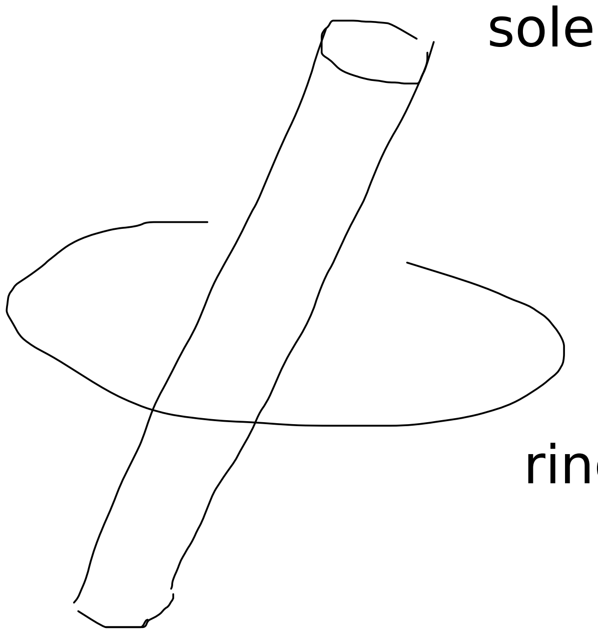
04/08/2011

mutual inductance

$\Phi = M I$

examples



solenoid

area A

turn density n

ring

1. current in solenoid, flux through the ring

$$B = \mu_0 n I$$

$$\Phi = BA = \mu_0 n A I$$

$$\boxed{M = \mu_0 n A}$$

what if the solinoid is tilted?

2. current in the ring, flux through the solinoid

ring makes the magnetic field.

$$\vec{B}(\vec{r})$$

take a piece of solinoid of length dr

this piece has $n\,dr$ turns.

the flux through these turns is

$$d\Phi = \vec{B}\cdot\vec{A}\,n\,dr =$$

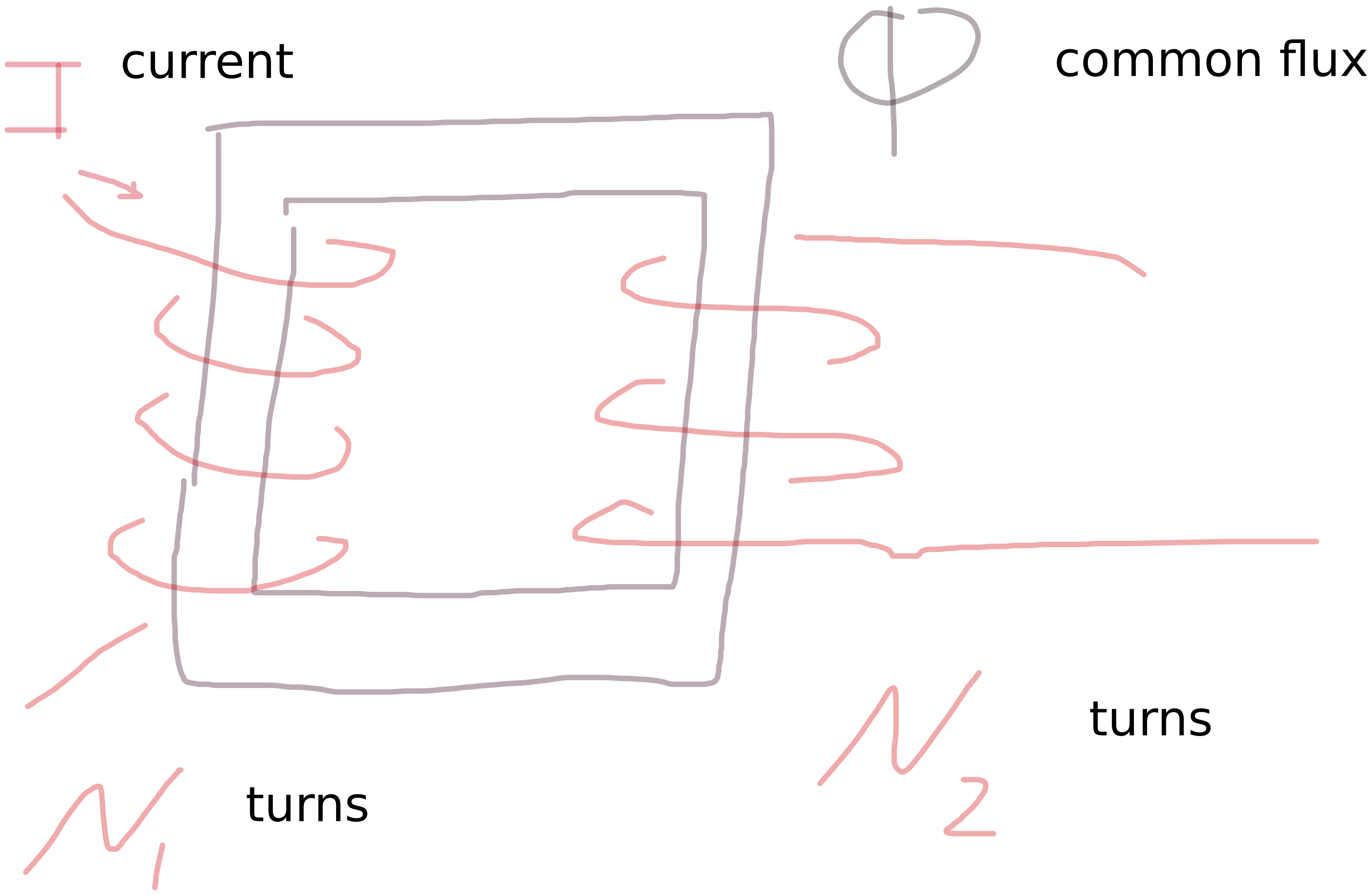
$$= \vec{B}\,d\vec{r}\;\;A\,n$$

the total flux through the solinoid is

$$\Phi = An\int \vec{B}\cdot d\vec{r} = \mu_0 n A I$$

$$\boxed{M = \mu_0 n A}$$

transformer



$$\Phi_1 = N_1 \phi$$

$$\Phi_2 = N_2 \phi$$

$$V_1 = N_1 \frac{d\phi}{dt} = \frac{N_1}{N_2} N_2 \frac{d\phi}{dt} = \frac{N_1}{N_2} V_2$$

$$\boxed{\frac{V_1}{N_1} = \frac{V_2}{N_2}}$$

$$P = I_1 V_1 = I_2 V_2$$

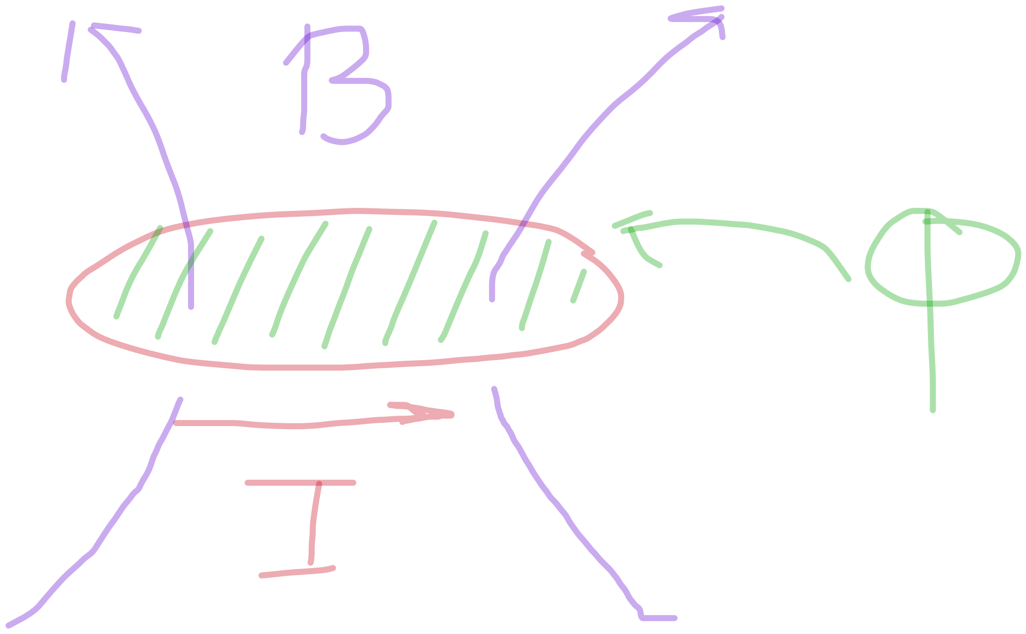
$$\boxed{I_1 N_1 = I_2 V_2}$$

for AC current only!!!

The end!

