Transactions Cost and Interest Rate Rules

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Abstract

This paper evaluates quantitatively the effect of real money balances in a New Keynesian framework. Money in our model facilitates transactions and is introduced through a transactions cost technology. This technology acts like a distortionary consumption tax which varies endogenously with the nominal interest rate. In this setup the resultant Phillips curve becomes a function of the nominal interest rate. Our analysis has important policy implications. First, we find, unlike Woodford (2003), accounting for real-balance effects does not result in the policy maker’s loss function having an interest rate smoothing term. Second, we show that in the case of a temporary shock to productivity the optimal policy response under discretion is to allow for a trade-off between inflation and the output gap. This trade-off arises endogenously in our model. The quantitative effects on the macroeconomic variables are found to be significant.

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1 Introduction

The role of money in business cycles has long been studied by macroeconomists. This paper evaluates quantitatively the effects of real money balances in a New Keynesian framework. Recently, the forward looking models with nominal price rigidity assumption have increasingly been used to carry out monetary policy analysis. Rotemberg and Woodford (1997), Clarida, Gali, and Gertler (1999), Woodford (2000), and McCallum and Nelson (1999) among others have popularized this simple model for use in monetary policy analysis. In most of these models, monetary policy affects aggregate demand through the effects of real interest rates on the desired timing of private expenditure. A common feature in the models is that they assign minimal role to the changes in the stock of money. Rotemberg and Woodford (1997) make no reference to money in their model. Others simply include money to derive a money demand equation that determines the amount of money that needs to be supplied given levels of output and interest rate. Changes in money play a limited part in determining the dynamics of real variables. Ireland (2000, 2002) constructs a small structural model with money in the utility framework and performs maximum likelihood estimations to conclude that real balance has negligible effects on output and inflation dynamics.

In this paper, we introduce money as an asset which facilitates transactions. Specifically, higher average real money balances, for a given volume of transaction, lowers the transactions costs. Our setup allows us to distinguish between the distortionary and wealth effects associated with holding real money balances. Given that there is evidence to suggest that the wealth effect is negligible, we focus on the distortionary effects. We show that the transactions cost function acts like a distortionary consumption tax that fluctuates endogenously with the nominal interest rate. This is because these costs in our model vary directly with the velocity of money, which in turn is a function only of the nominal interest rate. An increase in the nominal interest rate, for example, results in the households economizing on their cash balances, thereby increasing the velocity of money and consequently the transaction costs. The increase in transaction
costs increases the effective tax rate on consumption and alters the labor-leisure choice. The presence of these costs therefore has direct implications for the real marginal costs of the firm. We show that the resultant aggregate supply curve or the New Keynesian Phillips curve now becomes a function of the nominal interest rate. This result is akin to the one derived by Ravenna and Walsh (2003). In their model, however, firms are required to borrow money from financial intermediaries at a prevailing nominal interest rate to pay their wage bill. Due to this assumption, the marginal cost of the firm then becomes a function of the nominal interest rate and the resulting Phillips curve also becomes a function of the nominal interest rate.

Our analysis has some interesting implications for the design of optimal monetary policy. Following Woodford (2003), we derive the appropriate welfare based loss function and show that it is possible to express the loss function in terms of a measure of the output and the inflation gap. The loss function derived here differs from the one in Woodford (2003), when there are transaction frictions. Woodford (2003) shows that in the presence of transaction frictions the policy maker’s loss function has an interest rate smoothing term. In our analysis which emphasizes the distortionary effects of real money balances, there is no interest rate smoothing term. We then proceed to evaluate optimal policy under discretion when the economy is subject to productivity shocks. We find that it is optimal for the policy maker to allow for a trade-off between inflation and output gap in response to such a shock. This is unlike the standard New Keynesian result where it is optimal to completely neutralize the shock and keep output and inflation gap at their targeted levels. It is also worth noting that our model generates this trade-off endogenously. This is again in contrast to the standard New Keynesian models which require an exogenous cost-push shock to generate any meaningful trade-off. An examination of the impulse responses shows that the response of inflation and the output gap to this shock is quantitatively significant.

The rest of the paper proceeds as follows. Section 2 develops the basic model with cash. Section 3 and 4 examines the economy under flexible and sticky prices respectively. Section
5 formulates and analyzes the optimal policy rules and Section 6 summarizes the results and concludes.

