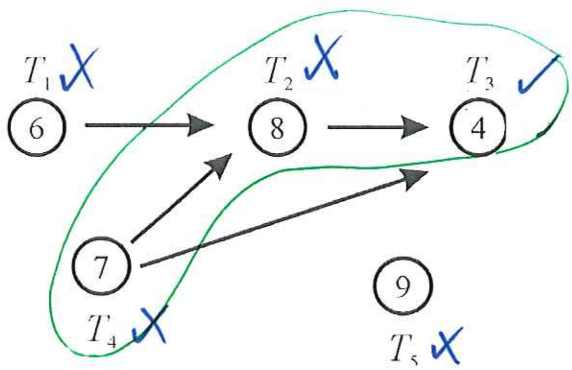




List Processing Algorithm

1. **Assignment of Processors:** The lowest numbered idle processor is assigned to the highest priority ready task until either all processors are assigned or all ready tasks are being worked on.
2. **Status Check:** When a processor completes a task, that processor becomes idle. Check for ready tasks and tasks not yet completed and determine which of the following applies
   a. If there are ready tasks, repeat step 1.
   b. If there are no ready tasks but not every task has been completed, the idle processor remains idle until more tasks are completed.
   c. If all tasks are completed, the job is done.

Using the order-requirement digraph below, schedule the tasks on two processors using the priority list T_5, T_4, T_3, T_2, T_1. What is the completion time if the times for each task are given in minutes? Is it optimal?

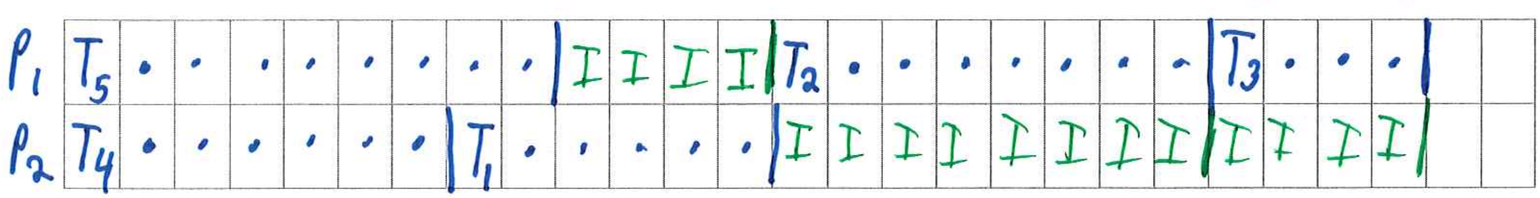


critical path

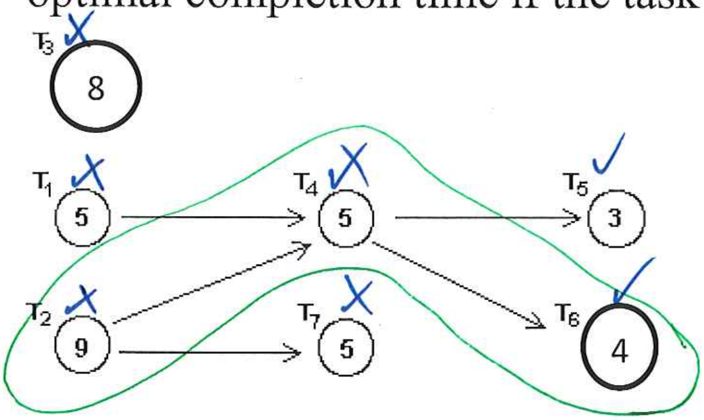
$T_4\ T_2\ T_3 = 19$ min

not optimal

25 min



Using the order-requirement digraph below, schedule the tasks on three processors using the priority list $T_1, T_2, T_3, T_4, T_5, T_6, T_7$. What is the optimal completion time if the task times are in hours? 18 hr



critical path

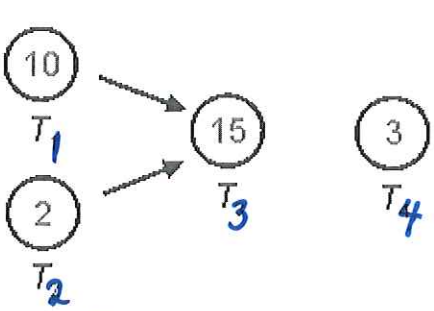
$T_2\ T_4\ T_6 = 18$ hours

18 hours

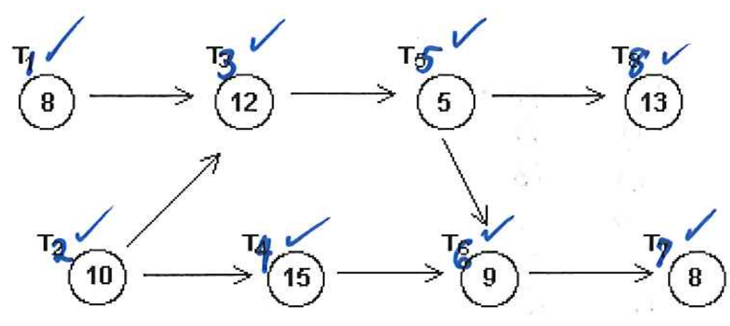
P1	T1	•	•	•	•	I	I	I	I	T4	•	•	•	•	T5	•	•	I					
P2	T2	•	•	•	•	•	•	•	•	T7	•	•	•	•	T6	•	•	•					
P3	T3	•	•	•	•	•	•	•	I	I	I	I	I	I	I	I	I	I					

