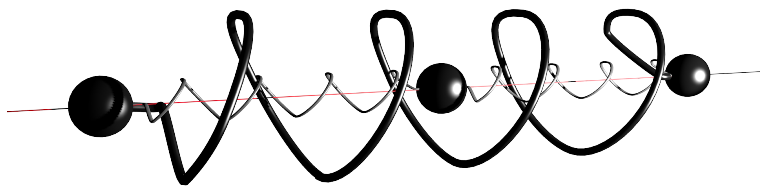
# EXAM 1. Tuesday, March 18-20, 2025, take home. Due on Thursday, March 20, 12:45pm.

**Problem 1.** *2001-Fall-CM-U-3*

Three spheres of equal mass $m$ are constrained to move in one dimension along the line connecting their centers. The three spheres are connected by three springs, as shown in the figure. The three springs have equal spring constants $k$ In equilibrium, all three of the springs are at their respective natural lengths.

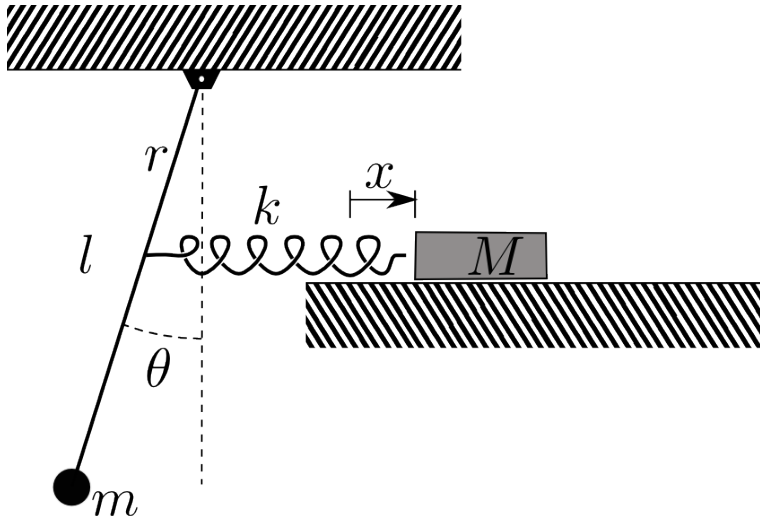
1. Choose a reasonable set of coordinates and find the equations of motion.
2. Find the normal-mode frequencies.



**Problem 2.** *2001-Spring-CM-U-2*

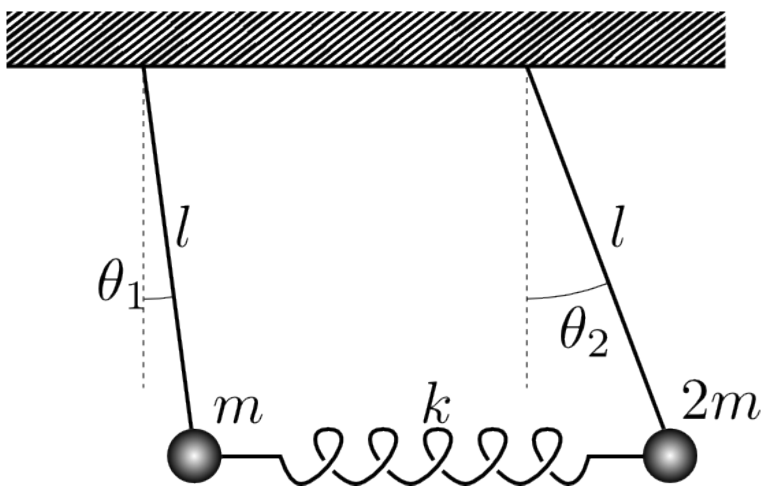
Consider the system, pictured below, which consists of a ball of mass $m$ connected to a massless rod of length $l$. This is then joined at point $r$ to a spring of spring constant $k$ connected to a block of mass $M$ which rests on a frictionless table. When $\theta = 0$ and $x = 0$ the spring is unstretched.

1. Write the Lagrangian for the system in terms of the coordinates $\theta$ and $x$ assuming small displacements of the pendulum
2. Write the equations of motion for the system.
3. Making the simplifying assumptions that $M = m$, $l = 2r$, and setting $k/m = g/l = \omega_0^2$, find the normal-mode frequencies of this system for small oscillations in terms of $\omega_0$
4. Assuming the same conditions, calculate the ratio of amplitudes (for each of the two masses) of the two normal modes of oscillation. In other words, find the normal modes.



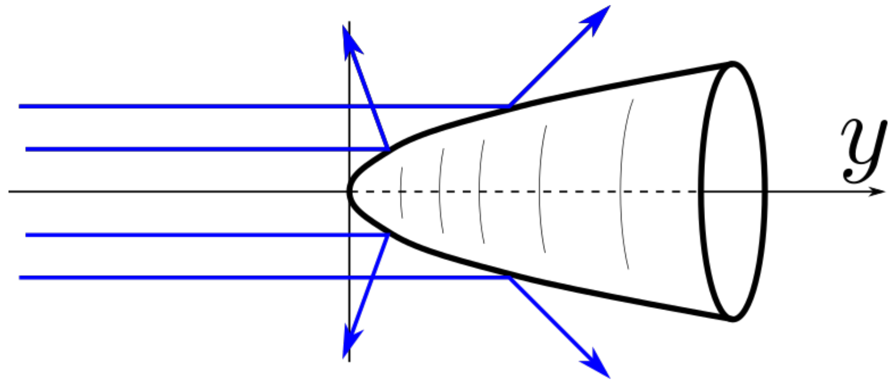

**Problem 3.** *1983-Spring-CM-G-6*

Two pendula made with massless strings of length $l$ and masses $m$ and $2m$ respectively are hung from the ceiling. The two masses are also connected by a massless spring with spring constant $k$. When the pendula are vertical the spring is relaxed. What are the frequencies for small oscillations about the equilibrium position? Determine the eigenvectors. How should you initially displace the pendula so that when they are released, only one eigen frequency is excited. Make the sketches to specify these initial positions for both eigen frequencies.




**Problem 4.** *Paraboloid scattering.*

An immovable paraboloid (which was produced by rotating a parabola $y = \frac{x^2}{4R}$ around the $y$ axis), elastically scatters particles that are coming from $y = -\infty$. Find the differential scattering cross-section $d\sigma$.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