Fall 2015 Math 151
Final Exam Practice

(covering Sections 1.1 - 6.4)

courtesy: Amy Austin

NOTE: These problems are to serve merely as practice for your final exam. The final exam for Math 151 is NOT a common exam. Each instructor makes up his or her own final exam. In addition to working this problem set, it is advised that you review the lecture notes and work the first three exams as well.

1. Given the vectors $\mathbf{a} = \langle -1, -3 \rangle$ and $\mathbf{b} = \langle 1, 1 \rangle$, compute:

   a.) $\mathbf{a} - 4\mathbf{b}$

   b.) Find a unit vector in the direction of $\mathbf{a}$

   c.) Find the angle between $\mathbf{a}$ and $\mathbf{b}$

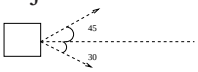
   d.) Find the vector and scalar projection of

   $\mathbf{b}$ onto $\mathbf{a}$

2. Find a vector equation and a set of parametric equations for the line passing thru the points $A(1, -2)$ and $B(3, 8)$.

3. Find a vector equation of the line passing throuth the point $(8, 5)$ and perpendicular to the line $4x + 9y = 2$.

4. An object on the ground is pulled with two forces as shown below. Find the magnitude of each force if the object is moved horizontally with a total force of 50 N.



5. Using the definition of the derivative, find $f'(x)$ for:

   a.) $f(x) = \sqrt{1 + x}$

   b.) $f(x) = \dfrac{1}{x - 3}$

6. Compute the following limits without the aid of a calculator.

   a.) $\lim\limits_{x \to -1^+} \dfrac{x - 3}{x^2 + x}$

   b.) $\lim\limits_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

   c.) $\lim\limits_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

   d.) $\lim\limits_{x \to 3} f(x)$ where

   $$f(x) = \begin{cases} \sqrt{x^2 + 16} & \text{if } x \leq 3 \\ x^3 - 10 & \text{if } x > 3 \end{cases}$$

   e.) $\lim\limits_{x \to 2^-} \dfrac{|\, 3x - 6 \,|}{x - 2}$

   f.) $\lim\limits_{x \to 0^-} \dfrac{|\, x \,|}{x^2 - 5x}$

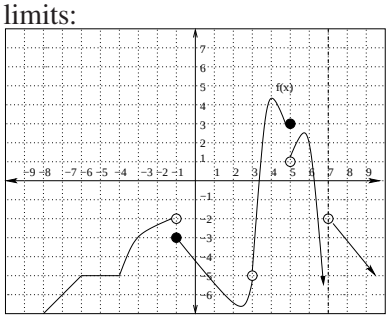
   g.) $\lim\limits_{x \to 0^+} \dfrac{|\, x \,|}{x^2 - 5x}$

   h.) $\lim\limits_{x \to 0} \dfrac{|\, x \,|}{x^2 - 5x}$

   i.) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2 + 3x + 1}}{1 - 2x}$

   j.) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 + 3x + 1}}{1 - 2x}$

7. Use the graph of $f(x)$ below to compute the following limits:



   a) $\lim\limits_{x \to 5^-} f(x)$

   b) $\lim\limits_{x \to 7^+} f(x)$

   c) $\lim\limits_{x \to 3} f(x)$

   d) $\lim\limits_{x \to -1} f(x)$

   e) Discuss the continuity and differentiability of the function whose graph is given above.

8. Find $f'(x)$ if $f(x) = |x^2 - 2x|$

9. Discuss the continuity of the following functions:

   a.) $f(x) = \begin{cases} x^2 - 2 & \text{if } x < 0 \\ \dfrac{x^2 - 4}{x - 2} & \text{if } 0 \leq x < 2 \\ x^3 - 4 & \text{if } x \geq 2 \end{cases}$

   b.) $f(x) = \begin{cases} x & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 5x^3 - 4 & \text{if } x > 1 \end{cases}$

   c.) Find the value of a and b that make $f(x)$ differentiable everywhere.

   $$f(x) = \begin{cases} 3x^2 & \text{if } x \leq 1 \\ ax + b & \text{if } x > 1 \end{cases}$$

10. Find the equations of both tangent lines to the parabola $f(x) = x^2 + x$ that pass thru the point $(2, -3)$.

11. Find all horizontal and vertical asymptotes for $f(x) = \dfrac{x}{x^2 - x}$.

12. If $f(x) = \sqrt[3]{5x - 3}$, what is $\lim_{h \to 0} \dfrac{f(-1+h) - f(-1)}{h}$?

13. Find $f''(0)$ for $f(x) = e^{2x}$

14. What is the domain of $f(x) = \ln(e^x - 4)$?

15. Compute the inverse of $f(x) = \dfrac{e^x}{1 + e^x}$

16. Find $\dfrac{dy}{dx}$ for:
    a.) $x^3y - 3x^3 + y^4 = x + y$
    b.) $\cos(x - y) = 2xy - 4x$

17. Compute the derivative:
    a.) $y = \dfrac{4x^3 - 8}{1 - x^2}$
    b.) $f(t) = t^3 \cos(1 - t^2)$
    c.) $G(x) = \tan^3(4x - 1)$

18. If $F(x) = f(x^2 + 2x)$ and $f'(15) = 7$, compute $F'(3)$.

19. Find the tangent line to the graph of $y = xe^x$ at $x = \ln 3$.

20. Given $x = t^3$, $y = 16t - t^2$, find the cartesian equation for the tangent line at $t = -1$. Also, find the equations of all vertical and horizontal tangent lines.