A ***decreasing time priority list*** is created by listing all the tasks from the longest to the shortest completion time. If there is a tie, the lower numbered task has the higher priority.

Create a decreasing time priority list for the digraphs below:



$T_3\ T_1\ T_4\ T_2$

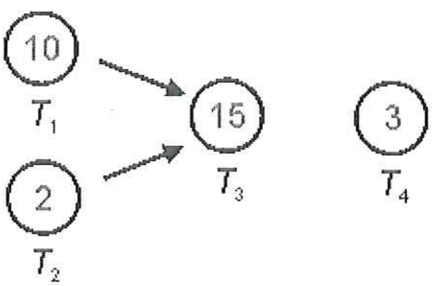


$T_4\ T_8\ T_3\ T_2\ T_6\ T_1\ T_7\ T_5$

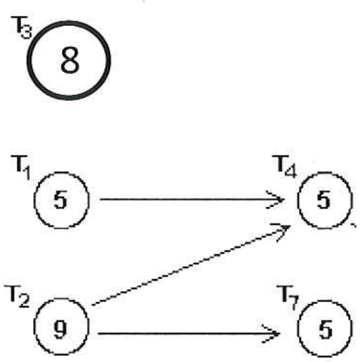
Creating a Priority List for Critical Path Scheduling

1. Find a task that heads a critical (longest) path in the order-requirement digraph. If there is a tie, chose the lowest task number.
2. Place the task found in step 1 next in the priority list.
3. Remove the task found in step 1 from the digraph. Remove all edges attached to the removed task to form a new diagraph.
4. If all tasks have been removed, the list is completed. If tasks remain, return to step 1.

Create a critical path priority list for the digraphs below

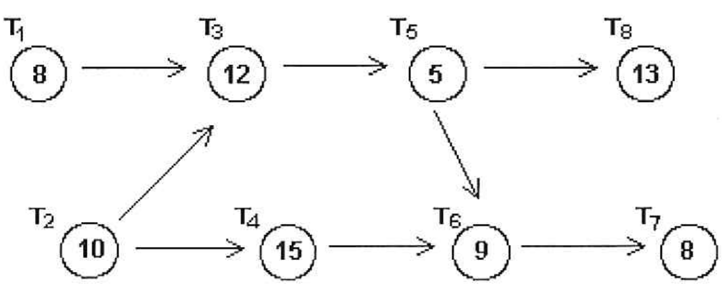


$T_1\ T_2\ T_3\ T_4$



$T_2\ T_1\ T_3\ T_4\ T_7$

		After T_2	After T_1	After T_3
T_3	8	8	8	0
$T_1\ T_4$	10	10	5	5
$T_2\ T_4$	14	5	5	5
$T_2\ T_7$	14	5	5	5



$T_2\ T_1\ T_3\ T_4\ T_5\ T_6\ T_8\ T_7$

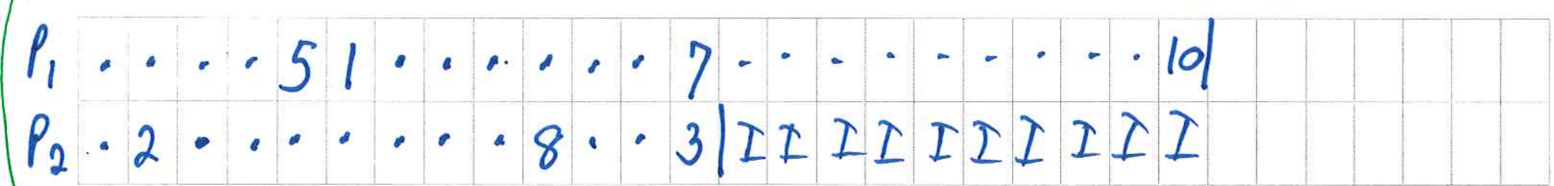
	After T_2	After T_1	After T_3	After T_4
$T_1 T_3 T_5 T_8 = 38$	38	30	18	18
$T_1 T_3 T_5 T_6 T_7 = 42$	42	34	22	22
$T_2 T_3 T_5 T_8 = 40$	30	30	18	18
$T_2 T_3 T_5 T_6 T_7 = 44$	34	34	22	22
$T_2 T_4 T_6 T_7 = 42$	32	32	32	17

When tasks are independent they can be done in any order. When the tasks are scheduled in the order they are listed, this is called a ***list processing algorithm***.