Self-inductance

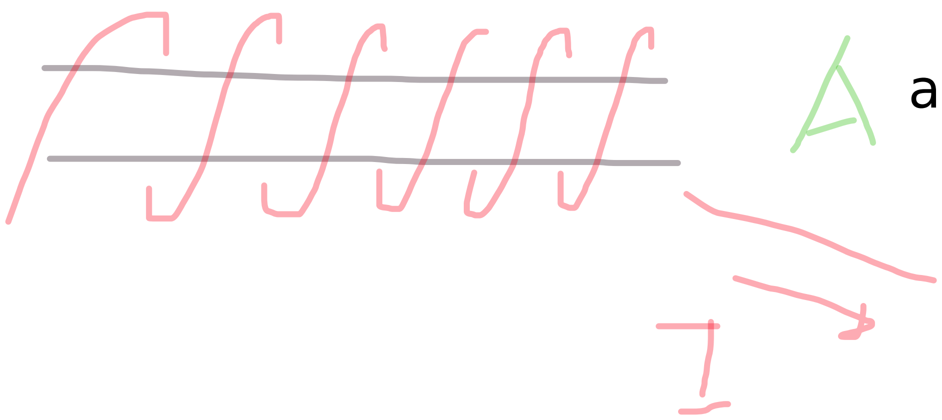
04/11/2011



$$\Phi \sim I$$

$$\Phi = L I$$

self inductance

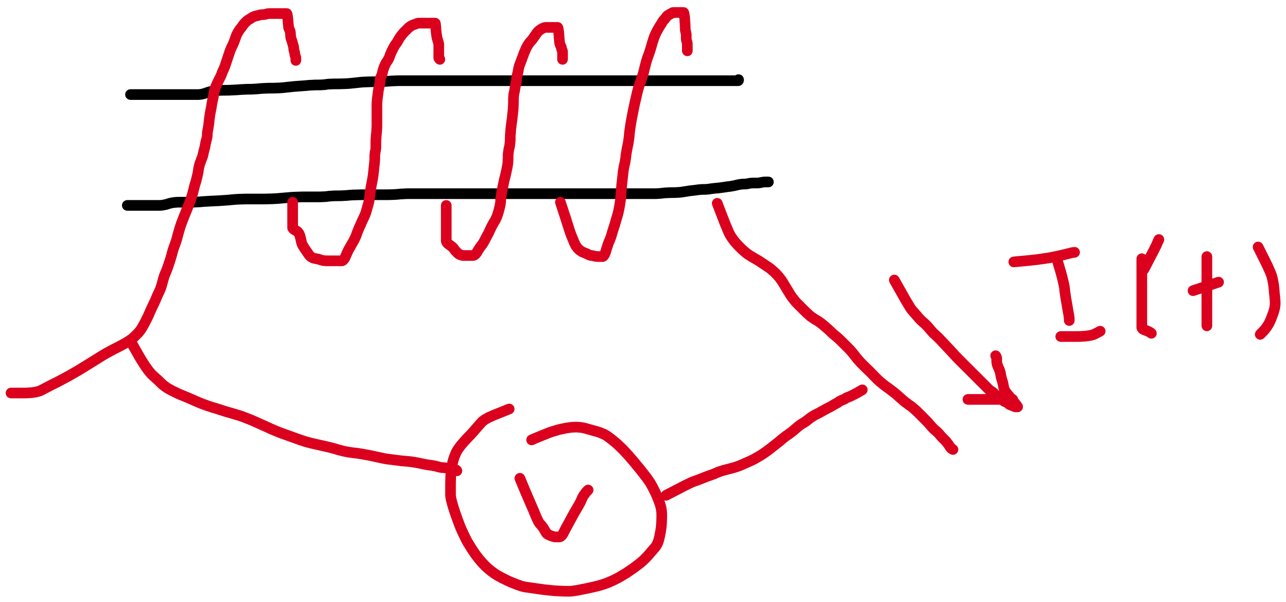


area

$$\Phi = NBA = n\ell\mu_0 nA]$$

$$\boxed{L = n^2 A \ell \mu_0}$$

for any inductor

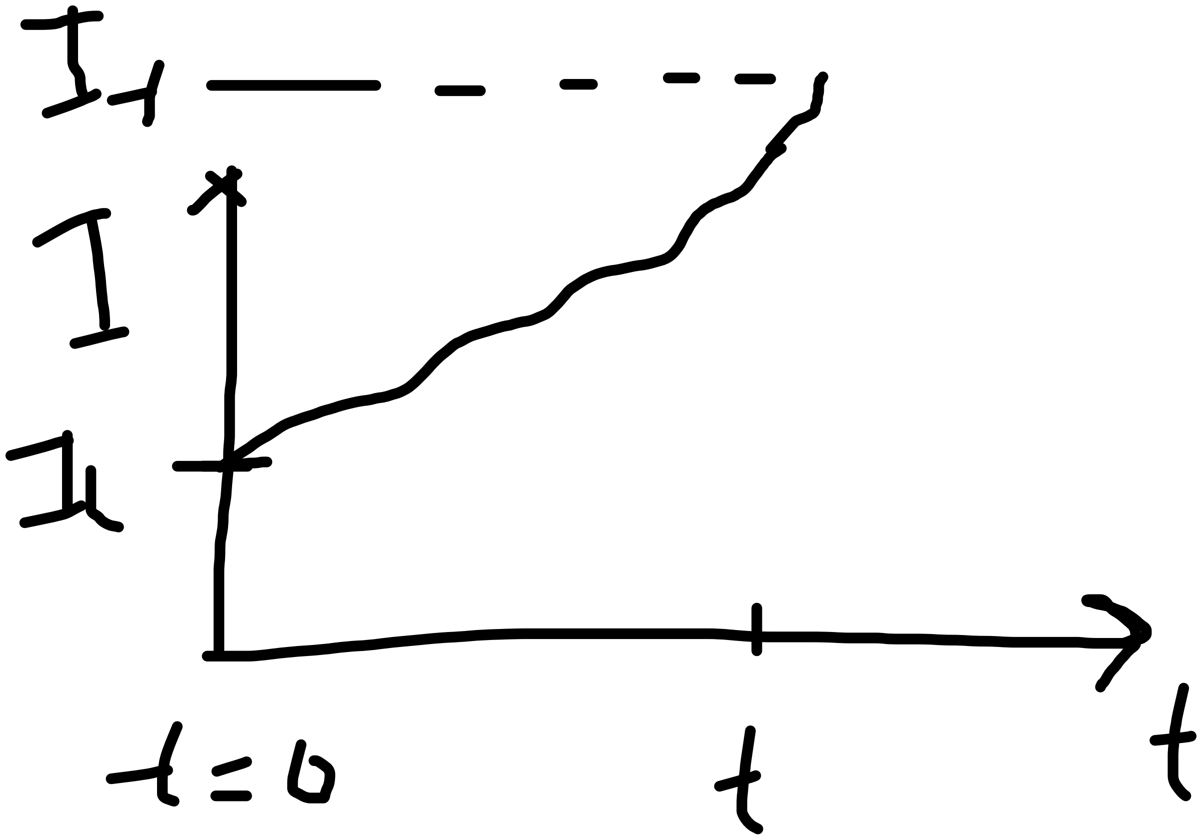


$$V = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

energy of an inductor

1. increase I induces V
2. V conteracts.

It takes wotk to increase I.



increase current from I to I+dI during time dt

$$V = -L\frac{dI}{dt}$$

$$P = IV = LI\frac{dI}{dt}$$

$$W = \int_{t_1}^{t_f} P\,dt = \int_{t_1}^{t_f} L I \frac{dI}{dt}\,dt =$$

$$= \int_{I_1}^{I_f} L I\,dI = \frac{L I_f^2}{2} - \frac{L I_1^2}{2}$$

$$\boxed{E_L = \frac{L I^2}{2}}$$

Any inductor

for a solenoid

$$L = n^2 A l \mu_0$$

$$B = \mu_0 I n$$

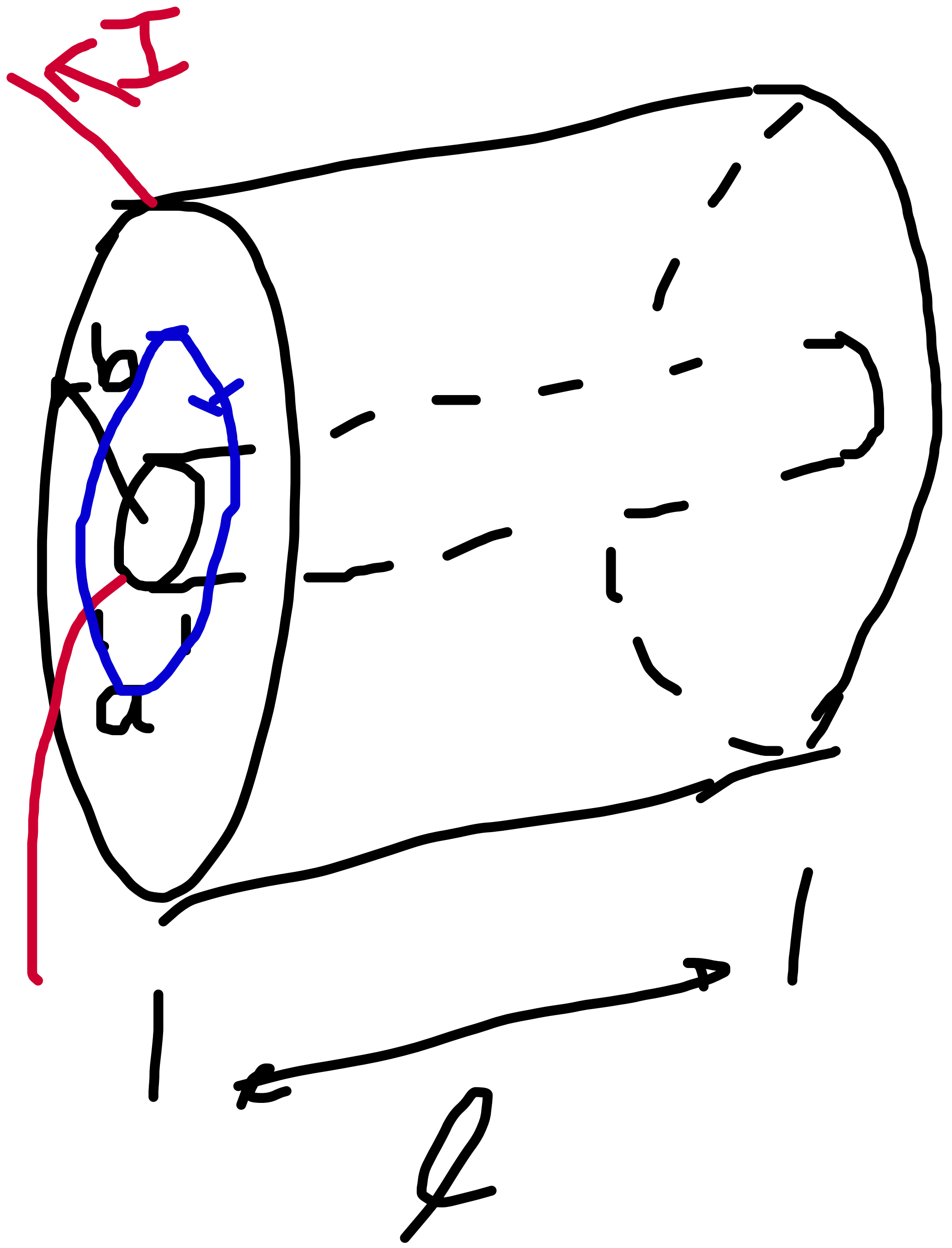
$$E = \frac{L I^2}{2} = \frac{n^2 \mu_0 I^2}{2} l A =$$

$$= \frac{B^2}{2\mu_0} v$$

volume

$$\boxed{\varepsilon_B = \frac{B^2}{2\mu_0}}$$

energy density of magnetic field



What is L?

$$B \, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi} \frac{1}{r}$$

$$E = \int \frac{B^2}{2\mu_0}\, dv =$$

$$= \frac{\mu_0^2 I^2}{4\pi^2} \frac{1}{2\mu_0} \int \frac{1}{r^2}\, dx\, dy\, r\, dr$$

$$= \frac{\mu_0 I^2}{4\pi} l \ln b/a$$

$$L_0 = \frac{\mu_0 l}{2\pi} \ln b/a$$

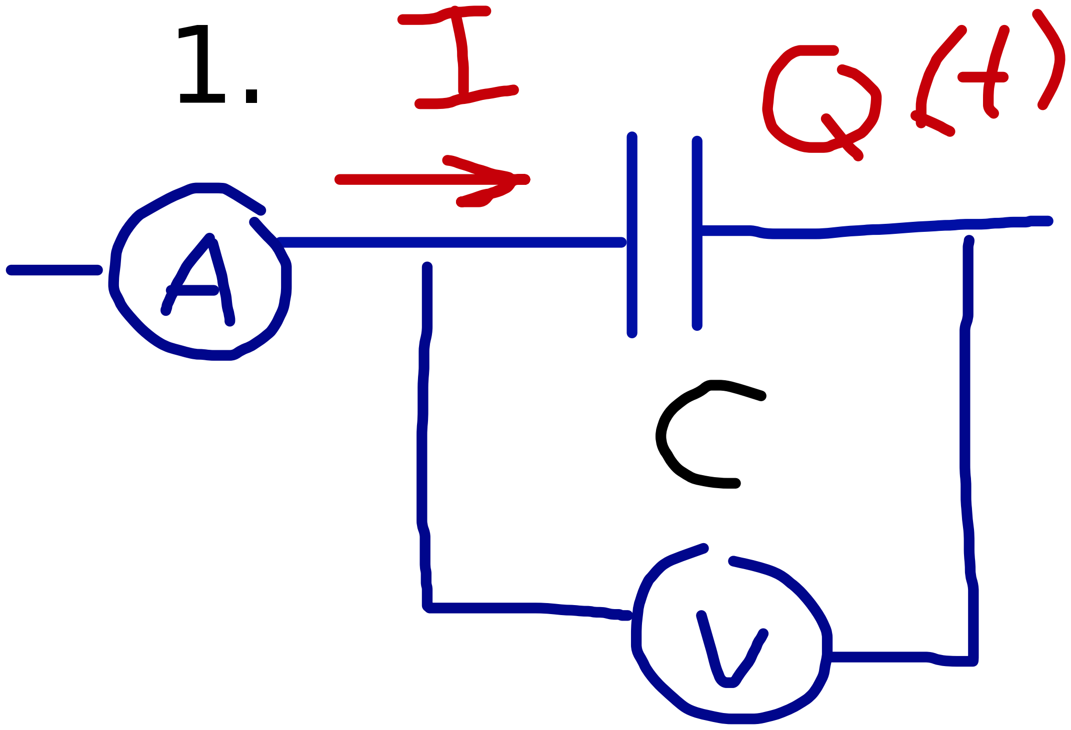
The End!