2 Model

The model consists of households that supply labor, purchase goods for consumption, hold money and bonds. Firms, hire labor, produce and sell differentiated products in monopolistically competitive goods markets. Households and firms behave optimally; households maximize the present value of expected utility, and firms maximize profits.

2.1 Households

The preferences of the representative household are defined over a composite consumption good $C_t$, and leisure, $1 - N_t$, where $N_t$ is the time devoted to market employment. Households maximize the expected present discounted value of utility:

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right]$$

The parameter $\theta$ governs the price elasticity for the individual goods. The households decision problem is a two-stage problem. First, regardless of the level of $C_t$, it will always be optimal for the household to purchase the combination of individual goods that minimize the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of $C_t$, the household chooses $C_t$ and $N_t$ optimally.
Dealing with the first problem of minimizing the cost of buying $C_t$, the household’s decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$  \hspace{1cm} (3)$$

subject to

$$\left[ \int_0^1 c_{jt}^{\theta-1} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t$$  \hspace{1cm} (4)$$

Solving the above problem the households demand for good $j$ can be written as

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$  \hspace{1cm} (5)$$

where $P_t$ is the aggregated price index for consumption and is given by

$$P_t = \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$  \hspace{1cm} (6)$$

Given the definition of the aggregate price index in (6), we can now define the budget constraint of the households. First, we specify the way money is introduced. In this model, agents hold money to reduce transactions cost. An increase in the volume of goods exchanged would lead to a rise in transactions cost, while higher average real money balances would, for a given volume of transactions, lower costs. The transactions cost $S_t$ is defined as

$$S_t = C_t k_0 \left( \frac{M_t}{P_t C_t} \right)^{1-k_1}$$  \hspace{1cm} k_0 > 0, \hspace{0.5cm} k_1 > 1$$  \hspace{1cm} (7)$$

It is a function of $C_t$ and $\frac{M_t}{P_t}$. The technology implies that the transactions cost is homogenous of degree one in consumption and real money balances. This would mean that the consumption elasticity of money demand is equal to unity, a fact which is empirically supported. We can now write the households budget constraint as

$$C_t + \frac{M_t}{P_t} + B_t - \frac{N_t}{P_t} + C_t k_0 \left( \frac{M_t}{P_t C_t} \right)^{1-k_1} = \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t}$$

$$+ (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t + TR_t$$  \hspace{1cm} (8)$$
where \((1 + i_{t-1})\) is the gross nominal interest rate paid on bonds, \(\Pi_t\) is the profits received from firms, and \(TR_t\) is lump-sum transfer from the government.

In the second stage of the household’s decision problem, consumption, labor supply, money holdings and bond holdings are chosen to maximize (1) subject to (8). Denote a new term \(q\) as \(\frac{M}{PC}\), the inverse of the consumption velocity of money, then we can write the first order conditions as

\[
\beta^t C_t^{-\sigma} = \lambda_t \left[ 1 + k_1 k_0 q_t^{1-k_1} \right] 
\]

(9)

\[
C_t^{-\sigma} = \beta (1 + i_t) E_t \left( \frac{P_t}{P_{t+1}} \right) \left[ 1 + k_1 k_0 q_t^{1-k_1} \right] C_{t+1}^{-\sigma} 
\]

(10)

\[
q_t^{-k_1} = \frac{1}{k_0 (k_1 - 1)} \left( \frac{i_t}{1 + i_t} \right) 
\]

(11)

\[
\chi \frac{N_t^\eta}{C_t^{-\sigma}} \left[ 1 + k_1 k_0 q_t^{1-k_1} \right] = \frac{W_t}{P_t} 
\]

(12)

It is clear from (9) that transactions cost introduces a wedge between the marginal utility of consumption and the marginal utility of wealth. An increase in \(q_t\) (decrease in the consumption velocity of money) will tend to decrease the marginal utility of consumption and hence increase consumption. (10) is the standard Euler equation. (11) implicitly defines the household’s money demand function. (11) implies that a rise in the interest rate will lead to an increase in the velocity of money. The log-log elasticity of money demand with respect to nominal interest rate is given by \(\frac{1}{k_1}\). Further, as the parameter \(k_1\) approaches two, the transactions cost function becomes linear in velocity and the demand for money adopts the Baumol-Tobin square-root form with respect to the opportunity cost of holding money, \(\frac{i}{1+i}\). (12) shows that the velocity of money will distort the consumption/leisure margin. Given a level of real wage, a higher \(q_t\) will
make people work and consume more. It is clear from (9) and (11) that the transactions cost acts like a distortionary consumption tax that fluctuates endogenously with the nominal interest rate. That is, (9) says that the implicit consumption tax rate is \( k_1 k_0 q_t^{1-k_1} \), where \( q_t \) is a function of only the nominal interest rates. From (11) we know that as nominal interest rates rise, the household reduces its real money balances so that velocity rises and \( q_t \) falls. (9) shows that this increases the implicit tax on consumption purchases. We find from (12) that this increase in the implicit consumption tax alters the marginal rate of substitution between consumption and leisure. This effect turns out to be crucial in deriving the Phillips curve relation.

