

Collusion Facilitating and Collusion Breaking Power of Simultaneous Ascending Price and Descending Price Auctions*

Alexander L. Brown, Charles R. Plott, and Heidi J. Sullivan
California Institute of Technology

Abstract

This paper demonstrates that a robust, tacit collusion evolves quickly in a “collusion incubator” environment, but is destroyed by the simultaneous, descending price auction. Theories of collusion-producing behavior, along with the detail of the states on which strategies are conditioned, lead to a deeper understanding of how tacit collusion evolves and its necessary conditions. These theories explain how the descending price auction destroys the collusion. The experiments proceed by conducting simultaneous ascending price auctions in the collusion incubator. Then, once the tacit collusion developed, changing to the descending auction. The change moved prices from collusive levels to near competitive levels.

* The financial support of the National Science Foundation, the Caltech Laboratory for Experimental Economics and Political Science and the Lee Center for Advanced Networking is gratefully acknowledged. The authors benefited from comments of Jin Li, Katerina Sherstyuk, Ian Krajbich, and an anonymous referee.

1. INTRODUCTION

This paper explores the relationship between the market institutional (auction) environments, the preferences of multiple buyers over multiple items, and the outcomes that can be expected to evolve from the institutions. The paper focuses on how tacit collusion develops and institutions that might prevent it. The “collusion incubator,” a very special economic environment, was constructed to facilitate “tacit collusion” under the continuous, simultaneous ascending price auction. It was successful. Tacit collusion developed and the process of tacit collusion development was studied. Once tacit collusion was firmly developed, the institution was changed to a simultaneous descending price auction, while keeping the underlying, collusion incubator environment constant. The change in institution resulted in the unraveling of the tacit collusion, and the market prices evolved to near competitive levels. The process of “collusion unraveling” is studied and tied to particular features of descending price auctions.

Three features of the study are emphasized. (1) The power of the “collusion incubator” environment to foster tacit collusion under the continuous, simultaneous ascending price auction is studied. The tacit collusion facilitating power of this environment was first established by Li and Plott (this volume) and the property strongly replicates here. (2) The “tacit collusion breaking power” of the continuous, simultaneous descending price auction is identified and established for the first time. (3) The behavior of the market is modeled as a process of equilibrium selection. The data are examined for clues about how the selection takes place and what features might give it robustness properties.

The paper consists of seven sections including this introduction. The second section below reviews background literature. The third section describes the economic and institutional environment. The fourth section describes the experimental design. The fifth section addresses

issues of theory and predictions, beginning with a brief summary of previous results. The sixth section contains the results, and the final section is a summary of conclusions.

At the outset, an introductory comment about the special economic environment employed is needed. The environment is characterized by special “item aligned” and “folded” preferences that will be discussed in detail below. In essence, each buyer can be identified with an item for which the buyer has the unique highest value, thus, the term “item aligned.” In addition, the preferences involve a special type of interdependence in which pairs find themselves in competition over their most preferred items, thus the term “folded.” This pairing directly ties to theoretical models of collusion which generally involve only two buyers and not more. All of this is public information in the sense that unless otherwise stated as a treatment, all preferences were known to all participants. The Li and Plott discovery is that within this special environment tacit collusion evolves quickly under the continuous simultaneous ascending price auction and is remarkably robust in the sense that once developed it was not substantially altered by changes in the information structure.¹ The strategy of this research is to study how tacit collusion develops and to test its robustness against a substantial change in the institutional and informational environment.

For policy, one would like to understand how tacit collusion develops and dissolves in the hope that it would help identify the types of institutions that would cause the system to evolve from the tacit collusive equilibrium to the competitive equilibrium or equally important, evolve from competition to tacit collusion. For market policies, the “remedies” that discourage tacit collusion have desirable properties. On the other hand, cooperation among buyers is a type of solution to a “public goods” problem; a greater understanding of this cooperation may benefit

¹ Li and Plott found one collusion breaking intervention. It involved giving subjects identical valuations. This paper attempts to break collusion without drastically changing valuation structure.

public goods policy. Thus, regardless of the point of view, a deeper understanding of the process of “tacit” collusion and the institutions that discourage it or encourage it are of interest. That fundamental understanding motivates the research.

2. BACKGROUND LITERATURE

Several experiments have attempted to initiate and sustain collusive equilibria in various markets. The first experiments allow subjects to discuss strategy before they trade. Isaac and Plott (1981) find unsuccessful attempts at collusion when subjects were allowed to discuss strategy before entering an oral double auction. The success or failure of attempts to collude are related to the structure of the market institutions (Clauser and Plott, 1993). Isaac and Walker (1985) conclude communication “fosters bid rigging cartels” when subjects have time to communicate between trading periods. Kagel (1995) in his survey of auction experiments notes he has produced lower than competitive prices in auctions with an Isaac and Walker design. Kagel concedes that with one exception “outright collusion has not been reported under *standard* experimental procedures.” That exception concerns one of five trials, is fleeting and never unanimous among subjects.