04/13/2011

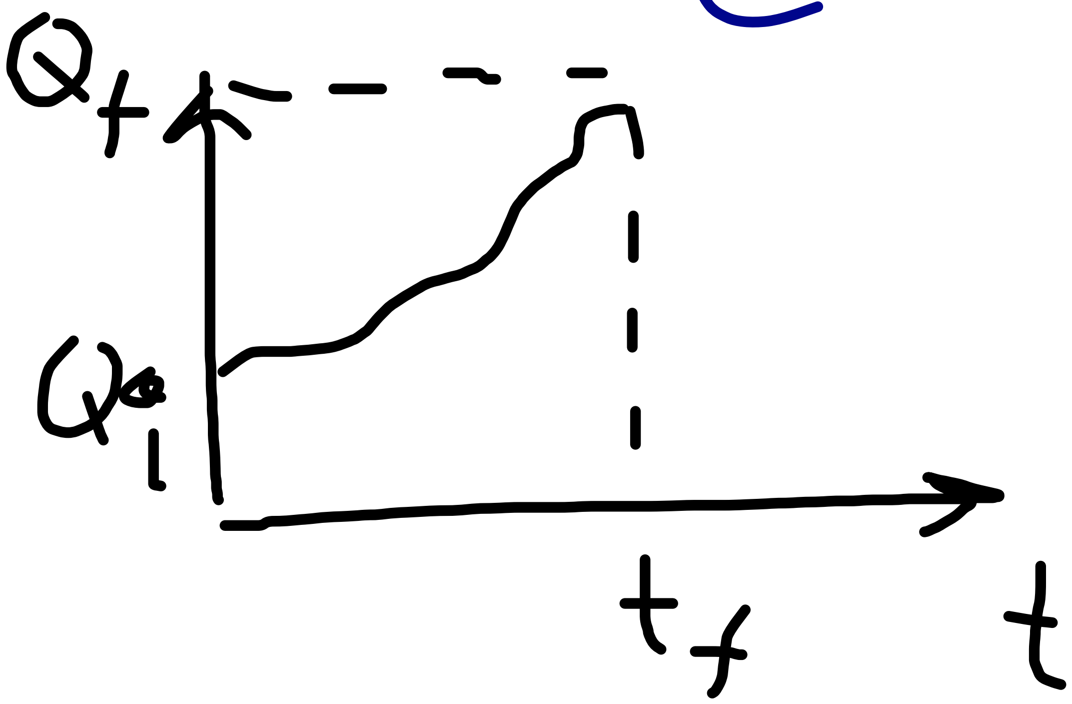
Plan

1. Capacitoror's energy. El. field energy density
2. Capacitor and inductor in a circuit
3. RC and RL circuits
4. Energy dissipation
5. LC circuit

1.



$$V = \frac{Q}{C}$$



$$I = \frac{dQ}{dt}$$

$$P = IV = \frac{Q}{C}\,\frac{dQ}{dt}$$

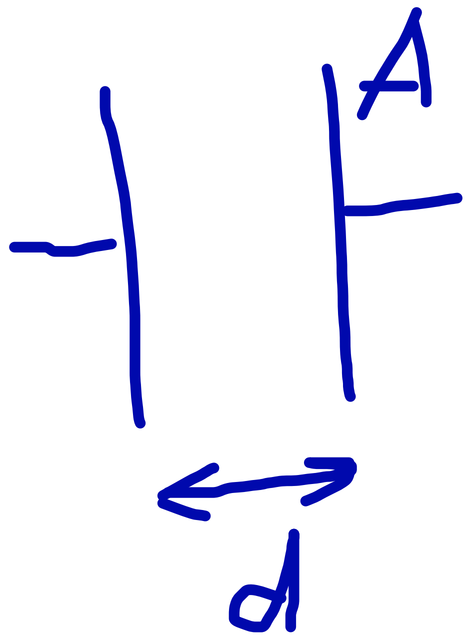
$$dW = P\,dt = \frac{1}{C} Q\,dQ$$

$$W = \int_{Q_i}^{Q_f} \frac{1}{C} Q\,dQ = \frac{Q_f^2}{2C} - \frac{Q_i^2}{2C}$$

$$\boxed{E_c = \frac{Q^2}{2C}}$$

Energy stored in a capacitor.

For a parallel plate capacitot

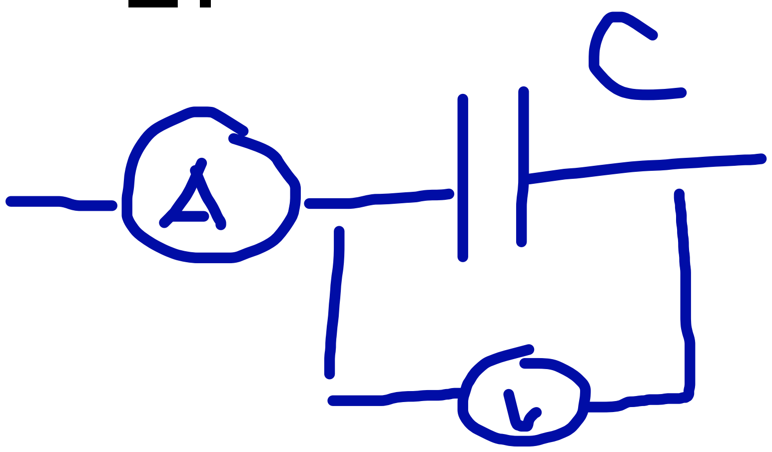


$$E = \frac{Q}{A\varepsilon_0} \; ; \quad C = \frac{A\varepsilon_0}{d}$$

$$E_c = \frac{Q^2}{2\varepsilon_0 A} d = \frac{\varepsilon_0 E^2}{2} A d$$

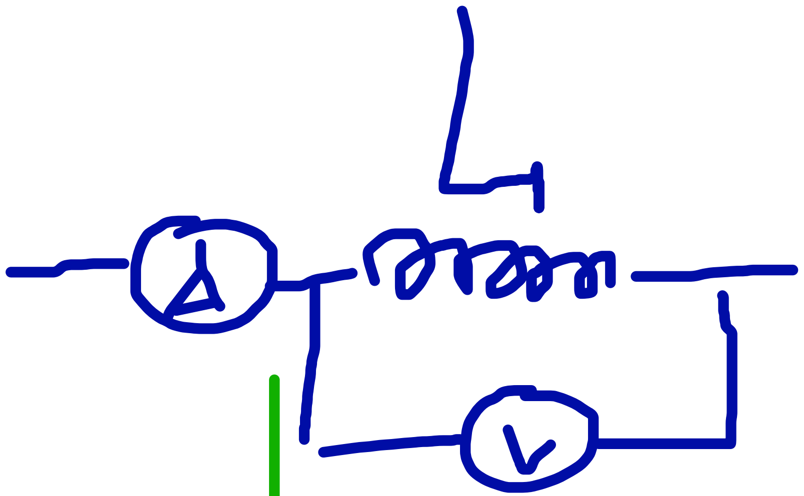
$$\boxed{\varepsilon_E = \frac{\varepsilon_0 E^2}{2}}$$

2.

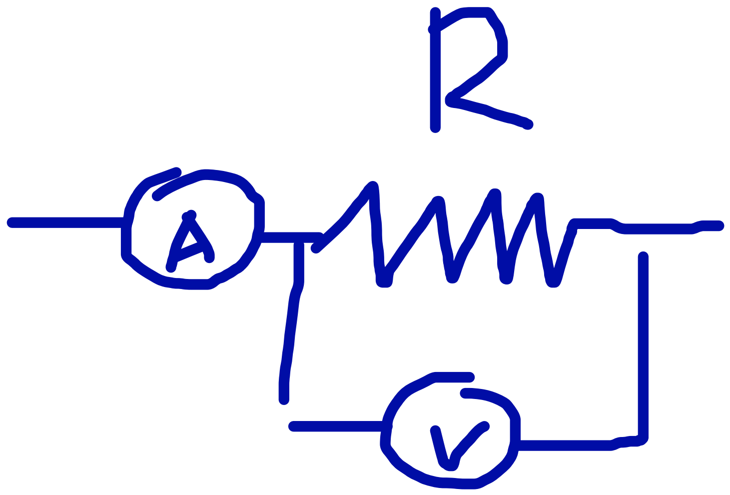


$$V = \frac{Q}{C}$$

$$I = \frac{dQ}{dt}$$

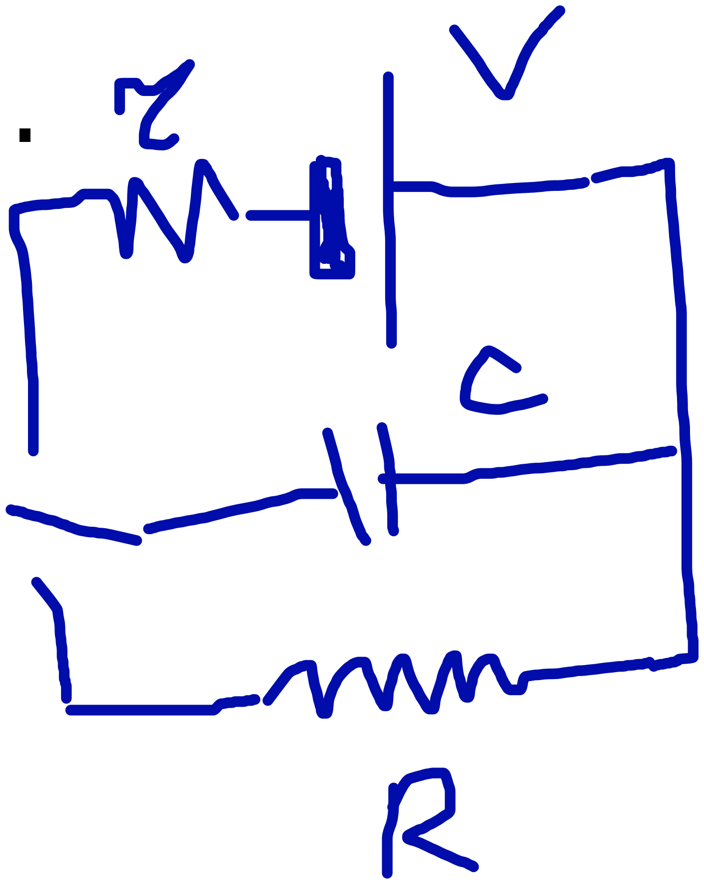


$$V = -L\frac{dI}{dt}$$



$$V = IR$$

3.



