RANKIN-SELBERG L–FUNCTIONS AND THE REDUCTION OF CM ELLIPTIC CURVES

SHENG-CHI LIU, RIAD MASRI, AND MATTHEW P. YOUNG

Abstract. Let f be an arithmetically normalized Hecke cusp form of weight 2 and prime level q. Let $K = \mathbb{Q}(\sqrt{-D})$ be an imaginary quadratic field of discriminant $-D$ such that q is inert in K, and let Θ_χ be the weight 1 theta series of level D corresponding to an ideal class group character χ of K. We establish a uniform asymptotic formula for the first moment of the Rankin-Selberg L–function $L(f \times \Theta_\chi, s)$ at $s = 1/2$ as χ varies over the ideal class group characters. We use this result to study various arithmetic problems, including the nonvanishing of these central values and the image of the reduction map on CM elliptic curves. In particular, if $\mathfrak{q}$ is any prime above q in the Hilbert class field of K, we prove that the reduction map modulo $\mathfrak{q}$ from the set of elliptic curves over $\overline{\mathbb{Q}}$ with complex multiplication by the ring of integers $\mathcal{O}_K$ to the set of supersingular elliptic curves over $\mathbb{F}_{q^2}$ is surjective for all $D \gg_\varepsilon q^{18+\varepsilon}$. This may be viewed as an analog of Linnik's theorem on the least prime in an arithmetic progression.