Defining \( \hat{X}_t \) as the percentage deviation of variable \( X_t \) from its steady state value \( \bar{X} \), we now express the variables in (10), (11), and (12) in terms of their percentage deviations from steady state. These equations help us to attain some of the model’s implications:

\[
\sigma (\hat{C}_{t+1} - \hat{C}_t) = (\hat{R}_t - E_t \hat{\pi}_{t+1}) + \frac{V_0}{1 + V_0} (k_1 - 1) (E_t \hat{q}_{t+1} - \hat{q}_t)
\]

\[
\hat{W}_t - \hat{P}_t = \sigma \hat{C}_t + \eta \hat{N}_t - \frac{V_0 (k_1 - 1)}{1 + V_0} \hat{q}_t
\]

\[
\hat{q}_t = -\frac{\hat{R}_t}{k_1 (R - 1)},
\]

where

\[ R_t \equiv (1 + i_t), \]
\[ V_0 \equiv k_0 k_1 q^{1-k_1}. \]
2.2 Firms

Firms employ labor to produce output using a constant returns to scale technology. The production function of the firm is given by

$$c_{jt} = A_t N_{jt}$$

where $c_{jt}$ is the output produced by firm $j$, $N_{jt}$ is labor hired by firm $j$ and $A_t$ is the available technology in the economy.

Following the Calvo-Yun setup, firms adjust their price infrequently. The opportunity to adjust follows a Bernoulli distribution. Define $\omega$ as the probability of keeping prices constant and $(1 - \omega)$ as the probability of changing prices. Each period, the firms that adjust their price are randomly selected, and a fraction $(1 - \omega)$ of all firms adjust while the remaining $\omega$ fraction do not adjust. Before analyzing the firm’s pricing decision, consider its cost minimization problem. This problem can be written as

$$\min_{N_{jt}} \left( \frac{W_t}{P_t} \right) N_{jt} + MC_t(c_{jt} - A_t N_{jt})$$

where $MC_t$ is the real marginal cost of the firm. The cost minimization problem implies

$$MC_t = \left( \frac{W_t/P_t}{A_t} \right)$$

Firms that adjust their price at time $t$ do so to maximize the expected discounted value of current and future profits. Profits at some point of time $t+i$ in the future are affected by the current choice of price at time $t$ given that firms cannot adjust their price in the intervening period. The firm $j$’s pricing decision then becomes

$$E_t \sum_{i=0}^{\infty} (\omega \beta)^i \lambda_{t,t+i} \left[ \frac{p_{jt}}{P_{t+i}} c_{jt} - MC_{t+i} c_{jt+i} \right]$$

subject to

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$
where

$$\lambda_{t,t+i} = \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} \left[ \frac{1 + e^{\pi t} k_1 k_0 q_t^{1-k_1}}{1 + e^{\pi t+1} k_1 k_0 q_{t+1}^{1-k_1}} \right]$$

(21)

When prices are flexible, real marginal cost is equal to the (constant) markup \(\frac{\theta}{\theta - 1} = \frac{1}{\mu}\), and

$$\left( \frac{W_t/P_t}{A_t} \right) = \frac{\theta}{\theta - 1} = \frac{1}{\mu}$$

(22)

As is well known (see Gali and Gertler (1999), Sbordone (2002), Walsh (2003)), this model leads to an inflation-adjustment equation of the form

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{m}c_t$$

(23)

where \(\hat{\pi}_t\) is the deviation of the inflation around the steady-state of \(\pi\) and \(\hat{m}c_t\) is the percentage deviation of the real marginal cost around its steady state value of \(\frac{\theta}{\theta - 1}\). The parameter \(\kappa\) is given by

$$\kappa = \frac{(1 - \omega)(1 - \omega \beta)}{\omega}$$

(24)