$t=0$

$$Q_0 = VC$$

t

$$\frac{Q}{C} + IR = 0$$

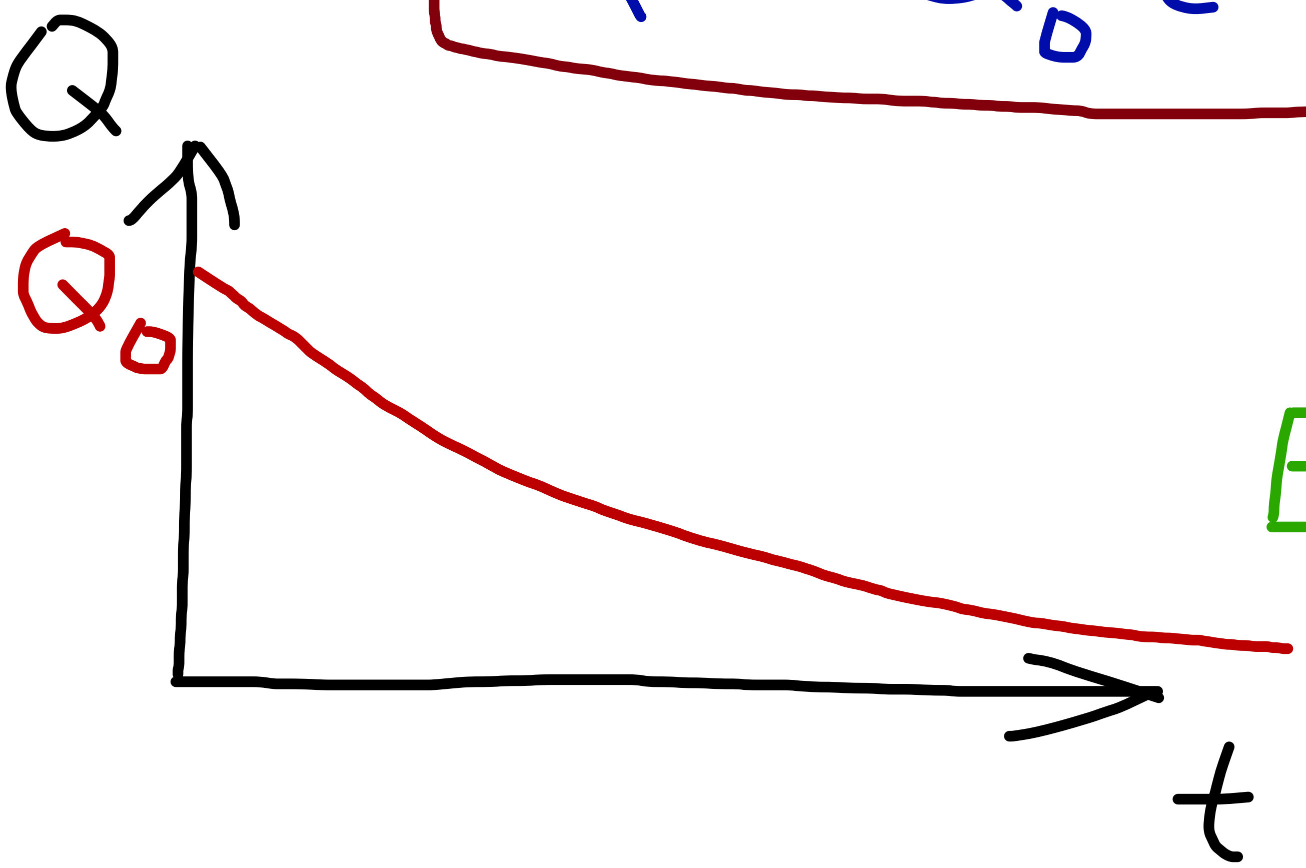
$$I = \frac{dQ}{dt}$$

$$\boxed{\frac{d}{dt} E_c = -I^2 R}$$

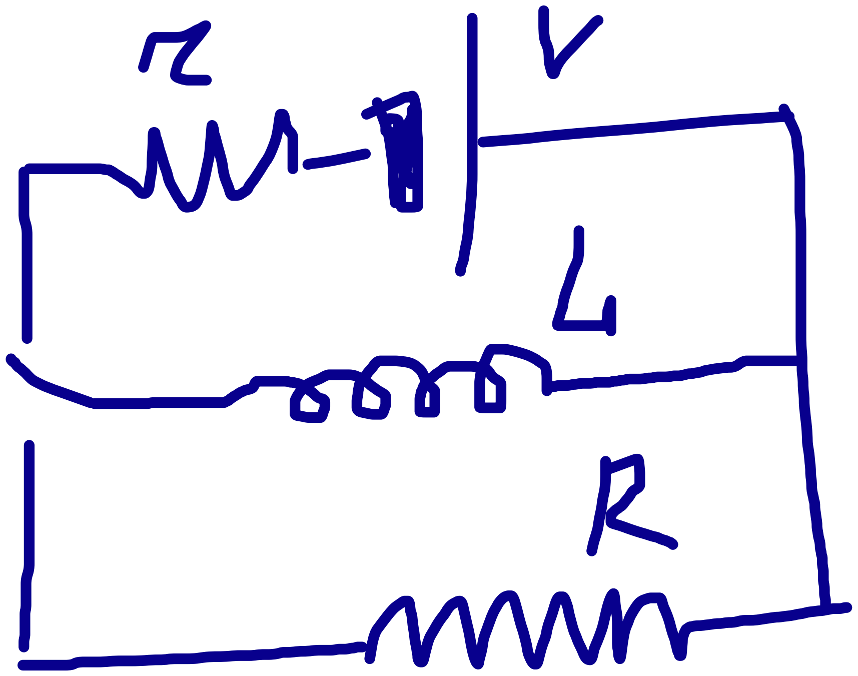
$$\frac{dQ}{dt} = -\frac{1}{RC} Q$$

$$\boxed{Q = Q_0 e^{-t/\tau}}$$

$$\tau = RC$$



$$E_c(t) = E_0 e^{-2t/\tau}$$



$t = 0$

$I_0 = V/r$

t

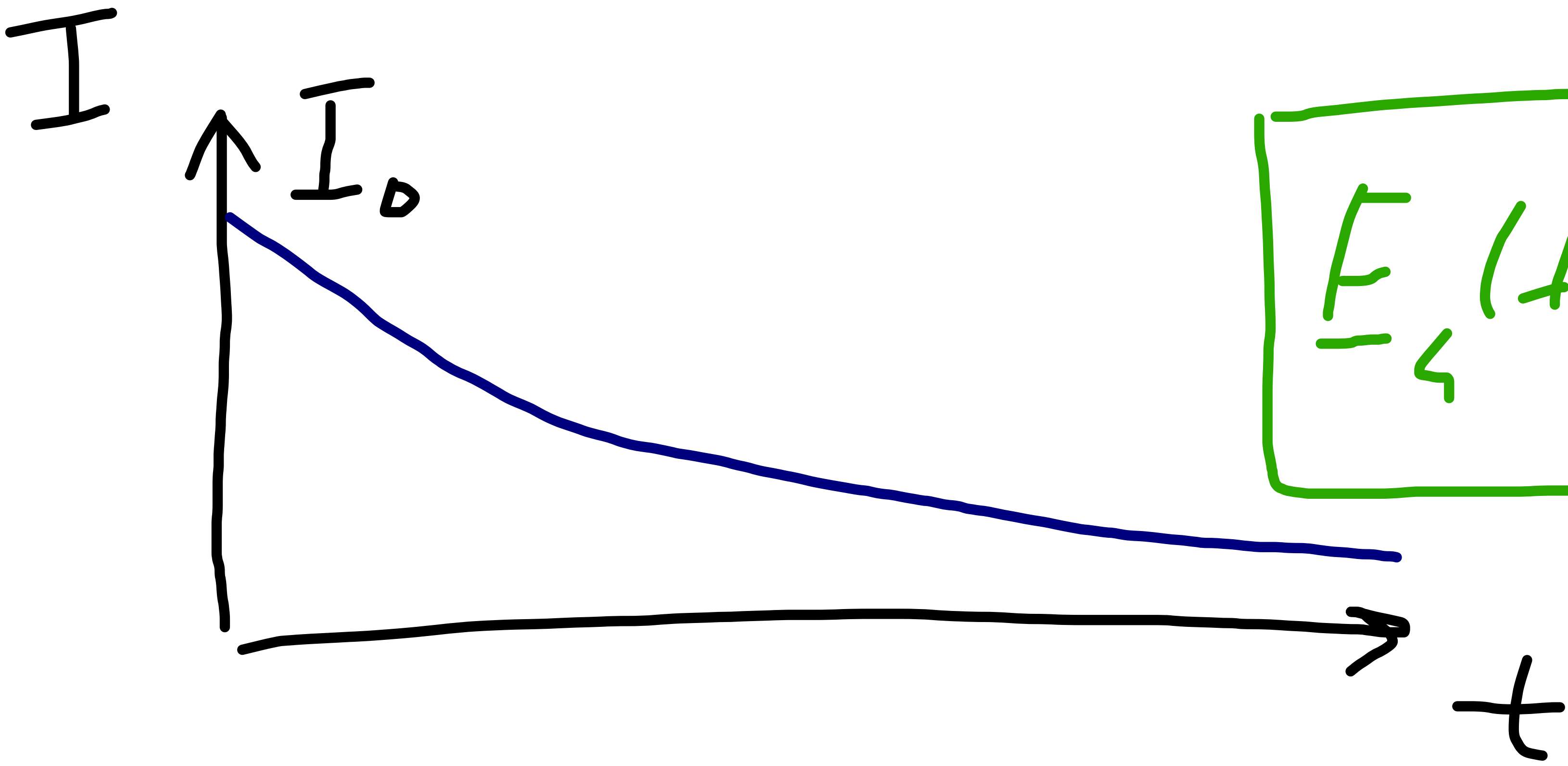
$$L\frac{dI}{dt} + RI = 0$$

$$\frac{d}{dt}E_L = -RI^2$$

$$\boxed{I(t) = I_0 e^{-t/\tau}}$$

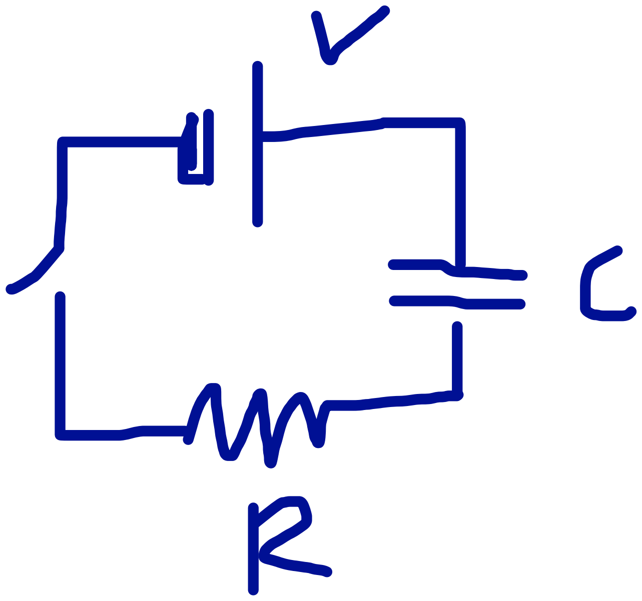
$\tau = L/R$

$I_0 = \frac{V}{r}$



$$\boxed{E_L(t) = E_0 e^{-2t/\tau}}$$

Charging

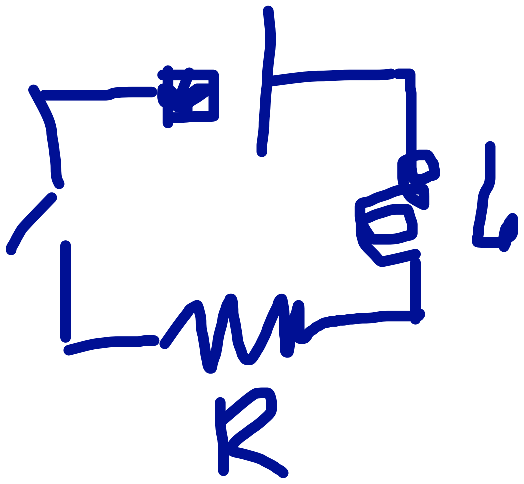


$t = 0 \quad Q = 0$

$$V - IR - \frac{Q}{C} = 0$$

$$Q = Q_f\left(1 - e^{-t/\tau_c}\right)$$

$$Q_f = VC$$



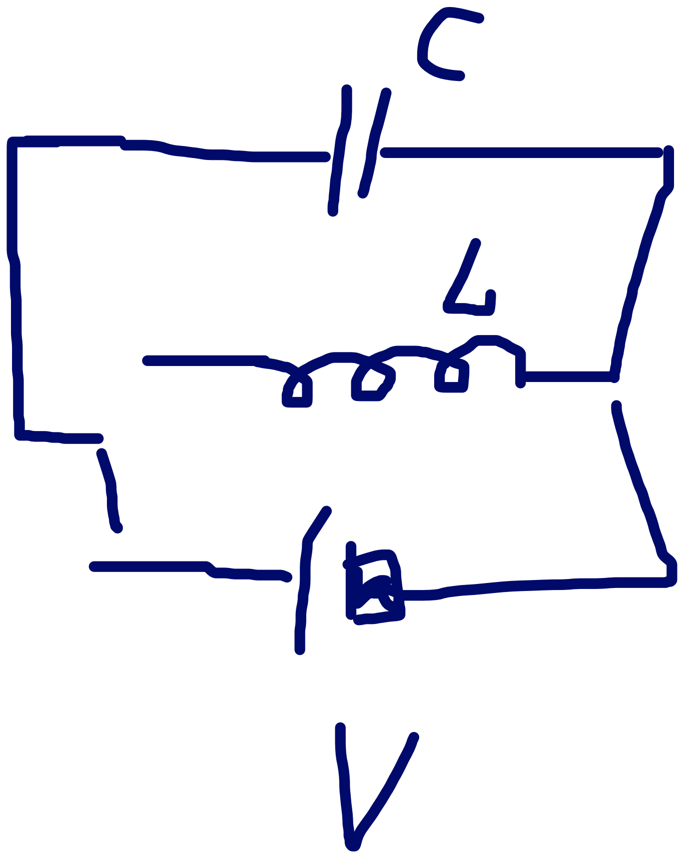
$t = 0 \quad I = 0$

$$V - IR - L\frac{dI}{dt} = 0$$

$$I = I_f\left(1 - e^{-t/\tau_L}\right)$$

$$I_f = \frac{V}{R}$$

lc ciruit



$t = 0$

$I = 0$

$Q_0 = CV$

$t: \quad -V_c + V_L = 0$