Using two processors, find the completion time for independent tasks of length 5, 2, 8, 1, 7, 3, 10 using the list processing algorithm and a decreasing time priority list. Are either completion times optimal?

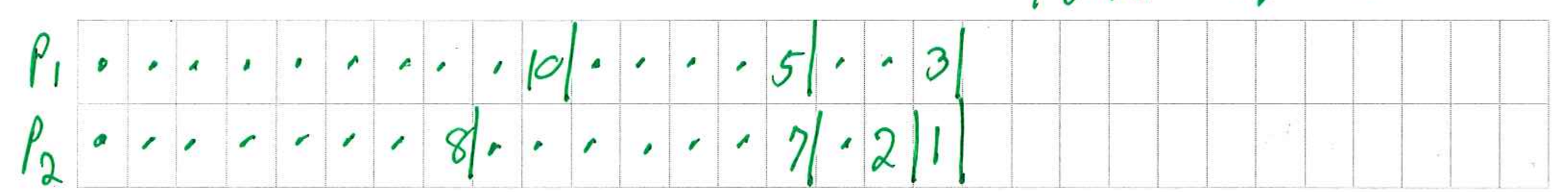
$$\text{optimal} = \frac{5+2+8+1+7+3+10}{\#\ \text{processors}} = \frac{36}{2} = 18 \text{ min}$$

23 min



10, 8, 7, 5, 3, 2, 1

18 min optimal



What is the minimum time required to perform nine independent tasks with a total task time of 45 minutes on three machines?

$$\frac{45 \text{ min}}{3 \text{ processors}} = 15 \text{ min/processor}$$

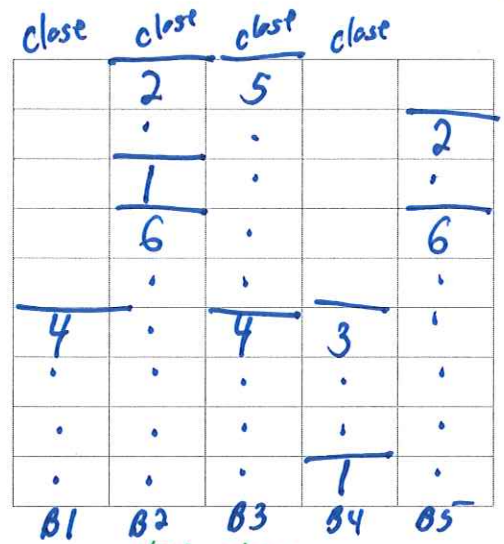
You have the following weights (in ounces) to pack into bins that hold no more than 9 ounces. How should this be done? What is the minimum number of bins needed? *4*

4, 6, 1, 2, 4, 5, 1, 3, 6, 2 *34 ounces*

optimal *34 ounces / 9 ounces/bin = 3.7 bins so need at least 4 bins*

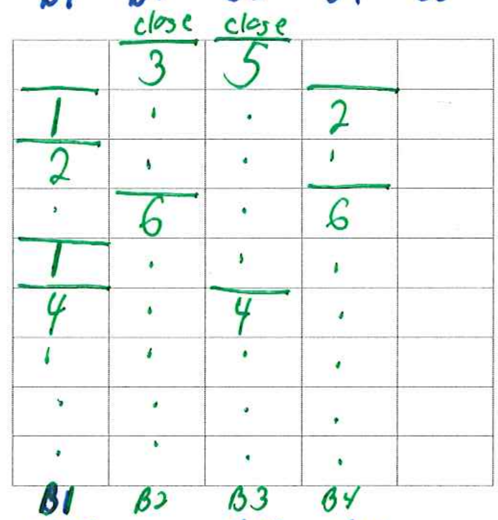
Next-fit Algorithm (NF): Put items into the open bin until the next item will not fit. Close the bin and open a new bin for the next item.

4, 6, 1, 2, 4, 5, 1, 3, 6, 2 *Not optimal*



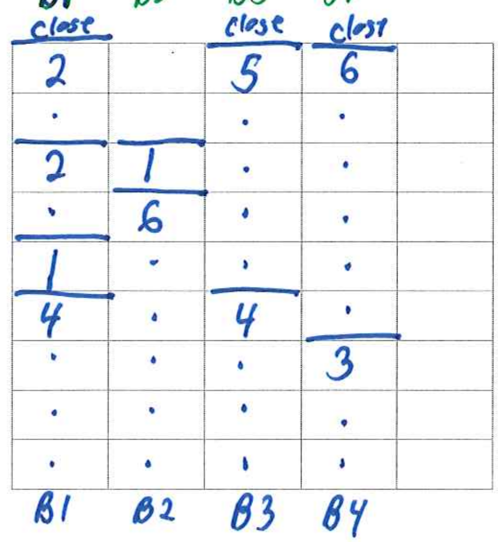
First-fit Algorithm (FF): Put items into the first already open bin that has space for it. If no open bin has space, open a new bin.