### 2.3 Government and Resource Constraints

The government plays no active role. It gives back to the households as lump sum transfers the proceeds from money creation and transaction costs.

$$TR_t = \frac{M_t - M_{t-1}}{P_t} + C_t k_0 \left( \frac{M}{PC} \right)^{1-k_1}$$

(25)

The fact that \(C_t k_0 \left( \frac{M}{PC} \right)^{1-k_1}\) appears in the government’s flow constraint reflects the assumption that it is a private cost for the consumer but not a social cost. Formally, the government is assumed to provide shopping services to the consumer and the proceeds from such an activity are then transferred back to the consumer in a lump sum way (See Vegh (2002) for a similar interpretation). This assumption is made to eliminate wealth effects. Given that there is broad consensus that these wealth effects are small, we feel that this is a reasonable assumption. Since
our transactions cost works like a distortionary tax, this assumption is reminiscent of the one frequently seen in public finance literature. This also differentiates our framework from other models (Ireland (2000), Woodford (2000)) which use a money in the utility framework and do not have this rebate. We show in Section 5 that this assumption modifies the policy maker’s loss function in a non-trivial way.

Now to close the model, let us state the market equilibrium conditions. since bonds are inside money, aggregate bond holdings in this economy must be zero:

$$B_t = 0. \quad (26)$$

Substituting (25), and (26) into the households budget constraint, we get the goods market equilibrium

$$Y_t = C_t. \quad (27)$$

### 3 Flexible-Price Equilibrium

When prices are flexible, all firms charge the same price and real marginal cost is constant, such that $MC_t = \frac{1}{\mu}$. $\mu$ is the constant markup charged by firms under flexible prices. From (18), this would imply that

$$\frac{W_t}{P_t} = \frac{A_t}{\mu} \quad (28)$$

We also know from (12) that real wage must be equal to the marginal rate of substitution between leisure and consumption. This condition implies that

$$\chi \frac{N_t^\eta}{C_t^{-\sigma}} \left[ 1 + k_1 k_0 q_t^{1-k_1} \right] = \frac{W_t}{P_t} = \frac{A_t}{\mu} \quad (29)$$

We now proceed to derive the flexible price level of output. Following Ravenna-Walsh we can define a new term $Y_t^*$ as the output level that one would obtain under flexible prices conditional on a given level of interest rate $R_t^*$ (in other words we assume $\hat{R}_t^* = 0$)
Substituting into (29), goods market clearing condition $Y_t^* = C_t$, the production function $Y_t^* = A_t N_t$, we can express the flexible price level output $Y_t^*$ in linearized form as

$$\tilde{Y}_t^* = \frac{1 + \eta}{\sigma + \eta} \tilde{A}_t$$

(30) indicates that the flexible price output level is affected only by shocks to productivity.

4 Sticky Price Equilibrium

As is well known, when prices are sticky, output can differ from the flexible-price equilibrium level. Since firms are not allowed to adjust prices every period, the firm must take into account expected future marginal cost as well as current marginal cost whenever it has an opportunity to adjust price. (23) implies that inflation depends on the real marginal costs faced by the firm. We also know from (18) that the firm’s real marginal cost is equal to the real wage it faces divided by the marginal product of labor. In case of flexible prices, this is simply equal to $\frac{1}{\eta}$. Meanwhile for the sticky price case, this is not true and the real marginal cost, and thus the markups are endogenous variables. Expressed in terms of percentage deviations this can be written as

$$\tilde{MC}_t = \tilde{W}_t - \tilde{P}_t - \tilde{A}_t$$

(31)

(14) tells us that the real wage is related to the marginal rate of substitution between consumption and leisure. Substituting for $\tilde{W}_t - \tilde{P}_t$ from (14), we can express (31) as:

$$\tilde{W}_t - \tilde{P}_t - \tilde{A}_t = (\sigma + \eta)\tilde{Y}_t - (1 + \eta)\tilde{A}_t - \frac{V_0(k_1 - 1)}{(1 + V_0)}\tilde{q}_t$$

(32)

We can use (30) and (15) to rewrite $\tilde{MC}_t$ as

$$\tilde{MC}_t = (\tilde{Y}_t - \tilde{Y}_t^*)(\sigma + \eta) + \frac{V_0(k_1 - 1)}{k_1(1 + V_0)(R - 1)}\tilde{R}_t$$