$$\frac{Q}{C} + L\frac{dI}{dt} = 0$$

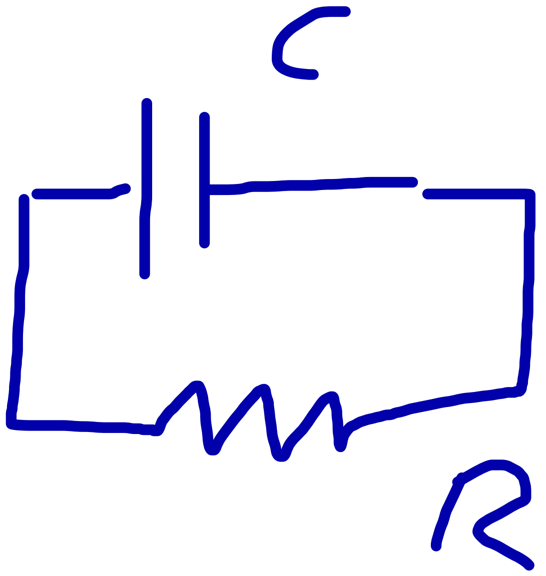
$$I = \frac{dQ}{dt} \qquad \frac{d}{dt}\left(E_c + E_L\right) = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \quad \Bigg| \quad Q = Q_0\cos(\omega t)$$

$$\omega = 1/\sqrt{LC}$$

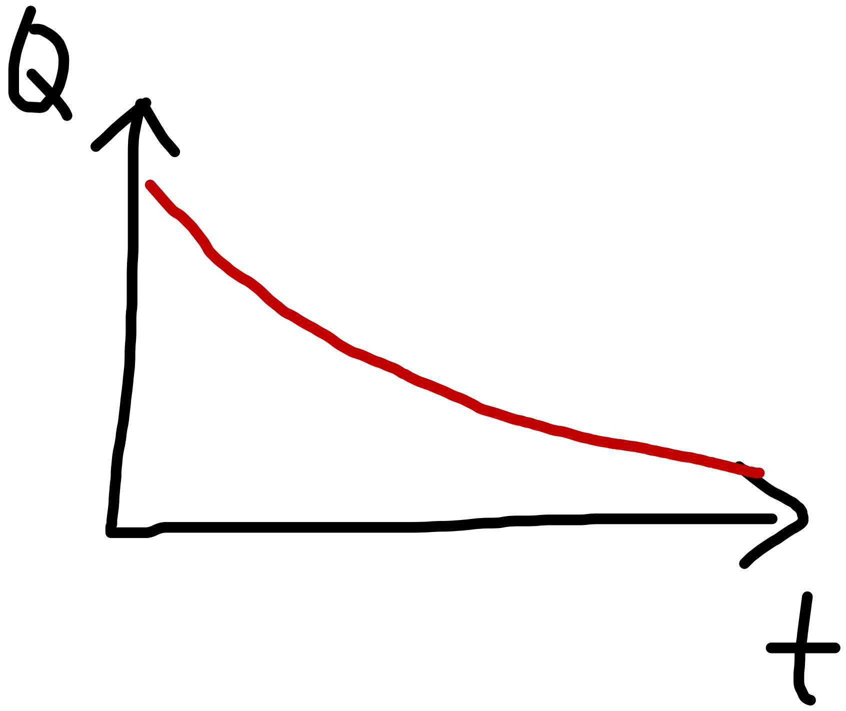
04/15/2011

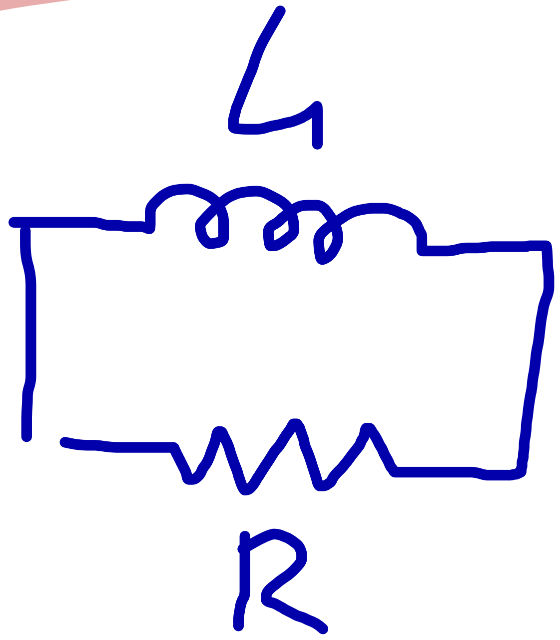
RLC circuit



$$Q = Q_0 e^{-t/\tau}$$

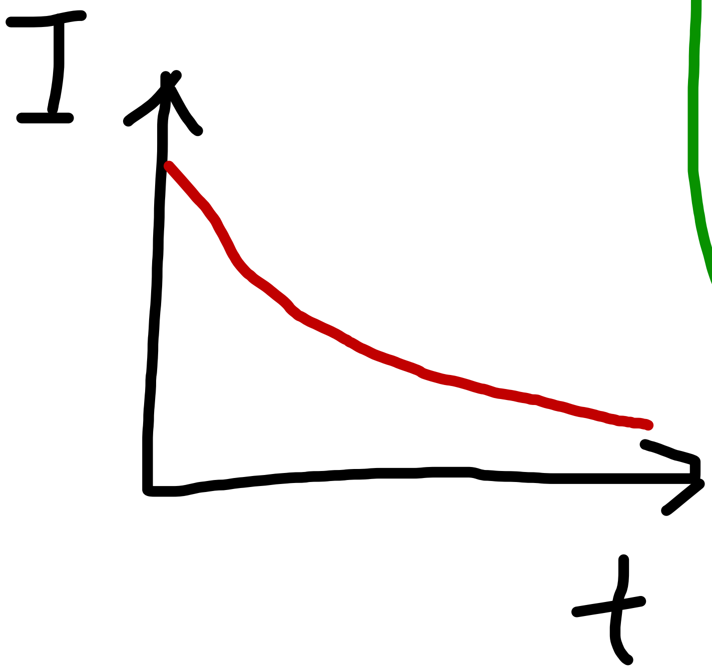
$$\tau = RC$$

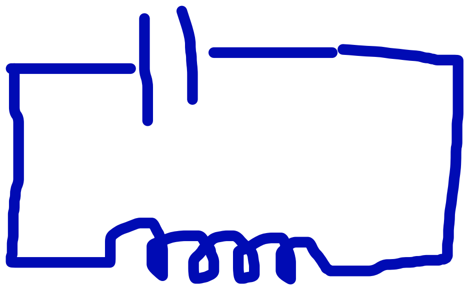




$$I = I_0 e^{-t/\tau}$$

$$\tau = L/R$$





$$Q(t=0) = Q_0$$

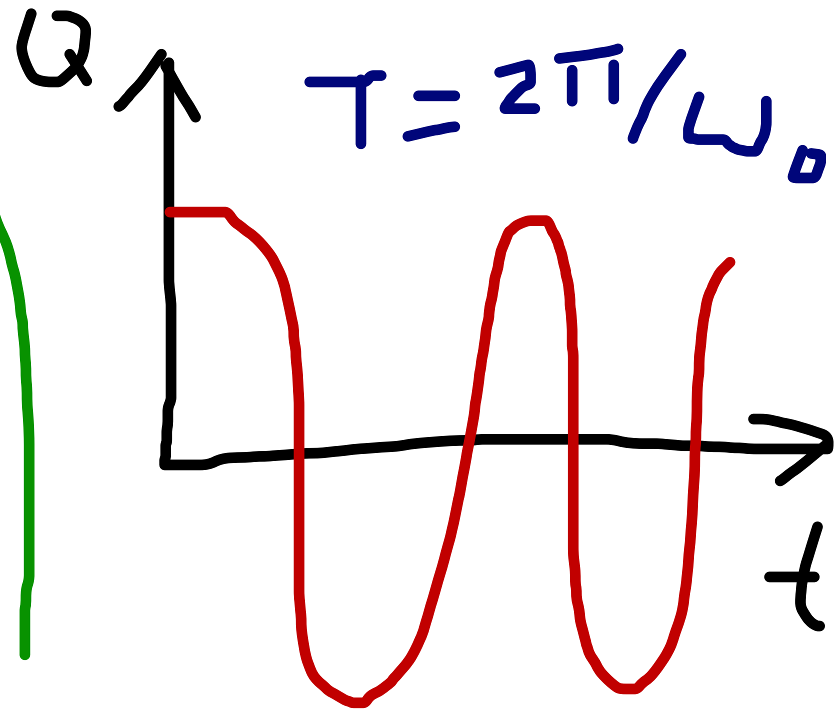
$$I(t=0) = 0$$

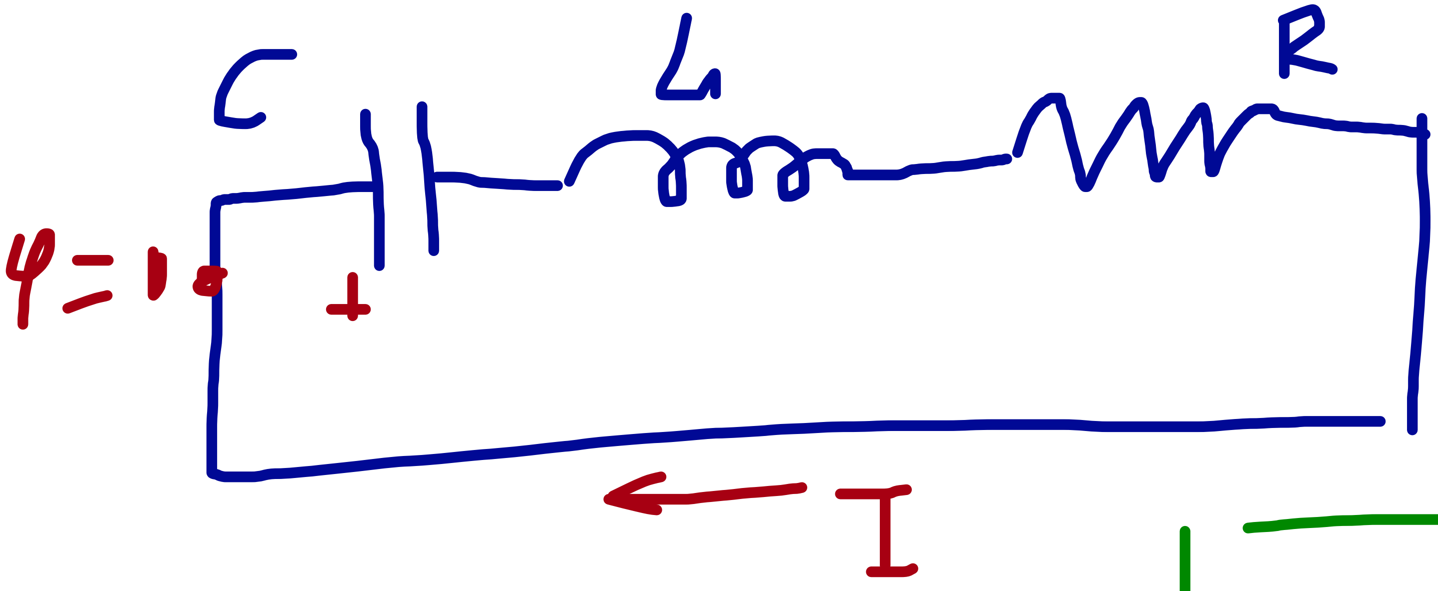
$$Q = Q_0 \cos(\omega_0 t)$$

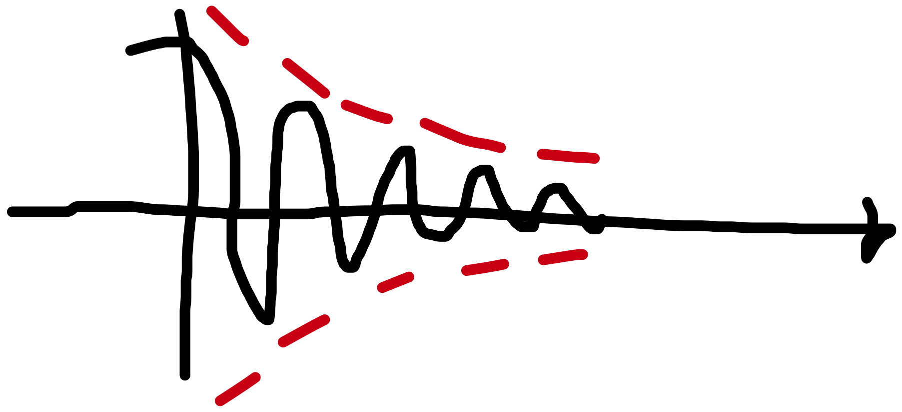
$$I = -\omega_0 Q_0 \sin(\omega_0 t)$$

$$\omega_0^2 = 1/LC$$

$$T = 2\pi/\omega_0$$







$$V_c + V_L + V_R = 0$$

$$\boxed{I = \frac{dQ}{dt}}$$

$$-\frac{Q}{C} - L\frac{dI}{dt} - RI = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