4, 6, 1, 2, 4, 5, 1, 3, 6, 2 *optimal*



Worst-fit Algorithm (WF): Put items into an already open bin that has the most space for it. If no open bin has space, open a new bin.

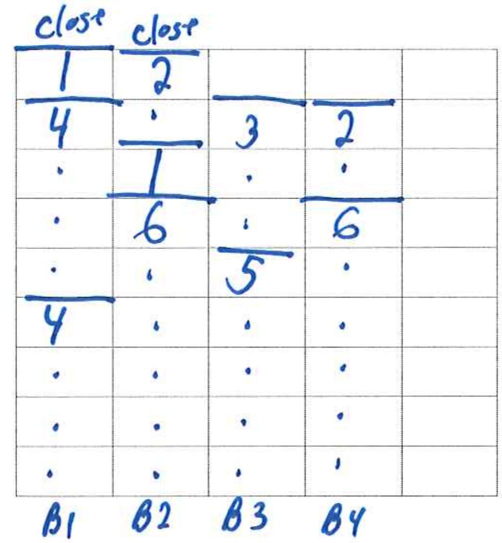
4, 6, 1, 2, 4, 5, 1, 3, 6, 2 *optimal*



Best-fit Algorithm (BF): Put items into an already open bin that has the least space for it. If no open bin has space, open a new bin.

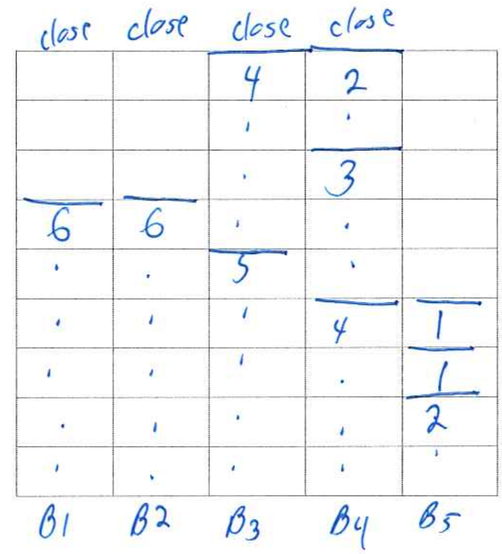
4, 6, 1, 2, 4, 5, 1, 3, 6, 2

optimal



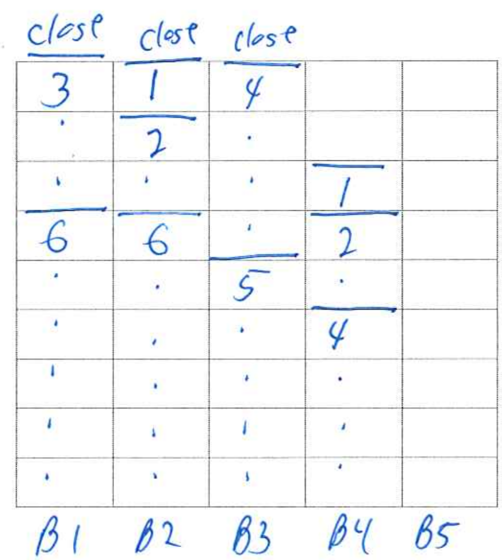
Next-fit Decreasing Algorithm (NFD): Arrange the items from largest to smallest. Then put items into the open bin until the next item will not fit. Close the bin and open a new bin for the next item.

6, 6, 5, 4, 4, 3, 2, 2, 1, 1



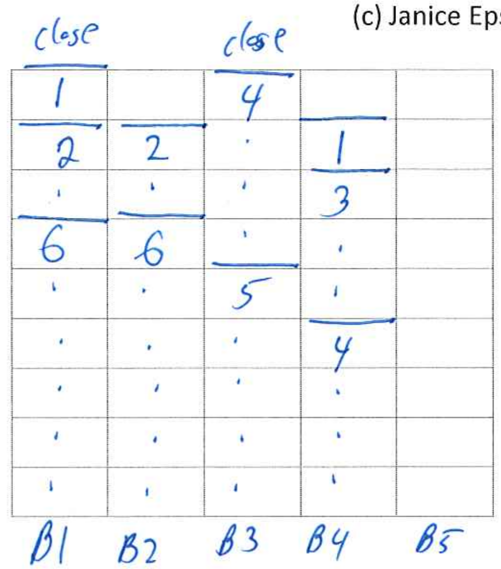
First-fit Decreasing Algorithm (FFD): Arrange the items from largest to smallest. Then put items into the first already open bin that has space for it. If no open bin has space, open a new bin.