21. Find the linear and quadratic approximation for $f(x) = \ln x$ at $a = 2$. What is the usefullness of the linear and quadratic approximations?

22. A filter in the shape of a cone is $7cm$ high and has a radius of $3cm$ at the top. A solution is poured in at the rate of $1cm^3/min$. Find the rate at which the height of the residue is increasing when the height is $2cm$.

23. A kite is flying 100 feet above the ground at the end of a string. The girl flying the kite lets out the string at a rate of 1 foot per second. If the kite remains 100 feet above the ground, how many feet per second is the horizontal distance from t he girl increasing when the string is 125 feet long?

24. The length of a rectangle is decreasing at the rate of 1 foot per second, but the area remains constant. How fast is the width increasing when the length is 10 feet and the width is 5 feet?

25. Solve for x: $\log x + \log(x + 1) = \log 6$

26. Solve for x: $\ln x - \ln(x + 2) = 1$

27. Solve for t: $2^t = 4^{3t-1}$

28. If $\mathbf{r(t)} = \langle \ln t, te^t \rangle$, find parametric equations for the tangent line to the curve at the point $(0, e)$.

29. Find the derivative of $f(x) = \ln(\cos x)$

30. Find y' for $y = (1 + x + 3x^3)^{\sqrt{x}}$

31. A cup of coffee 130° is placed in a 65° room. If the temperature of the coffee is 115° after 2 minutes, when will the coffee be 90°?

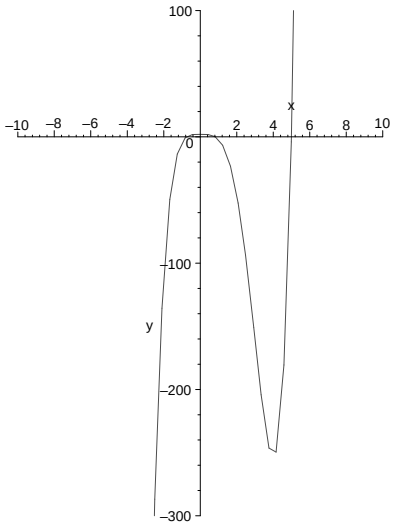
32. A bacteria culture starts with 1500 bacteria, and the population triples every 2 hours. How long will it take for the population to reach 2520 bacteria?

33. If $f(x) = x^4 - 4x^3 - 6$, Find the intervals where $f(x)$ is increasing/decreasing, locate all local extrema, intervals of concavity and points of inflection. Sketch the curve.

34. Where is $f(x) = x \ln x$ concave up?

35. Find the absolute extrema for $f(x) = x^3 - 3x^2 - 1$ over the interval $[-1, 1]$.

36. Below, the graph of f' is given. Using the graph of f', determine all critical values of f, where f is increasing and decreasing, local extrema of f, where f is concave up and concave down, and the x-coordinates of the inflection points of f. Assume f is continuous.



37. Compute the following limits using L'Hospital's rule:
    a.) $\lim_{x\to 0} \dfrac{x^3 - 3x^2}{x^2 + x - \sin x}$
    b.) $\lim_{x\to\infty} \left(1 + \dfrac{3}{x} + \dfrac{5}{x^2}\right)^x$
    c.) $\lim_{x\to 0^+} \csc x \ln(1 + \sin(7x))$
    d.) $\lim_{x\to\infty} \dfrac{\ln 2x}{x^2}$
    e.) $\lim_{x\to 1^+} (x-1)\tan\dfrac{\pi x}{2}$

38. Find the most general antiderivative of $\dfrac{x^2 - 2x + 1}{\sqrt{x}}$

39. Approximate $\displaystyle\int_0^1 e^{x^2}\,dx$ using 4 equally spaced intervals, and take x_i^* to be the midpoint of each subinterval.

40. Set up the limit that gives the area bounded by $f(x) = x^2 + 5x$ on the interval $[3, 4]$.

41. Compute $\cos(\tan^{-1}\dfrac{2}{5})$

42. Compute $\sin^{-1}(\sin\dfrac{2\pi}{3})$

43. Find the derivative of $y = \cos^{-1}(3t) - 2\tan^{-1}(\sqrt{t})$

44. What is the domain of $\arcsin(4x - 5)$?

45. A rectangular poster is to have an area of 180 square inches with 1.5 inch margins at the top and bottom, and a 1 inch margin on the sides. What poster dimensions will give the largest printed area?

46. A company wants to manufacture a box with a volume of 36 cubic feet. The box has no top, and the length is twice the width. Find the dimensions of the box that minimizes the amount of material used.

47. Find $f(x)$ if it is known that $f'(x) = \sin x + 5e^x + x$ and $f(0) = 7$

48. A stone is thrown downward with a speed of 5 m/s off a building 350 m high. Find the height of the stone at time t.

49. Find $\dfrac{d}{dx}\displaystyle\int_2^{x^2} \sqrt{1 - t^4}\,dt$

50. Find $\displaystyle\int_{-1}^{3} |x - 2|\,dx$

51. $\displaystyle\int_{-2}^{0} \sqrt{4 - x^2}\,dx$

52. $\displaystyle\int_1^4 (2x^2 + \sqrt{x} - 4x^{-1})\,dx$

53. $\displaystyle\int_0^1 \frac{3}{x^2 + 1}\,dx$

54. $\displaystyle\int_0^{1/2} \frac{1}{\sqrt{1 - x^2}}\,dx$