(33)
The Phillips curve given by (23), can then be rewritten as

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \kappa (\sigma + \eta) (\tilde{Y}_t - \tilde{Y}^*_t) + \kappa \left( \frac{V_0 (k_1 - 1)}{k_1 (1 + V_0) (R - 1)} \right) \tilde{R}_t$$  \hspace{1cm} (34)

The Phillips curve we just obtained is different from the standard New Keynesian Phillips curve found in the literature. (34) states that the Phillips curve is not just a function of the output gap, but also a function of the nominal interest rates. The nominal interest rate behaves very much like the exogenously added cost push shock in the literature used to derive a trade off between inflation and output. In this framework, however, this term is endogenous. This result is similar to the one derived by Ravenna and Walsh (2003). However, they use a cash-in-advance framework with the assumption that firms should borrow money from financial intermediaries at a prevailing nominal interest rate to pay their wage bill. This assumption links real marginal cost to nominal rate by construction.

\section{5 Optimal Monetary Policy}

In this section, we examine the optimal policy problem. We first show that the presence of the shock to the transactions cost results in the policy maker’s objective function being different from the standard objective function in the New Keynesian framework. We also derive optimal policies and show the existence of trade-off between output and inflation even in the absence of the traditional cost push shock. Then, we simulate the alternative regimes of optimal monetary policy to examine the effects of productivity shocks.

\subsection{5.1 Loss Function}

Following Woodford (2003), Ravenna and Walsh (2003), we obtain our policy objective function by taking a second order approximation of the utility function. It can be shown that the present discounted value of utility of the representative household can be approximated by
\[
\sum_{t=0}^{\infty} \beta^t U_t \approx U - \Theta \sum_{t=0}^{\infty} \beta^t L_t
\]  

(35)

where

\[
\Theta = \frac{1}{2} U_c \bar{\omega} \left[ \frac{\omega}{(1 - \omega)(1 - \omega \beta)} \right] \theta
\]  

(36)

\[
L_t = \alpha (\hat{Y}_t - \hat{Y}_t^*)^2 + \hat{\pi}_t^2
\]  

(37)

The parameter \( \alpha \) is given by

\[
\alpha = \left[ \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \right] \left( \frac{\sigma + \eta}{\theta} \right)
\]  

(38)

\( z^* \) is the gap between flexible price steady state and the efficient steady state level of output where there are no distortions. Following much of the literature we will assume that there are fiscal subsidies available so that these efficiency distortions are eliminated and \( z^* = 0 \).

We can now define \( x_t = \hat{Y}_t - \hat{Y}_t^* \) as the output gap- the gap between output and the flexible price output under a constant nominal interest policy. The policy maker’s problem can then be written as

\[
\max_{x_t, \hat{\pi}_t, \hat{R}_t} \frac{1}{2} E_r \sum_{t=0}^{\infty} \beta^t \left( \hat{\pi}_t^2 + \alpha x_t^2 \right)
\]  

(39)

The loss function derived here differs from the loss function in Woodford (2003) when there are transaction frictions. In his analysis, Woodford has an interest rate smoothing term in the loss function. In our analysis since we assume that the wealth effect due to transactions costs are eliminated, as they are rebated to the consumer, we do not have the interest rate smoothing term\(^3\).
We can now consider (13). Imposing the goods market clearing condition given by (27), the money market clearing condition given by (15), we get

\[ x_t = E_t x_{t+1} - \frac{1}{\sigma}(\hat{R}_t - E_t \hat{\pi}_{t+1}) + \frac{V_0(k_1 - 1)}{k_1 \sigma (1 + V_0)(R - 1)}(E_t \hat{R}_{t+1} - \hat{R}_t) + g_t \quad (40) \]

Where \( g_t \) is given by

\[ g_t = \frac{1 + \eta}{\sigma + \eta} \left( \hat{A}_{t+1} - \hat{A}_t \right) \]

We can similarly rewrite the Phillips curve given by (34) as

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa(\sigma + \eta)x_t + \frac{\kappa V_0(k_1 - 1)}{k_1 (1 + V_0)(R - 1)} \hat{R}_t \quad (41) \]

The policy maker’s problem is thus reduced to maximizing (39) subject to (40), and (41).