6, 6, 5, 4, 4, 3, 2, 2, 1, 1



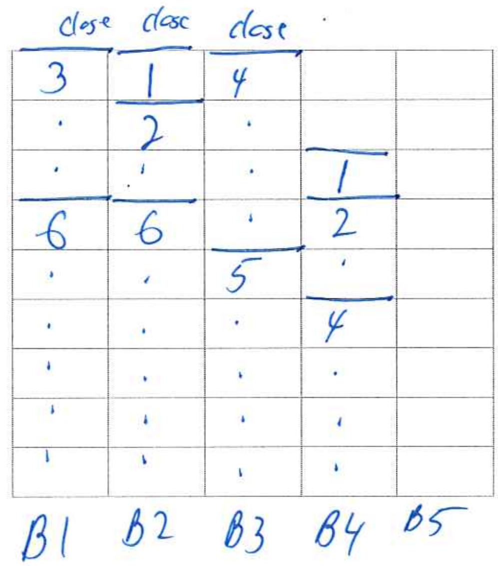
Worst-fit Decreasing Algorithm (WFD): Arrange the items from largest to smallest. Then put items into an already open bin that has the most space for it. If no open bin has space, open a new bin.

6, 6, 5, 4, 4, 3, 2, 2, 1, 1



Best-fit Decreasing Algorithm (BFD): Arrange the items from largest to smallest. Then put items into an already open bin that has the least space for it. If no open bin has space, open a new bin.

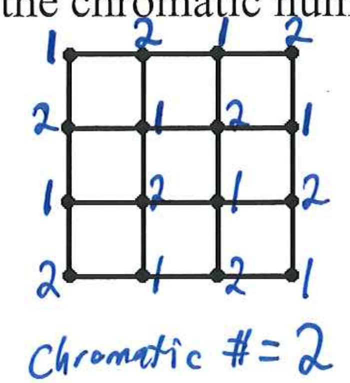
6, 6, 5, 4, 4, 3, 2, 2, 1, 1

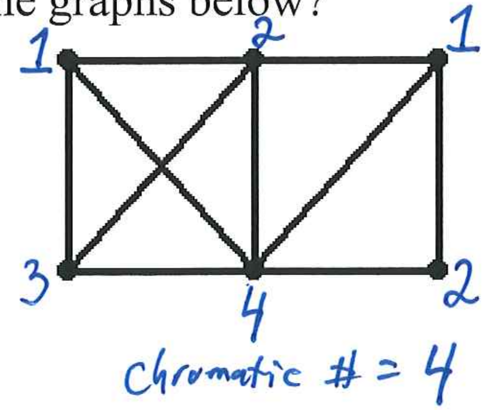


The ***chromatic number*** of a graph is the minimum number of colors needed to label the vertices of the graph so that no two vertices joined by an edge have the same color.

<u>EXAMPLE</u> vertex coloring

What are the chromatic numbers for the graphs below?



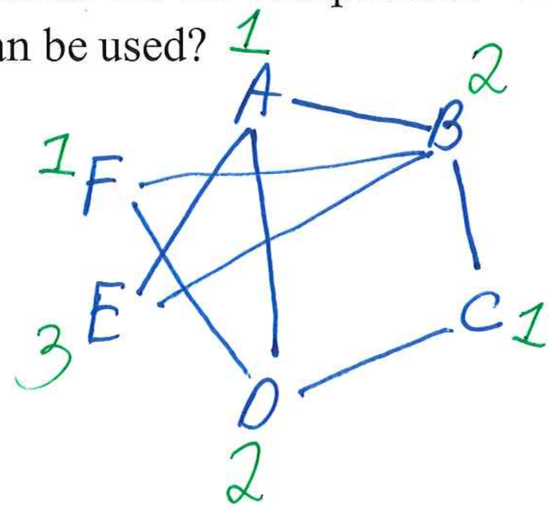


The ***vertex coloring*** problem for a graph requires assigning each vertex of the graph a color (or label) such that two vertices joined by an edge are assigned different colors.

EXAMPLE

The table below shows chemical compounds that cannot be mixed without causing dangerous reactions. Find graph would be used to facilitate scheduling of disposal containers for the compounds. What is the fewest number of containers that can be used?

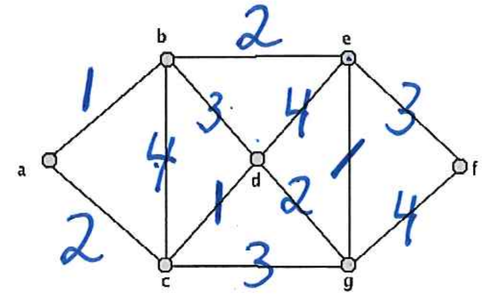
	A	B	C	D	E	F
A		X		X	X	
B	X		X		X	X
C		X		X		
D	X		X			X
E	X	X				
F		X		X		



3 Containers

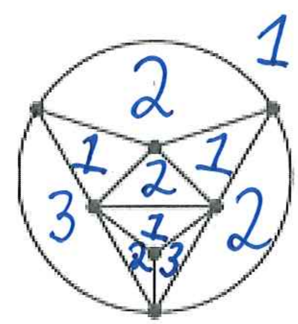
The ***edge-coloring number*** of a graph is the minimum number of colors needed to color the edges of the graph so that edges that share a common vertex get different colors.