$$Q(t) = Q_0\, e^{-t/\tau} \cos(\omega t)$$

$$\tau = 2L/R \qquad \omega^2 = \frac{1}{LC} - \frac{1}{\tau^2}$$

04-22-2011

Plan

1. Exam discussion
2. Driven oscillator

Count: 83

Ave: 44.8

Std: 19.4

Max: 91

$100 > A > 64$

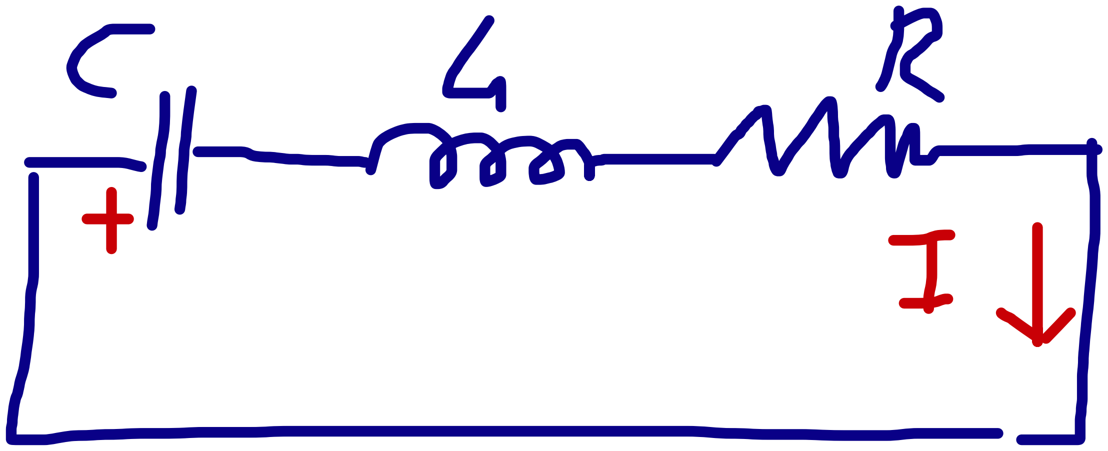
$64 > B > 44$

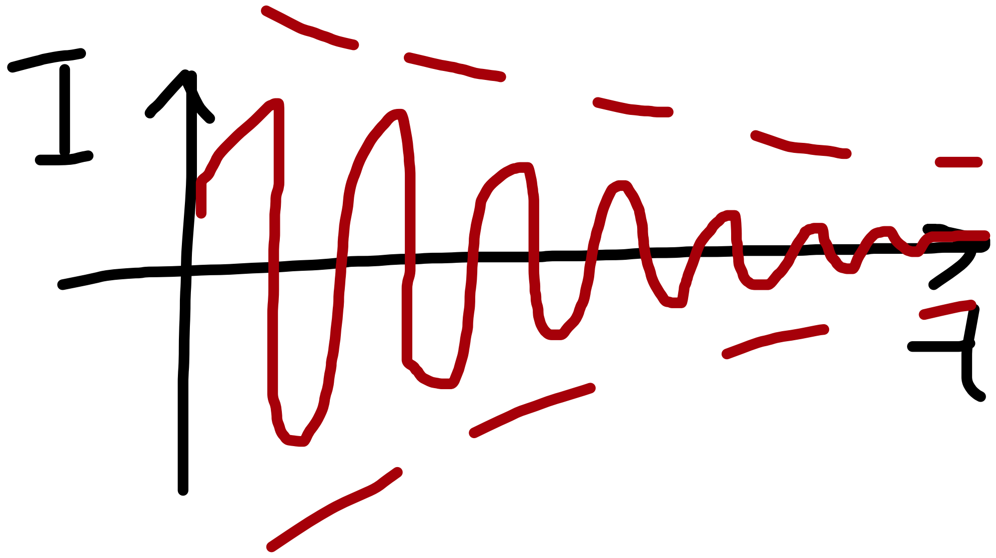
$44 > C > 24$

$24 > D > 10$

Solution of the Exam problems

2. Driven Oscillator





$$-\frac{Q}{C}-L\frac{dI}{dt}-RI=0$$

$$I=\frac{dQ}{dt}$$

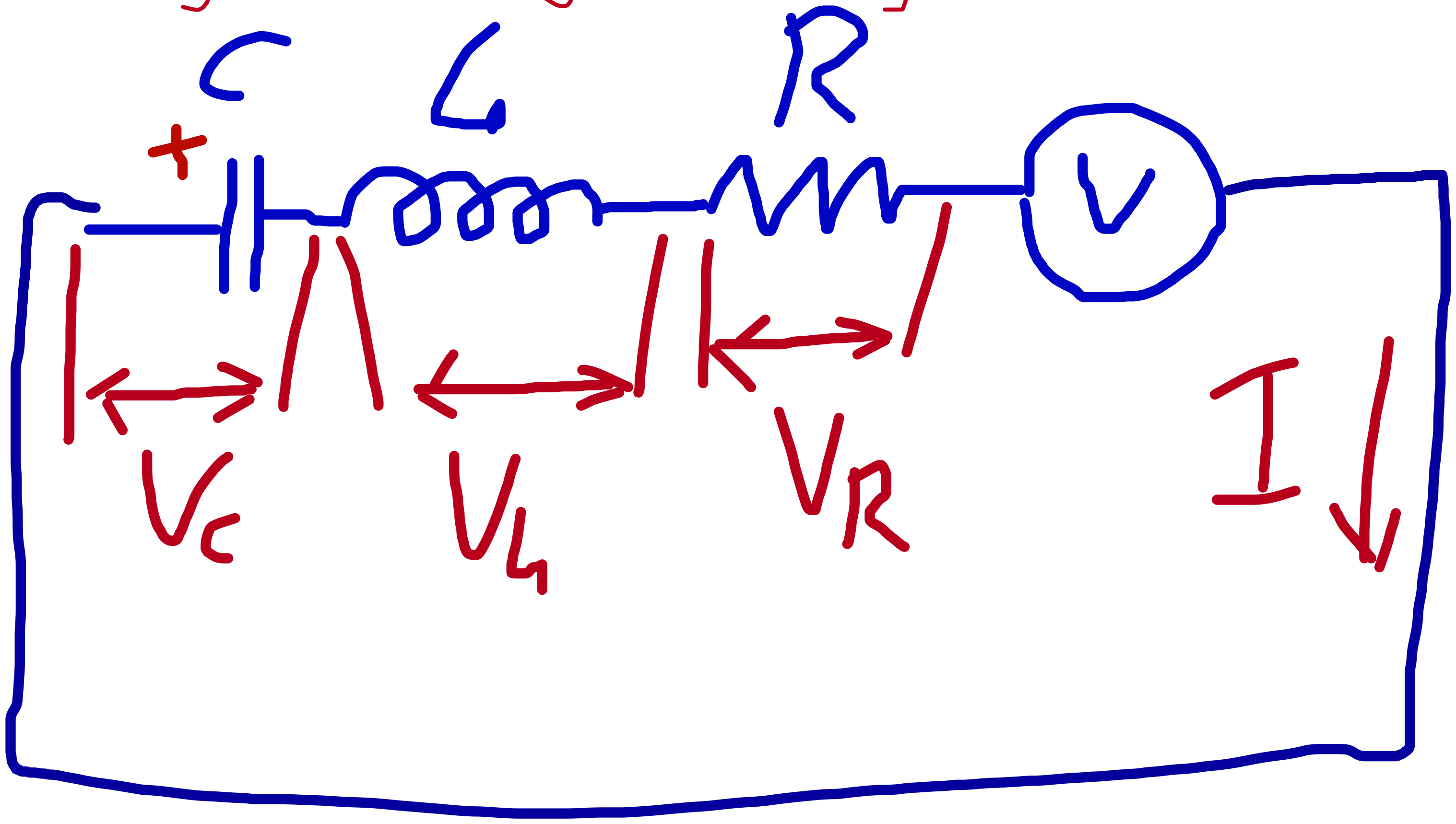
initial cond.

$$Q_0(t)=\underline{A}\,e^{-t/\tau}\cos(\omega_0 t+\underline{\varphi})$$

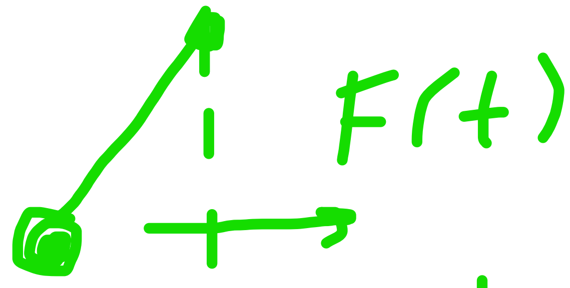
$$\boxed{\tau=\frac{2L}{R}}\;;\;\boxed{\omega_0^2=\frac{1}{LC}-\frac{1}{\tau^2}}$$

independent of init. cond.

