Homework 1. Due Tuesday, January 20, 12:45pm

Problem 1. *Problem 5.13 from Classical Mechanics by J. R. Tylor*
The potential energy of a one-dimensional mass m at a distance r from origin is

$$U(r) = U_0 \left(\frac{r}{R} + \lambda^2 \frac{R}{r} \right)$$

for $0 < r < \infty$, with U_0, R, and λ all positive constants.

1. Find the equilibrium position r_0.
2. Let x be the distance from equilibrium and show that, for small x, the potential energy has the form $U = \text{const.} + \frac{1}{2}kx^2$.
3. What is the angular frequency of the oscillations in terms of U_0, R, and λ (and, of course, m)?

Problem 2. *Problems 7.30 from Classical Mechanics by J. R. Tylor*
Consider a pendulum of length l suspended inside a railroad car that is being forced to accelerate with a constant acceleration a.

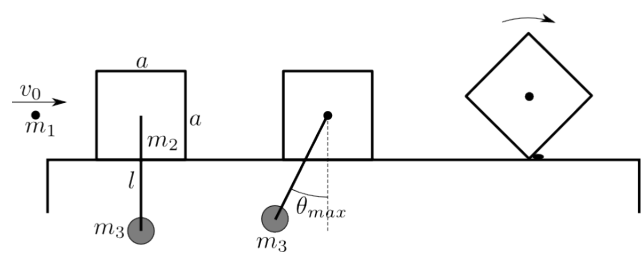
1. Write down the Lagrangian for the system and the equation of motion for the angle ϕ — the angle between the pendulum and the vertical.
2. Find the equilibrium angle ϕ_0.
3. Show, that the equilibrium is stable.
4. Find the angular frequency of the small oscillations around the equilibrium.

Homework 2. Due Tuesday, January 27, 12:45pm

Problem 1. *1994-Spring-CM-U-1*
A bullet of mass m_1, is fired with velocity v_0 at a solid cube of side a on the frictionless table. The cube has mass of m_2 and supports a pendulum of mass m_3 and length l. The cube and pendulum are initially at rest. The bullet becomes embedded in the center of the cube instantaneously after the collision.

1. If θ_{max} is the maximum angle through which the pendulum swings, find the velocity v_0 of the incident bullet.

2. When the pendulum's swing reaches the maximum angle, the pendulum's string is cut off. Therefore the solid cube slides and hits a small obstacle which stops the leading edge of the cube, forcing it to begin rotating about the edge. Find the minimum value of v_0 such that the cube will flip over. Note that the moment of inertia of the cube about an axis along one of its edge is $\frac{2}{3}Ma^2$. Assume the bullet is a point located at the center of the cube.



Problem 2. *2001-Spring-CM-U-3*
A particle with mass m, confined to a plane, is subject to a force field given by

$$\vec{F}(\vec{r}) = \begin{cases} -k\vec{r}, & \text{for } |r| \leq a \\ 0, & \text{for } |r| > a \end{cases}$$

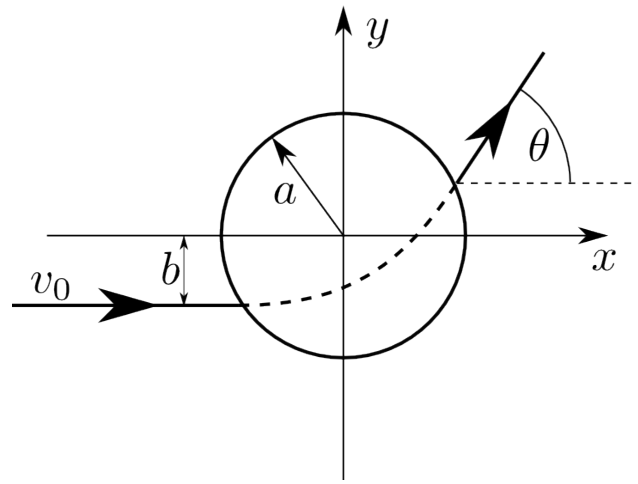
The particle enters the field from the left horizontally with an initial velocity v_0 and a distance $b < a$ below the force center as shown.

1. Write down the equation of motion for $r \leq a$ in Cartesian coordinates in terms of x, y, and $\omega = \sqrt{k/m}$

2. Give the trajectory of the particle when $r < a$.

3. For $v_0 = a\omega$ find the coordinates of the particle as it exits the region of non-zero force field.

4. For $v_0 = a\omega$, find the deflection angle θ of the departing velocity at the exit point.


y
θ
a
x
b
v_0

Homework 3. Due Tuesday, February 3, 12:45pm

Problem 1.
A random process has a probability density for a single measurable x given by

$$\rho(x) = Ae^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

where x_0 and σ are known numbers.

1. Find the constant A.
2. Find the average value of x, $\langle x \rangle$.
3. Find the standard deviation $\sqrt{\langle (x - \langle x \rangle)^2 \rangle}$ (be careful with the order of operations).

Problem 2.
A particle in a beam has velocity V and internal energy ϵ. The particles randomly disintegrate inside the detector into two particles of mass m_1 and m_2. The detector measures the kinetic energy of a particles of mass m_1 irrespective of the direction of their velocities.

1. What is the average kinetic energy measured by the detector?.
2. Find the standard deviation of the kinetic energy measured by the detector?

Problem 3.
A particle in a beam has velocity V and internal energy ϵ which is less than the kinetic energy of the particles in the beam. The particles randomly disintegrate inside the detector into two particles A and B of equal mass m. The detector measures the angle θ_L the velocity of the particle A makes with the velocity V of the beam.

Find the average of the tangent of the angle measured by the detector. Express the answer in therms of the speed of the particle A in the center-of-mass frame and the velocity V.