There were 7 teams and the remaining games to be played can be represented in the graph below. What is the fewest number of game days needed? 4 days



Face-Coloring Number

Find the fewest number of colors needed to color the map such that no two edges have the same color. 3

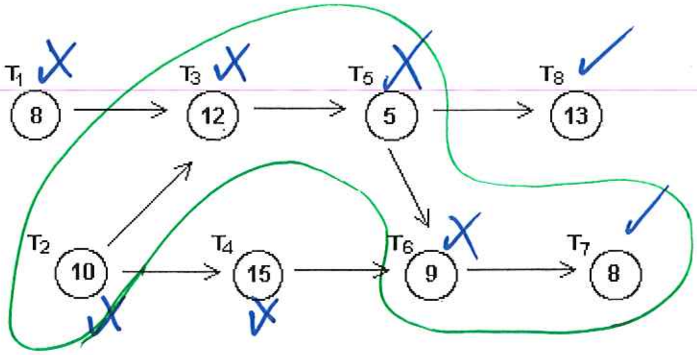


SAMPLE EXAM QUESTIONS FROM CHAPTER 3

1. Given the order-requirement digraph below (with time given in minutes) and the priority list

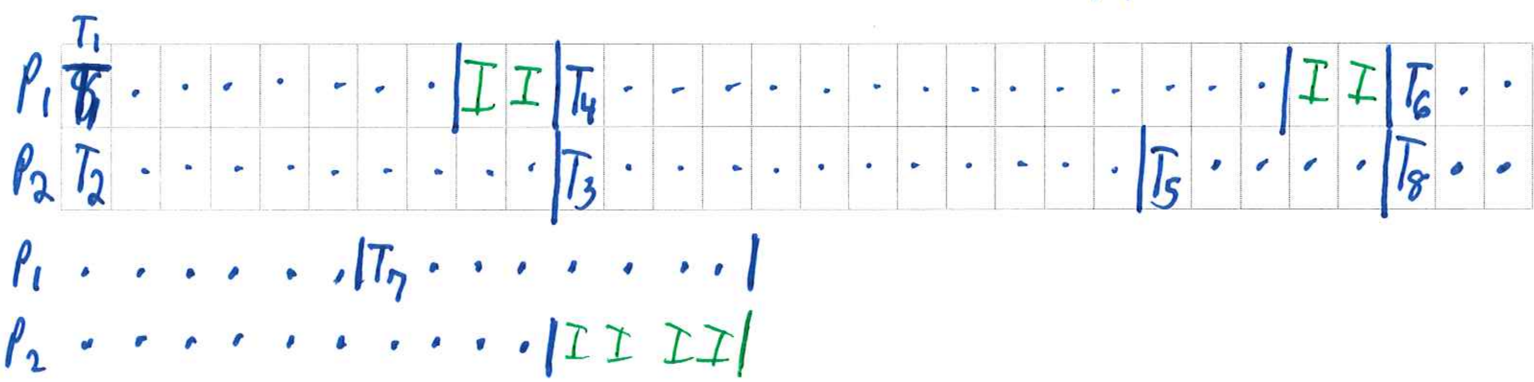
$$T_4, T_5, T_6, T_7, T_8, T_1, T_2, T_3,$$

apply the list-processing algorithm to construct a schedule using two processors. How much time does the resulting schedule require? Is the schedule optimal?

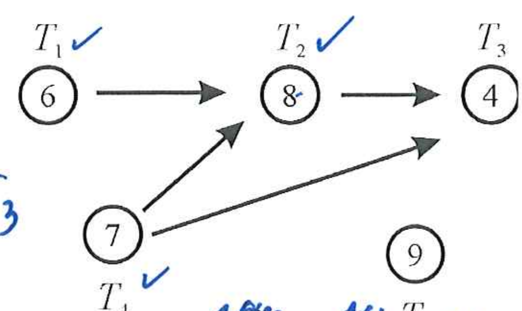


critical Path $T_2 T_3 T_5 T_6 T_7 = 44$ min

44 min min



2. Using the given graph, determine the critical path priority list.



(A) T_5, T_4, T_1, T_2, T_3

(B) T_5, T_4, T_2, T_1, T_3

(C) T_4, T_1, T_2, T_3, T_5

(D) T_4, T_1, T_2, T_5, T_3

(E) None of these/need more information

$T_4 T_1 T_2 T_5 T_3$

		After T_4	Aft T_1	Aft T_2
$T_1 T_2 T_3$	= 18	18	12	4
$T_4 T_2 T_3$	= 19	12	12	4
$T_4 T_3$	= 11	4	4	4
T_5	= 9	9	9	9

3. What is the minimum time required to perform six independent tasks with a total task time of 48 minutes on three machines?