Studies more recent than Kagel report collusion. By allowing bid matching in a simultaneous ascending auction, Sherstyuk (1999) finds sustained collusion with three buyers and two items with equal and commonly known values. In her next work, the same design produces collusion with private values (Sherstyuk, 2002). Surprisingly, that study also finds collusive equilibria in trials without bid-matching. Sherstyuk hypothesizes that subjects colluded over the repeated playing of the auction. Grimm and Engelmann (2006) identify some tacit collusion with two items and buyers in an ascending clock auction. Kwasnica and Sherstyuk

(2007) find the first systematic evidence of buyer collusion in simultaneous price auctions without communication or bid matching.² Their result does not hold for more than two buyers.

A few studies offer evidence of sustained collusive equilibria with more than two buyers. Kwasnica (2000) reports collusion with five buyers and five objects in an experiment with communication. Phillips et al., (2003) find collusion among six buyers in trials with and without communication. Curiously, buyers in groups of six were better coordinated than buyers in groups of two in their study. Li and Plott (2005) provide a robust collusive equilibrium using a specific valuation structure with eight buyers and eight items. This experiment will replicate this last design and attempt to disrupt the equilibrium.

3. EXPERIMENTAL ENVIRONMENT

3.1 Items and Preferences: the Collusion Incubator

This experiment used the preference environment invented by Li and Plott. For all rounds of all experiments, there were eight buyers and eight items to purchase. Buyers received valuations each round. Their profits were determined by the sum of the difference between their valuation of an item they obtained and its purchase price. In this way, valuations can be thought of as redemption values. Valuations would vary, but the following features were preserved in all rounds:

- i. Each buyer had a strict preference ordering for the eight items.
- ii. No item had the same nominal value for any two buyers.
- iii. For every item, there is exactly one buyer that valued it n th ($n=1,2,\dots,8$) in his preference ordering.

² Kwasnica and Sherstyuk also find some evidence of retaliation by buyers facilitating collusion. Retaliatory strategies will be a central theme in this paper and is discussed in detail in the theory (section 5) and results (section 6) sections of this paper.

- iv. (Buyer Aligned) If a buyer valued an item n th in his preference ordering, then he had the n th highest valuation of that item among all buyers. In particular, a buyer had the highest valuation for his most preferred item among all buyers.
- v. (Folded Preferences) If buyer i valued buyer j 's most preferred item n th, then buyer j valued buyer i 's most preferred item n th.

Table 1 provides a sample valuation table.

From the symmetry of the preference relations, it is possible to define “partners”. Buyer i is buyer j 's partner if he values j 's most preferred item second in his preference ordering. By condition (v), j will value i 's most preferred item second most in his preference ordering. By condition (iv), j will value his most preferred item at a higher nominal value than any other buyer. Buyer i will value that item higher than all buyers but j . Using condition (iv), an analogous statement can be made about buyer i 's most preferred item. In Table 1, buyers 121 and 122, 123 and 124, 125 and 126, and 127 and 128 are partners.

Interrelationships exist among the properties. The buyer-aligned property allows a type of “ownership” to an item that is public knowledge. In a sense, it is clear which item is buyer i 's.³ The folded preference structure interacts with partners to create a type of coordination of actions. Partners can achieve a better outcome by not bidding on each other's item. Next, those pairs of buyers who value each other's most preferred item third may evolve into a partner relationship and thus can recognize the possibility of improved outcome by not bidding on each other's item. Once this process unravels at the eighth level, total tacit collusion is reached.

Most likely, this pair-wise unraveling improves the chance of collusion among eight buyers. Most theory (Ausubel and Schwartz, 1999; Milgrom, 2000; Brusco and Lopomo, 2002;

³ This assumption appears elsewhere in the literature. Pesendorfer (2000) constructs a “ranking mechanism” to assign items to buyers in a cartel in an incentive compatible and asymptotically ex-ante efficient way. Trivially, that mechanism assigns each buyer in this experiment his most preferred item.

Engelbrecht-Wiggans and Kahn, 2005; Albano et al, 2006 [1]; Albano et al, 2006 [2]) and experiments (Kwasnica and Sherstyuk, 2007; Grimm and Engelmann, 2006) suggest collusion occurs between two buyers but not more. Experimental studies confirm the difficulty of coordination in groups of more than two (Van Huyck et al, 1990), albeit in different types of games. This collusion incubator's structure allows buyers to coordinate with their partner first, make real profits from that cooperation, and continue. The process towards total collusion may occur through steps of bilateral cooperation.

3.2 Institutions

This experiment will use both a simultaneous ascending price auction (SAPA) and simultaneous descending price auction (SDPA). Both auctions are called simultaneous because once either auction begins, a buyer can bid on any of the eight items at the same time. In the SAPA, a buyer can continue to bid on any of the eight items until the auction closes. Then he cannot bid on any item. In the SDPA, the descending price clock -- the price at which a buyer can purchase any of the eight items -- starts at the same price and counts down at the same rate for each item. (Thus, at any moment, any previously unpurchased item can be purchased at the same price.) Once a buyer bids on any of the eight items, it is purchased. Table 2 shows the procedural differences between each auction.