### 5.2 Optimal Discretionary Policy

We consider the case with discretionary policy regime when the economy is subject to productivity shocks. The Central Bank operating under discretion chooses the policy parameter, which in our case is the interest rate, by re-optimizing every period. The problem of the central banker is to choose the paths for \( \hat{R}_t, x_t \) and \( \hat{\pi}_t \). The policy maker thus maximizes (39) subject to (40) and (41). Let \( \lambda_1 \) and \( \lambda_2 \) be the Lagrangian multipliers associated with (40) and (41) respectively. Under optimal discretion the first order conditions are

\[ -\alpha x_t + \lambda_1 - \kappa(\sigma + \eta)\lambda_2 = 0 \quad (42) \]

\[ -\hat{\pi}_t + \lambda_2 = 0 \quad (43) \]
Equations (42) – (44), are the first order conditions with respect to $x_t$, $\hat{\pi}_t$, $\hat{R}_t$ respectively. Eliminating $\lambda_1$ and $\lambda_2$ from the above equations we get

$$x_t = -\frac{\kappa \hat{\pi}_t}{\alpha} \left[ (\sigma + \eta) - \frac{\sigma V_1}{1 + V_1} \right]$$

$$V_1 = \frac{V_0(k_1 - 1)}{k_1(1 + V_0)(R - 1)}$$

The above optimality condition implies that the central bank pursues a “lean against the wind” policy. This is very much in the spirit of the results with an exogenous cost push shock in Clarida, Gali, and Gertler (1999). In the standard New Keynesian literature, optimal policy is of the form $x_t = -\frac{\kappa \hat{\pi}_t (\sigma + \eta)}{\alpha}$. In our case the rule is given by (45). The optimal policy derived here will clearly be less aggressive in trading off output gap movements for inflation stability. In Clarida, Gali, and Gertler (1999), for example, any shocks to $g_t$ are always completely neutralized by adjusting the interest rate. In other words, interest rates are always set to ensure that output equals to its flexible-price level. This results in both inflation and output gap being stabilized. In our framework setting $x_t = 0$ is not consistent with zero inflation. This is because a shock to $g_t$ will require a movement in $R_t$ in the opposite direction. Given that the Phillips curve derived in our paper also has an interest rate term, a movement in $R_t$ will affect the Phillips curve directly.

**Result:** *In the case of a temporary shock to $A_t$, there exists a short run trade-off between inflation and output variability.*

This result is in direct contrast to the case obtained in the standard New-Keynesian literature which needs a cost-push shock to generate a meaningful trade-off. To illustrate our result, consider the following example. Let $A_t$ follow an AR(1) process such that $A_{t+1} = \rho_a A_t + \varepsilon_a$, where $\varepsilon_a = N(0, \sigma^2_\varepsilon) \ 0 < \rho_a < 1$. A system of equations (40), (41), and (45) can be solved to yield the equilibrium path of inflation which is given by
\[ \Gamma_1 \hat{\pi}_t = \Gamma_2 E_t \hat{\pi}_{t+1} + \Gamma_3 E_t \hat{\pi}_{t+2} + g_t \]  

(46)

Therefore the equilibrium path of inflation is a stochastic second order difference equation, where \( \Gamma_i \), where \( i = 1, 2, 3 \) are constants. A temporary rise in \( A_t \) will result in \( g_t \) falling (since \( \rho_a < 1 \)). From (46) we find that for reasonable parameter values this will result in a fall in \( \hat{\pi}_t \). It is clear from the optimal policy rule given by (45) that this will result in a rise in \( x_t \). To summarize, a temporary productivity shock results in a fall in inflation and a rise in the output gap. The model is thus able to generate the inflation-output gap trade off without explicitly resorting to an exogenous cost-push shock.

5.3 Numerical Results

All the results that follow are computed by calibrating the model and numerically solving it by using the approach described by Söderlind (1999). The model is interpreted as quarterly. The parameter values that we use are quite standard in the literature. We borrow these values from Galí and Gertler (1999), McCallum and Nelson (2000), Jensen (2002), and Ravenna and Walsh (2003).