(A) 2 minutes
(B) 8 minutes
(C) 16 minutes
(D) 18 minutes
(E) None of these/need more information

$\frac{48 \text{ min}}{3 \text{ processors}} = 16$ min/processor

4. Use the decreasing-time-list algorithm to schedule these independent tasks on two machines:

4 minutes, 5 minutes, 8 minutes, 3 minutes, 3 minutes, 7 minutes

8 7 5 4 3 3

How much time does the resulting schedule require?

(A) 15 minutes
(B) 16 minutes
(C) 17 minutes
(D) 18 minutes
(E) None of these/need more information

P_1 8+4 (12) +3 (15)
P_2 7+5 (12) +3 (15)

5. Use list processing algorithm to schedule these independent tasks on two machines:

9 minutes, 8 minutes, 7 minutes, 9 minutes, 2 minutes, 5 minutes.

How much time does the resulting schedule require?

(A) 19 minutes
(B) 20 minutes
(C) 21 minutes
(D) 22 minutes
(E) None of these/need more information

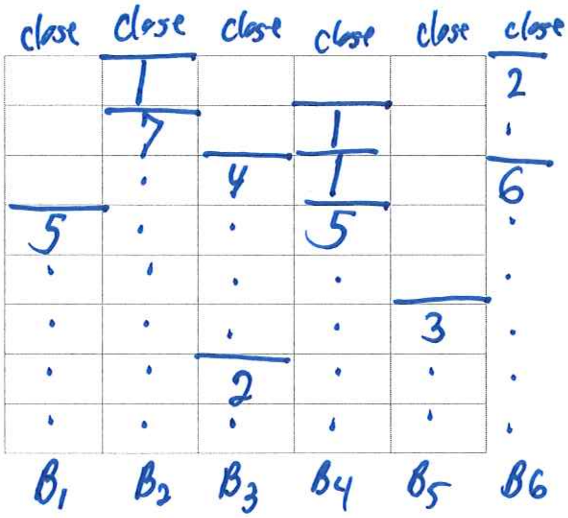
P_1 9+9 (18)
P_2 8+7 (15) +2 (17) +5 (22)

6. Find the packing that results from the use of given bin-packing algorithm to pack the following weights into bins that can hold no more than 8 lbs.

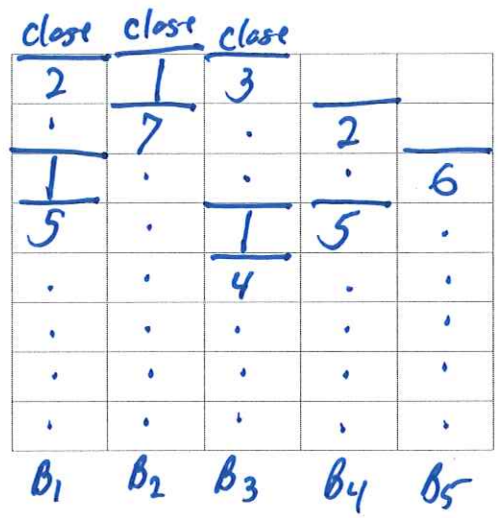
5 lbs, 7 lbs, 1 lb, 2 lbs, 4 lbs, 5 lbs, 1 lb, 1 lb, 3 lbs, 6 lbs, 2 lbs.

Are any of these packings optimal? $\frac{37\text{ lb}}{8\text{ lb/bin}} = 4.625$ bins optimal $= 5$ bins