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$



$$V = V_0 \cos(\omega t)$$



$$I = \frac{dQ}{dt}$$

Phase shift

$$-\frac{Q}{C} - L\frac{dI}{dt} - IR = V(t)$$

solution

$$\underbrace{Q(t) + Q_0(t)}_{\text{also solution!}}$$

Decays (Takes care of init. cond)

non Decaying solution!

$$-\frac{Q}{C} - L\frac{dI}{dt} - IR = V(t) \qquad \Big|\; I = \frac{dQ}{dt}$$

$$Q = Q_m \cos(\omega t + \theta)$$

$$\overbrace{\left(-\frac{1}{C} + \omega^2 L\right)}^{= L(\omega^2 - \omega_0^2)} Q_m \cos(\omega t + \theta) +$$

$$+\, \omega R Q_m \sin(\omega t + \theta) = V_0 \cos(\omega t + \theta - \theta) =$$

$$= V_0 \cos(\omega t + \theta)\cos\theta +$$

$$+\, V_0 \sin(\omega t + \theta)\sin\theta$$

$$\omega R Q_m = V_0 \sin\theta$$

$$L(\omega^2 - \omega_0^2) Q_m = V_0 \cos\theta$$

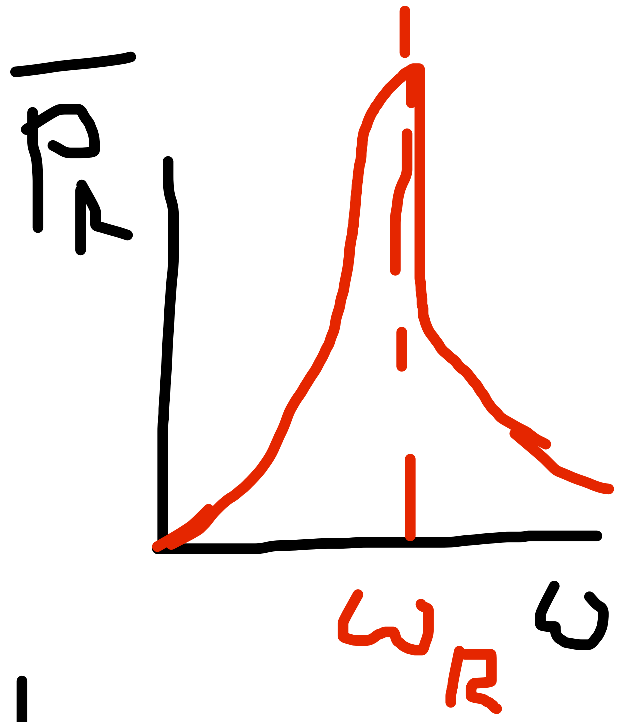
$$\tan\theta = \frac{\omega R}{L(\omega^2 - \omega_0^2)}$$

$$Q_m = \frac{V_0}{\left[\omega^2 R^2 + L^2(\omega^2-\omega_0^2)^2\right]^{1/2}}$$

$$I = -\overbrace{Q_m\,\omega}^{I_m}\cos(\omega t + \theta)$$

Power

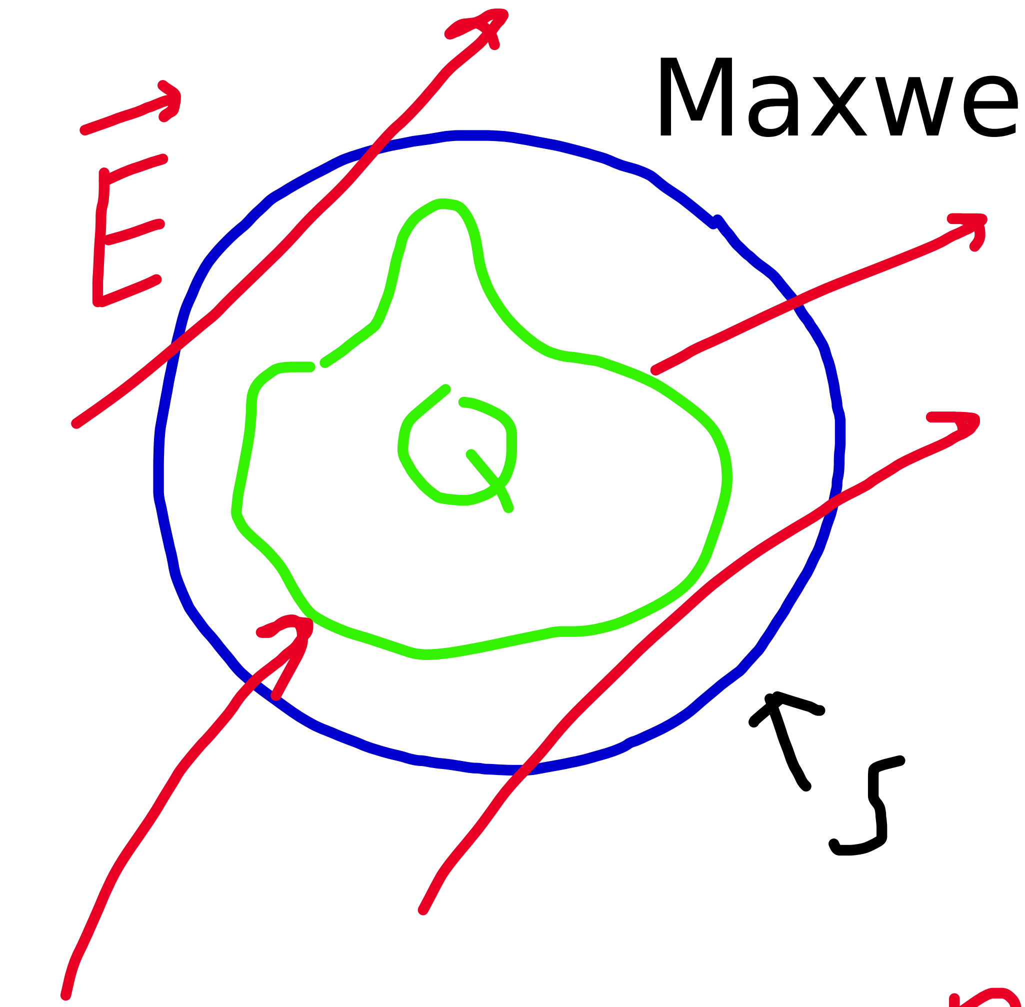
$$\bar{P} = \overline{IV} \qquad \bar{P}_L = \bar{P}_C = 0$$



$$\bar{P}_R = \frac{R I_m^2}{2} = \frac{R\omega^2 V_0^2}{2}\,\frac{1}{\omega^2 R^2 + L^2(\omega^2-\omega_0^2)^2}$$

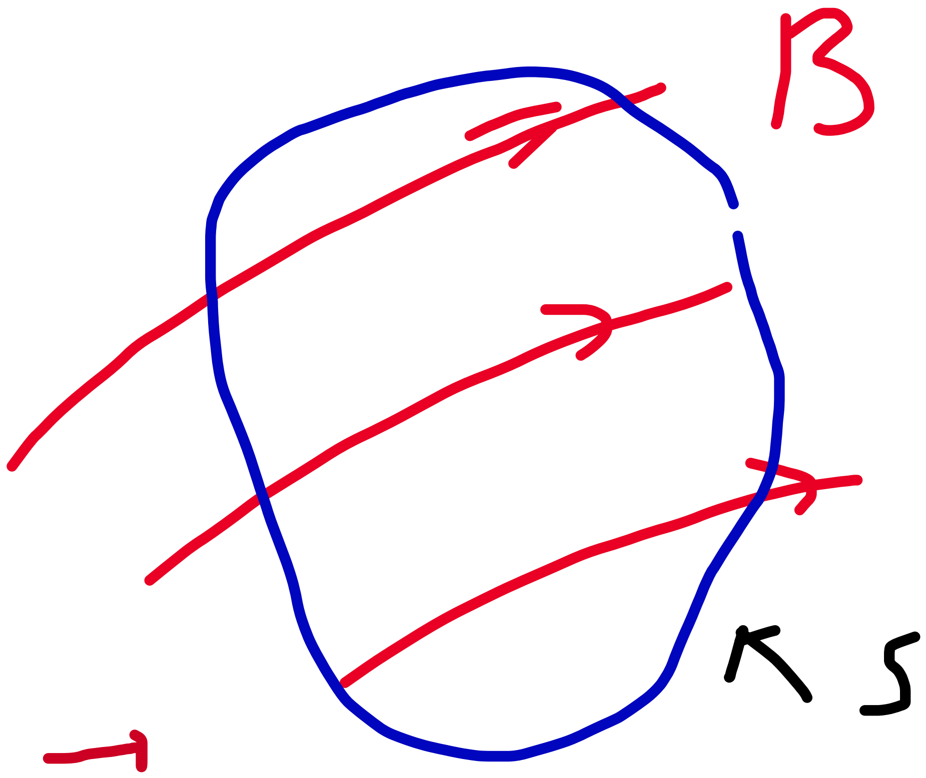
$$\omega_R^2 = \frac{1}{LC} - \frac{2}{t^2}$$

Maxwell's equations



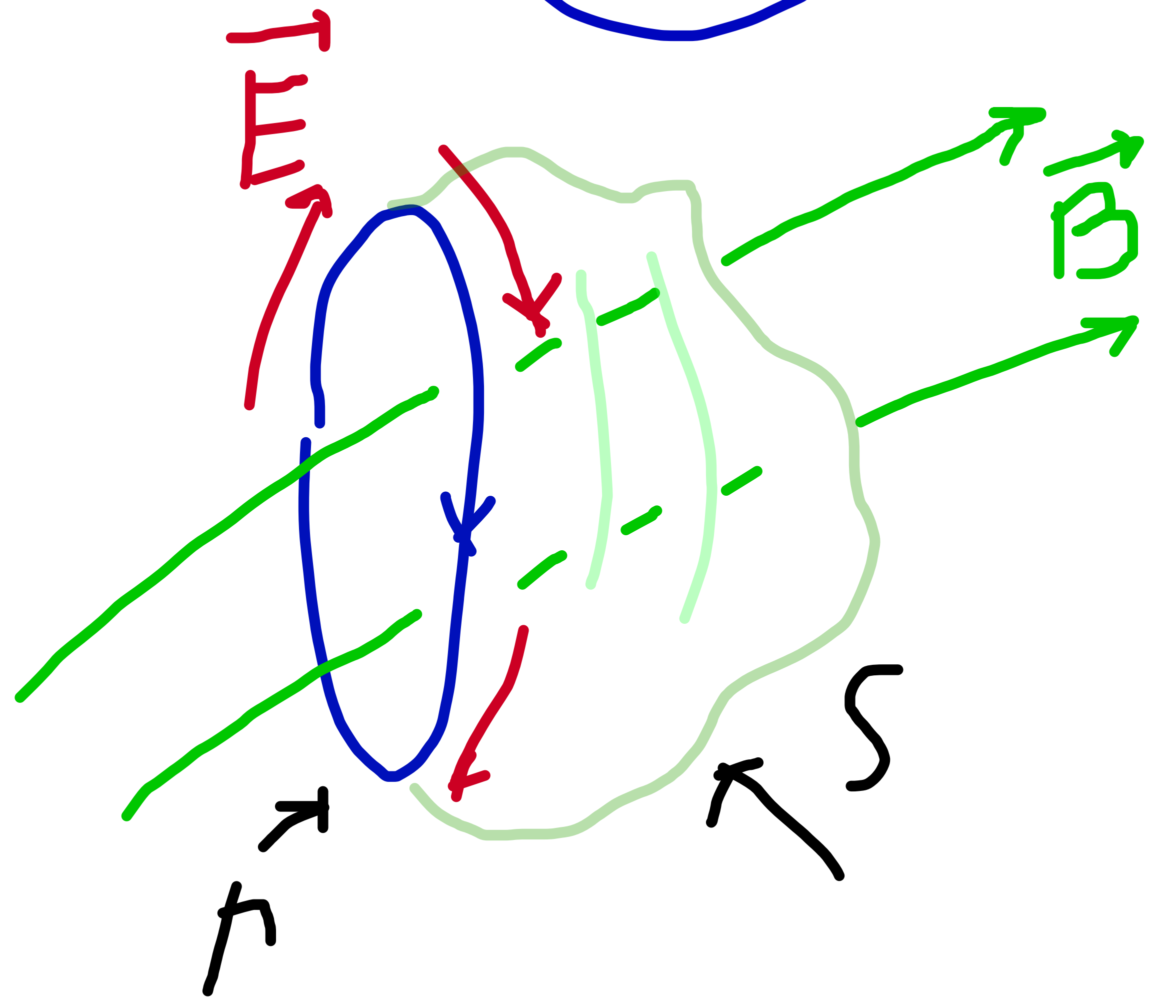
$$\oint_S \vec{E}\cdot d\vec{A} = Q/\varepsilon_0$$