In order to obtain a value for the interest elasticity of money demand we estimate (11), the equation for money demand using the Newey-West OLS estimator. For the data, we utilize the quarterly data set ranging between 1959 and 2000, provided by the St. Louis Fed. We try alternative definitions of \( q = M/PC \) the inverse of money velocity while estimating (11). For \( M \), we experiment with \( M1 \) as well as money base. For the aggregate consumption, we use GDP. For our benchmark case, we select \( q \) with \( M1 \). The estimated value of the interest rate elasticity of money demand is given by \( \frac{1}{k_1} = .42 \). Using the obtained value of the interest rate elasticity we calibrate \( k_0 \) to match a steady state value of \( M1 \) velocity given by 5.9913. This gives a value of \( k_0 = 1.1556 \times 10^{-4} \). The steady state level of transactions cost to GDP \( \frac{S}{Y} = .0013 \) and the value of \( V_0 = .003 \). In the case of money base (\( MB \)) we find \( \frac{1}{k_1} = .29 \), \( k_0 = 3 \times 10^{-7} \) and
velocity given by 19.88. The steady state level of transactions cost to GDP in this case is given by \( \frac{S}{V} = 2.5 \times 10^{-4} \) and the value of \( V_0 = 0.0012 \). Two important terms in our framework are the coefficients \( \frac{V_0(k_1-1)}{k_1\sigma(1+V_0)(\kappa-1)} \) in (40), and \( \frac{\kappa V_0(k_1-1)}{k_1(1+V_0)(\kappa-1)} \) in (41). These two terms would be zero in a cashless economy. Our calibrated values are (0.115, 0.015) for \( M_1 \) and (0.054, 0.007) for \( MB \) respectively. The rest of the parameters are summarized in the Table 1. We set \( \sigma \) and \( \eta \) at 1.5 and 1 respectively. \( \beta \) is set equal to 0.99 appropriate for interpreting the time interval as one quarter. The value of \( \omega = 0.75 \) is consistent with empirical findings of Gali and Gertler (1999). \( \theta \) is set equal to 11 and implies a steady state markup of 1.1. \( \kappa \) and \( \alpha \) are computed from (24) and (38).

Figure 1 displays impulse responses with \( M_1 \), \( MB \), and a limiting case in which there is zero transactions cost, i.e. a cashless economy. This cashless case corresponds to a standard New Keynesian model such as Clarida, Galí, and Gertler (1999). As discussed in the previous section, inflation-output gap trade-off arises with productivity shocks due to changes in real money balances. The results clearly indicate that a unit shock to the productivity parameter has a quantitatively significant impact on the inflation and the output gap variables. The effect is diminished when we switch from \( M_1 \) to \( MB \) to measure real money balances.

6 Conclusion

Standard New Keynesian models, which have become a popular tool for analyzing monetary policy assign minimal role to the changes in the stock of money. In this paper we attempt to quantify the effects of real money balances on macroeconomic variables such as inflation, output and interest rates.

In our framework money facilitates transactions and is introduced through a transactions cost technology. Transactions cost in our model acts like a distortionary consumption tax which varies endogenously with the nominal interest rate. The presence of this tax alters the labor-leisure trade-off and affects the marginal cost of the firm. The resultant IS and Phillips curves
therefore become functions of the nominal interest rate. Our analysis has some interesting policy implications. We find that accounting for transaction frictions does not result in the policy maker’s loss function having an interest rate smoothing term. Our model therefore suggests that there is a weak case for having an interest rate smoothing term in this class of models. We also show that in the case of a temporary shock to productivity the optimal policy response under discretion is to allow for a trade-off between inflation and the output gap. Here, our result differs from the standard New Keynesian prescription of completely neutralizing this shock by suitably adjusting interest rates. The trade-off in our model arises without the exogenous cost-push shock standard models need to generate similar effects. The impulse response suggests that the effects of productivity shocks in the presence of real money balances is quantitatively significant.

End Notes:

• 1. See Reinhart and Vegh (1995)
• 2. Note that $\hat{R}_t = 0$ corresponds to an interest rate peg in the flexible-price equilibrium, not a zero nominal interest rate.
• 3. We thank the referee for bringing this to our attention.
• 4. See Ravenna and Walsh (2003) for similar result.

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Table 1: Baseline Parameters
References


Figure 1: Impact of a unit productivity shock under optimal discretionary policy in a calibrated New Keynesian model in the cash and cashless case. First row refers to inflation (infla), the second, the output gap (ygap), and the last, interest rate (R). Column 1 represents the model with M1 (M1), Column 2, Monetary Base (MB), and the third, the limiting case, i.e. a cashless case. GDP is used to compute M1 and MB velocity. Parameters used are presented in Section 5.3 and the Table 1.