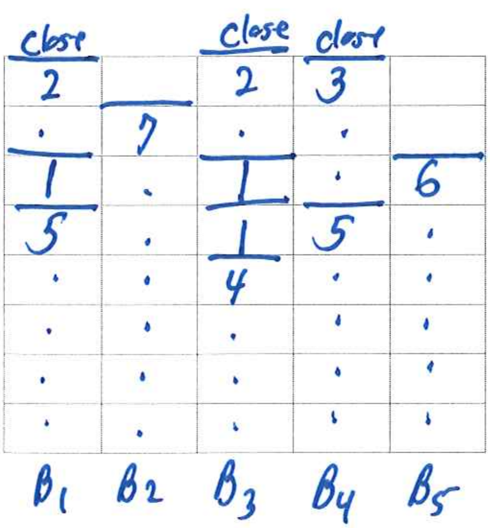
(a) Next fit Not optimal



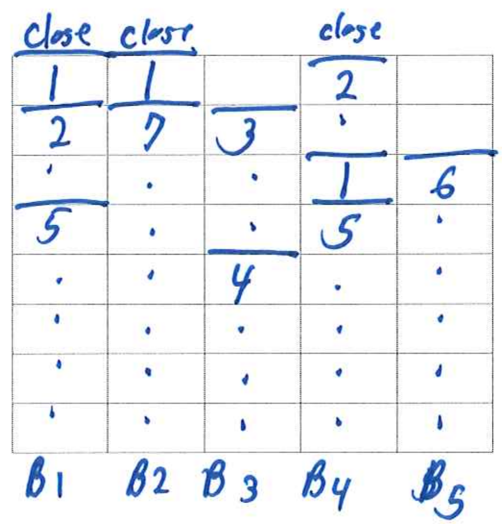
(b) First fit optimal



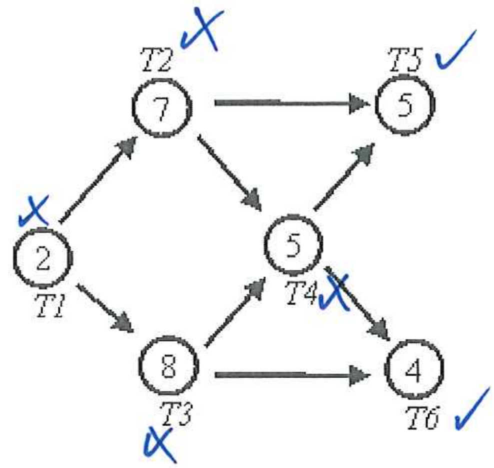
(c) Worst fit optimal



d) Best fit optimal

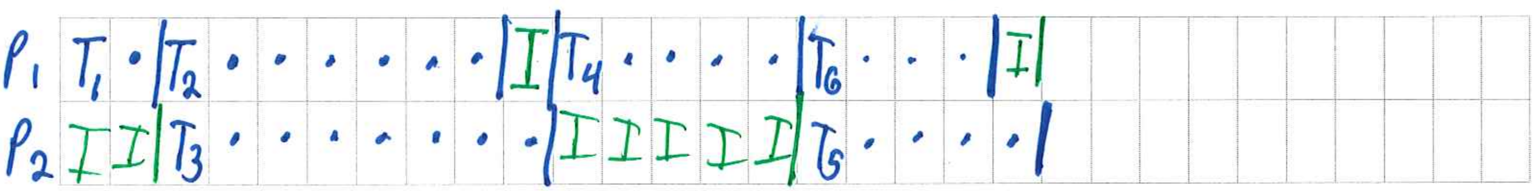


7. Use the digraph below and the priority list T_6, T_1, T_2, T_3, T_4, T_5 to schedule these tasks on two processors. Is the schedule optimal?



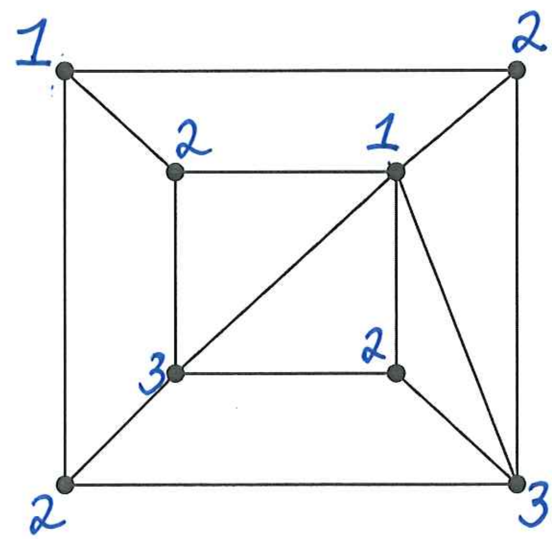
optimal
critical path
$T_1 T_3 T_4 T_5 = 20$ units

20 min



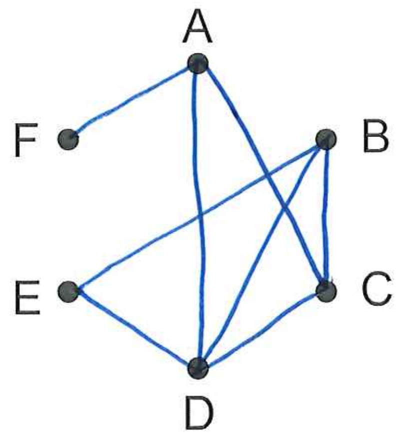
8. What is the chromatic number for the graph on the right? (vertex coloring)

(A) 2
(B) 3
(C) 4
(D) 5
(E) More than 5

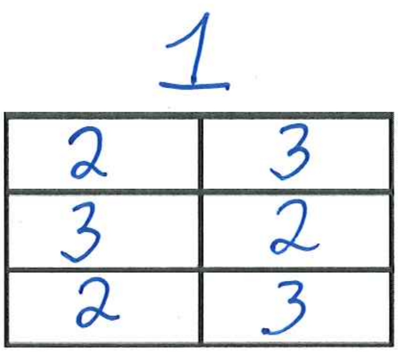


9. The chart below shows conflict. Represent this information in a graph

	A	B	C	D	E	F
A			X	X		X
B			X	X	X	
C	X	X		X		
D	X	X	X		X	
E		X		X		
F	X					



10. What is fewest number of colors needed to color the map below?



(A) 2 (B) 3 (C) 4 (D) 5
(E) more than 5