Gauss's law

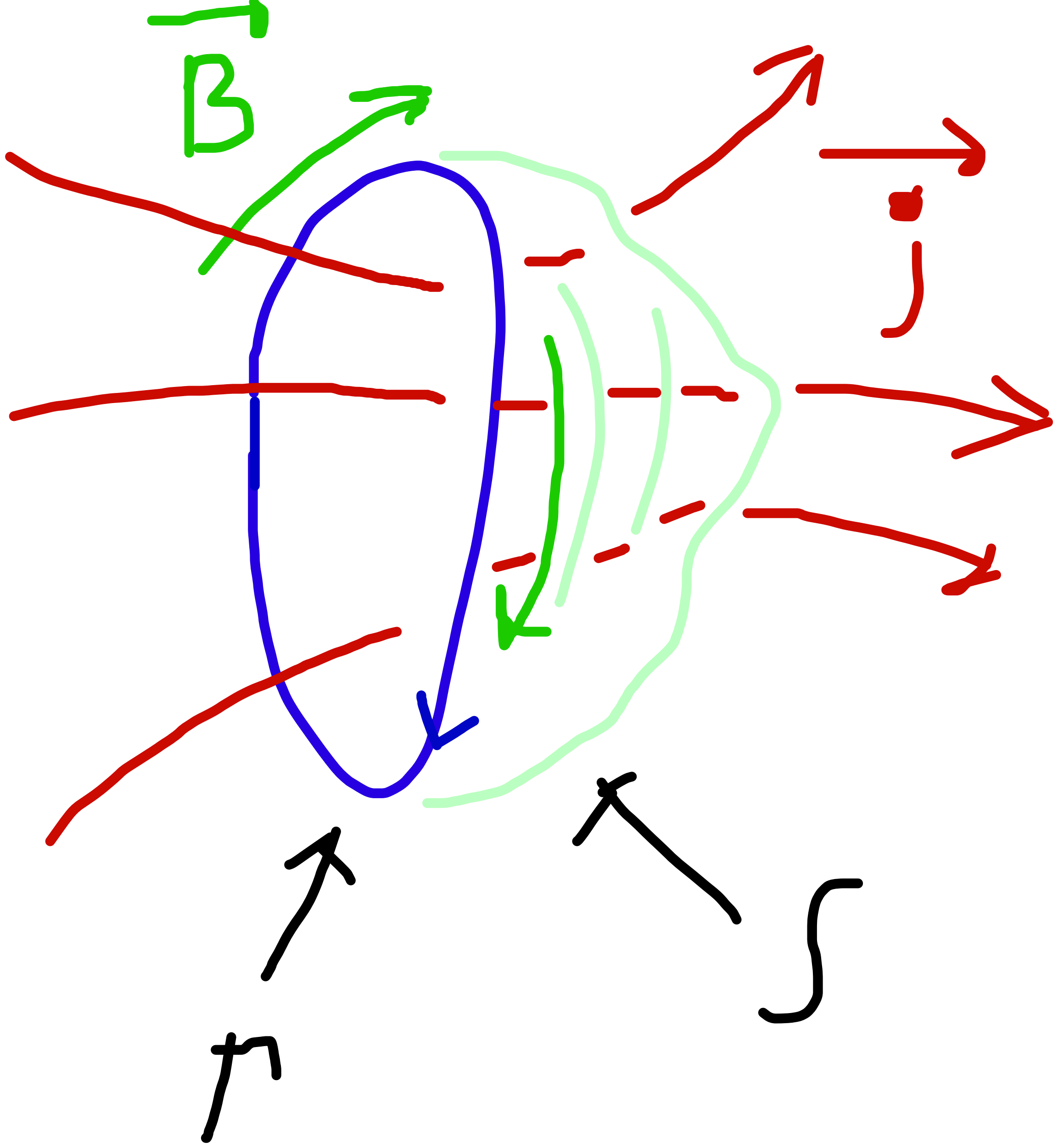


$$\oint_S \vec{B}\cdot d\vec{A} = 0$$

Faradey's law



$$\oint_\Gamma \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

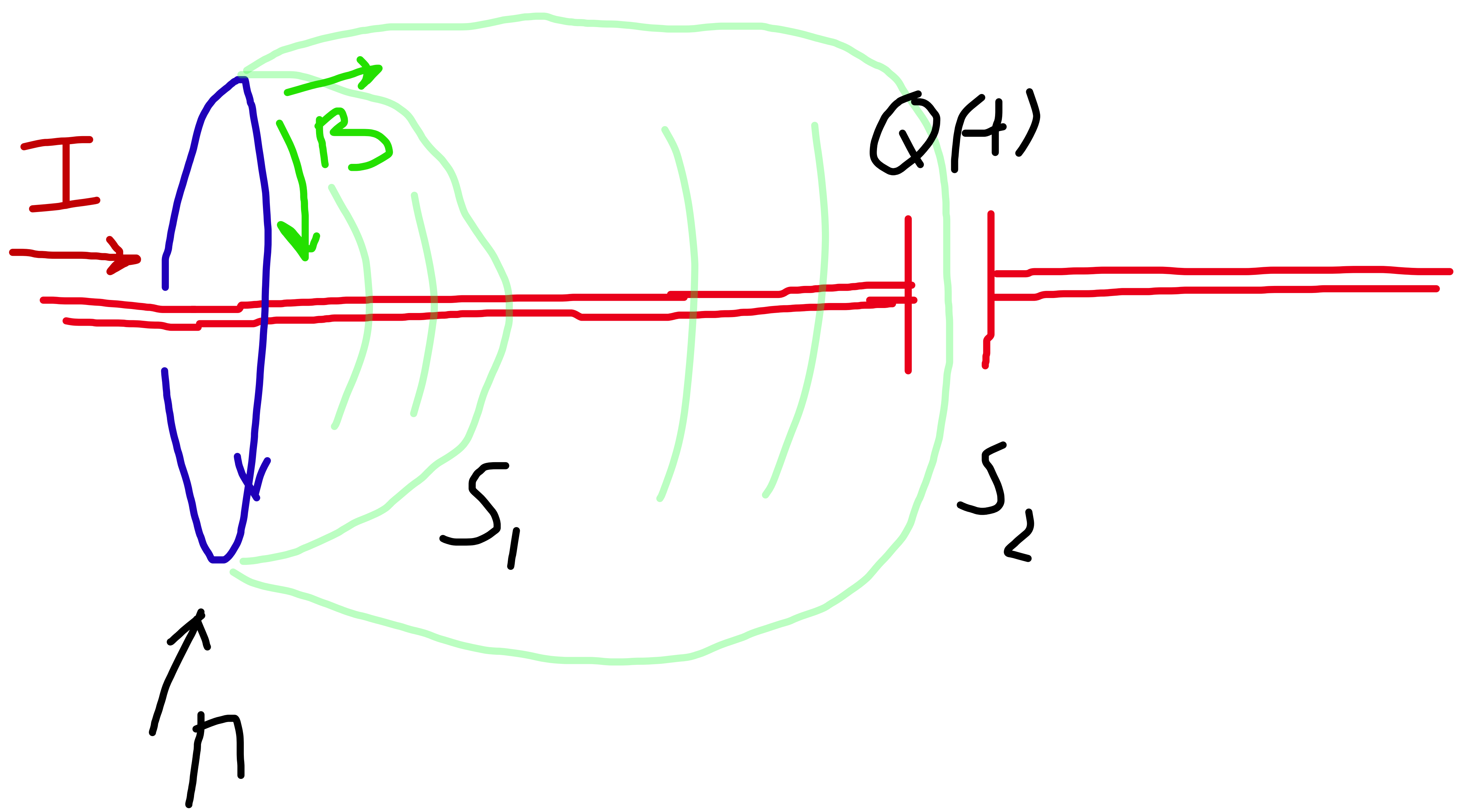


$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \mu_0 \int_S \vec{j}\cdot d\vec{A}$$

Ampere's law

no $\vec{E}$!

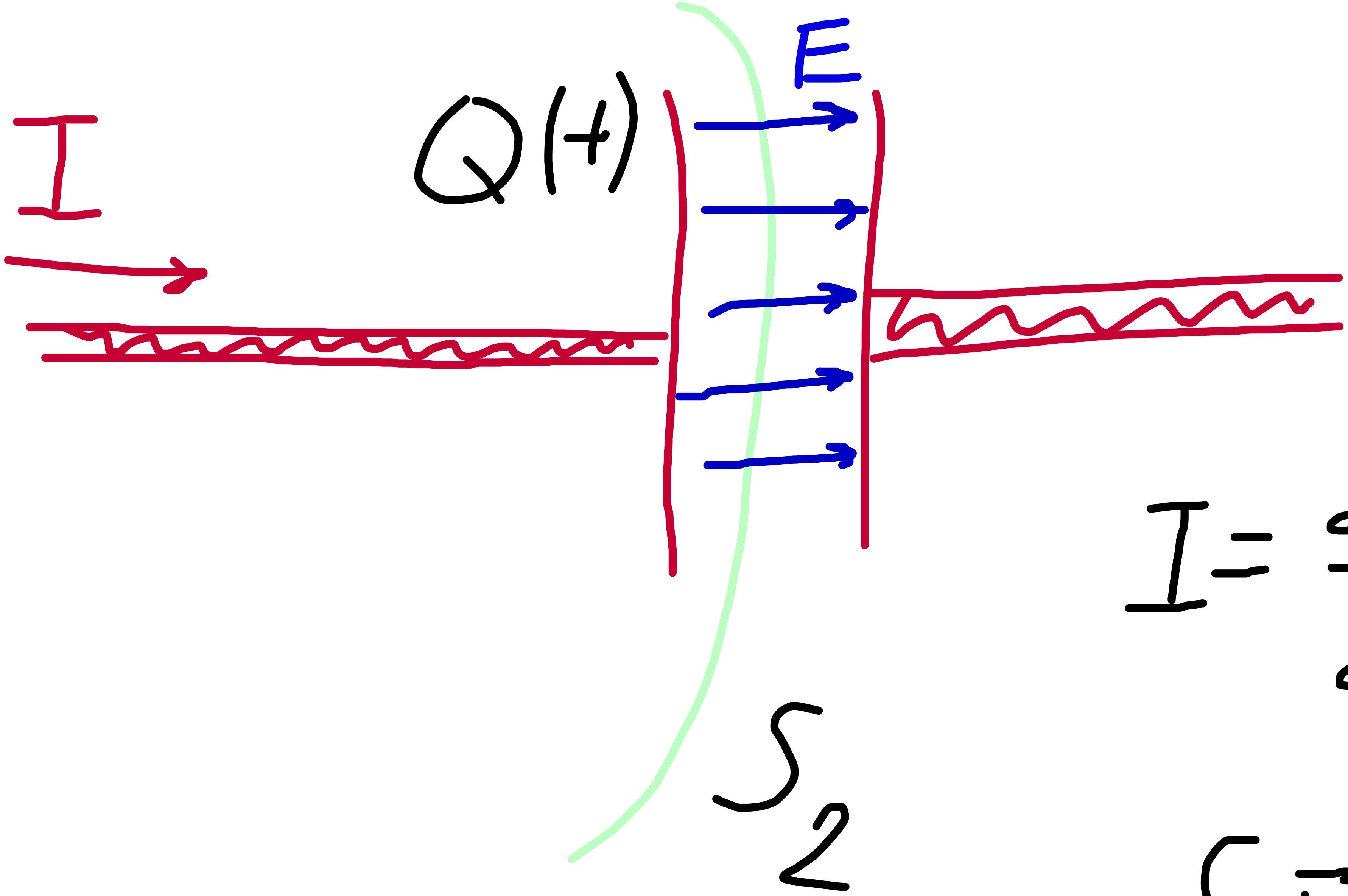
something is not Right



use S_1: $\oint_\Gamma \vec{B}\cdot\vec{dl} = \mu_0 I$

use S_2: $\oint_\Gamma \vec{B}\cdot\vec{dl} = ?$

no current through S_2



$$I = \frac{dQ}{dt}$$

Gauss

$$\int_{S_2} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$I = \varepsilon_0 \frac{d}{dt} \int_{S_2} \vec{E} \cdot d\vec{A}$$

$$\oint_{\Gamma} \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot \vec{dA} + \mu_0 \int_S \vec{j} \cdot \vec{dA}$$

In empty space

$$\boxed{\begin{aligned} &\oint \vec{E} \cdot \vec{dA} = 0 + Q/\varepsilon_0 \\ &\oint \vec{B} \cdot \vec{dA} = 0 \\ &\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int \vec{B} \cdot \vec{dA} \\ &\oint \vec{B} \cdot \vec{dl} = \varepsilon_0 \mu_0 \frac{d}{dt} \int \vec{E} \cdot \vec{dA} + \mu_0 \int \vec{j} \cdot \vec{dA} \end{aligned}}$$

material Eq. $\rightarrow$ $\boxed{\vec{j} = \sigma \vec{E}}$