Fall 2015 Math 151
Final Exam Practice - Answers *courtesy: Amy Austin*

Final Exam Practice: Sections 1.1 - 6.4

1. a.) $\langle -5, -7 \rangle$

   b.) $\left\langle \frac{-1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right\rangle$

   c.) $153°$

   d.) Vector projection: $\left\langle \frac{2}{5}, \frac{6}{5} \right\rangle$;

   scalar projection: $\frac{-4}{\sqrt{10}}$

2. A vector equation: $\langle 1+2t, -2+10t \rangle$;

   parametric equations: $x = 1+2t$, $y = -2+10t$

3. $x = 8+4t$, $y = 5+9t$

4. Magnitude of above force: $\frac{100}{(1+\sqrt{3})\sqrt{2}}$ N,

   Magnitude of below force: $\frac{100}{1+\sqrt{3}}$ N

5. Formula to use: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

   a.) $f'(x) = \frac{1}{2\sqrt{1+x}}$

   b.) $f'(x) = \frac{-1}{(x-3)^2}$

6. a.) ∞

   b.) $\frac{1}{4}$

   c.) $-\frac{1}{4}$

   d.) The limit does not exist because $\lim_{x \to 3^+} f(x) = 17$ and $\lim_{x \to 3^-} f(x) = 5$

   e.) -3

   f.) $\frac{1}{5}$

   g.) $-\frac{1}{5}$

   h.) DNE

   i.) $-\frac{1}{2}$

   j.) $\frac{1}{2}$

7. a.) 3

   b.) -2

   c.) -5

   d.) The limit does not exist

   e.) Not continuous at $x = 3$ (not in domain), not continuous at $x = -1$, $x = 5$ and $x = 7$ (the limit does not exist). Not differentiable at $x = -1$, $x = 3$, $x = 5$ and $x = 7$ (not continuous implies not differentiable). Also not differentiable at $x = -4$ and $x = -6$ because of sharp corners.

8. $f'(x) = \begin{cases} 2x - 2 & \text{if } x < 0 \text{ or } x > 2 \\ -2x + 2 & \text{if } 0 < x < 2 \end{cases}$

9. a.) Not continuous at $x = 0$ because $\lim_{x \to 0} f(x)$ does not exist. Continuous for all other values of x.

   b.) Not continuous at $x = 1$ because $\lim_{x \to 1} f(x) = 1$, yet $f(1) = 4$.

   c.) $a = 6$, $b = -3$

10. $x + y + 1 = 0$, $11x - y = 25$

11. horizontal asymptote: $y = 0$, vertical asymptote: $x = 1$.

12. $\frac{5}{12}$

13. 4

14. $x > \ln 4$

15. $f^{-1}(x) = \ln \frac{x}{1-x}$

16. a.) $y' = \frac{1 - 3x^2 y + 9x^2}{x^3 + 4y^3 - 1}$

    b.) $y' = \frac{\sin(x-y) + 2y - 4}{\sin(x-y) - 2x}$

17. a.) $f'(x) = \frac{12x^2 - 4x^4 - 16x}{(1-x^2)^2}$

    b.) $f'(t) = 3t^2 \cos(1-t^2) + 2t^4 \sin(1-t^2)$

    c.) $G'(x) = 12 \tan^2(4x-1) \sec^2(4x-1)$

18. 56

19. $y - \ln 27 = (3 + \ln 27)(x - \ln 3)$

20. At $t = -1$: $y + 17 = 6(x+1)$; horizontal tangent: $y = 64$; verical tangent: $x = 0$

21. $L(x) = \ln 2 + 1/2(x-2)$,
$Q(x) = \ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2$. The linear and quadratic approximations are useful in approximating the function for x sufficiently close to a.

22. $\frac{dh}{dt} = \frac{49}{36\pi}$ cm/min

23. $5/3$ feet per second

24. 0.5 feet per second

25. $x = 2$

26. $x = \frac{2e}{1-e}$ Since this is not in the domain, there is no solution.

27. $t = \frac{2}{5}$

28. $x = t, y = e + 2et$

29. $f'(x) = -\tan x$

30. $y' = y\left(\frac{\ln(1+x+3x^3)}{2\sqrt{x}} + \frac{\sqrt{x}(1+9x^2)}{1+x+3x^3}\right)$, where
$y = (1+x+3x^3)^{\sqrt{x}}$

31. 7.284 minutes

32. $t = .944$ hours

33. Inc: $(3,\infty)$, Dec: $(-\infty,3)$, Local Min: $(3,-33)$; Local Max: None; Concave up: $(-\infty,0)$ and $(2,\infty)$, concave down: $(0,2)$, points of inflection: $(0,-6)$ and $(2,-22)$

34. $(0,\infty)$

35. Absolute Max: -1; Absolute min: -5

36. critical values: $x = -1$, $x = 1$, $x = 5$, f inc: $(-1,1)$, $(5,\infty)$, ; f dec: $(-\infty,-1)$, $(1,5)$; local min: $x = -1$,$x = 5$; local max: $x = 1$; f cu: $(-\infty,0)$ and $(5,\infty)$; f cd: $(0,4)$; inflection points: $x = 0$, $x = 5$.

37. a.) -3
b.) e^3
c.) 7
d.) 0
e.) $\frac{-2}{\pi}$

38. $\frac{2}{5}x^{5/2} - \frac{4}{3}x^{3/2} + 2x^{1/2} + C$

39. $\frac{1}{4}\left(e^{1/64} + e^{9/64} + e^{25/64} + e^{49/64}\right)$

40. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left((3+i/n)^2 + 5(3+i/n)\right)$

41. $5/\sqrt{29}$

42. $\frac{\pi}{3}$

43. $y' = -\frac{3}{\sqrt{1-9t^2}} - \frac{1}{\sqrt{t}(1+t)}$

44. $\left[1, \frac{3}{2}\right]$

45. $2\sqrt{30}$ by $\frac{90}{\sqrt{30}}$

46. 3x6x2 cubic feet

47. $f(x) = -\cos x + 5e^x + \frac{x^2}{2} + 3$

48. $h(t) = -4.9t^2 - 5t + 350$

49. $2x\sqrt{1-x^8}$

50. 5

51. π

52. $\frac{140}{3} - 4\ln 4$

53. $\frac{3\pi}{4}$

54. $\frac{\pi}{6}$