In the SAPA, the reserve bid was 10 and each buyer was allowed to submit bids in increments of 10 on any item.⁴ The round ended when there were no bids placed for a certain amount of time (usually 30 seconds). The SDPA started at 900 and decreased at a rate of 2 francs per second. Subjects interacted with the auction software entirely using the mouse on their computer. Buyers bid by clicking arrows to determine the amount of their bid, clicked the

⁴ This value is important: the large difference between reservation price and competitive price creates large gains from collusion. Great gains foster collusion even when not predicted by theory (Sherstyuk, 2002).

items they wished to bid on, and clicked a button to confirm in the SAPA. In the SDPA, they only clicked on the items and a button to buy because the clock determined the bid in that auction. The computer's internal clock was precise enough to avoid any simultaneous bidding. Subjects learned immediately in either auction when an item was bid on or purchased.⁵

3.3 Information

Before the experiment began, subjects were briefed on the ascending price auction and given valuation sheets. They were told the valuation sheets were common to all subjects. They were told that each subject had a specific ID that would not change during the session. It was not disclosed that the experiment would switch to a descending price auction. That type of auction was not mentioned until the period before the auction occurred. In experiments #4 and #6 in which subject IDs were later removed, subjects were not told of this change until the period before the IDs were removed. In that condition, subjects were still given valuation sheets without subject numbers to show that the valuation structure was still intact.

4. PROCEDURES AND DESIGN

4.1 Experimental Procedures

Six experiments were run at the EEPS laboratory at Caltech. These experiments lasted between 1.5 and 2.5 hours. Subjects were recruited from Caltech undergraduate economics courses. Their pay ranged from \$25-\$75 for one experiment. Subjects participated in both auctions using computer software in the laboratory. Dividers were used to prohibit a subject from observing the screens of others. The subjects first learned how a SAPA auction worked and were not told there would be any changes to the design of the experiment. They were asked to record their purchases in the auction to reinforce the concepts of price and value. They were given a screenshot of the auction software with arrows so they could understand all aspects of the

⁵ See appendix 2 for screenshots of the bidding mechanisms.

software. The screenshot as well as subject instructions are available in Appendix 2. Before each round, subjects were given a valuation sheet with all subject values for each item (similar to Table 1). These values were in francs and subjects were paid in cash at the end of the experiment at the rate of 1 cent = 2 francs.

4.2 Experimental Design/Environment

Table 3 shows more detail about the experimental environment.

After experiments 1-3 had been run, it was suggested that some cooperative bids in the SDPA occurred because subjects earned reputations over multiple periods. To examine this belief, the software was modified to allow the removal of subject IDs. This modification was used in experiments #4 and #6. It was designed to be used after subjects had experienced a few periods in the SDPA with subject IDs.

After collusion, the outcome where each subject obtains his most preferred item at its reservation price, had occurred for three or more periods, subjects were told about the SDPA and the computers were switched to that auction. All remaining periods would be in the SDPA. If subjects were unable to sustain collusion and there was less than forty minutes remaining in the experiment, subjects were also switched to the SDPA. This occurred in experiments #1 and #6. In experiments #4 and #6, to examine effects of reputation, subject IDs were removed from the computer screen and the valuation sheets.

5. MODELS AND SOLUTION CONCEPTS

In the experiment, buyers encounter unique valuations of items in a complete information setting. Preference information is public (unless otherwise indicated for special exercise). This environment differs from other simultaneous ascending price models that study environments with private information (Brusco and Lopomo, 2002; Engelbrecht-Wiggans and Kahn, 2005),

budget constraints (Milgrom, 2000; Benoit and Krishna, 2001), and homogenous items (Ausubel and Schwartz, 1999). These aforementioned models have one equilibrium, while similar models applied to the environment studied here have an infinite number.⁶

Two allocations are of special significance and will become part of equilibria depending on the institution. In the simultaneous ascending price auction, a continuum of equilibria fall between the two allocations. In the simultaneous descending price auction, only one equilibrium exists. The competitive (seller preferred) allocation has each buyer receiving his most preferred option and paying its second highest value. The tacit collusive (buyer preferred) allocation has each buyer receiving his most preferred item and paying 10 francs (the minimum possible bid).⁷

Equilibrium strategies supporting allocations as equilibria depend on concepts of “retaliation” and “punishing” strategies, techniques which are available to buyers in the SAPA but not in the SDPA. Known preferences allow a description of “a buyer’s item” in the sense that it is theoretically clear that a buyer will end up buying the item that he prefers most. With this in mind, two concepts of retaliation can be developed for theoretical purposes. It is important to notice that these concepts do not depend on prices or the level of bids and instead depend only on the concept of “a buyer’s item”. First, narrow retaliation has a buyer placing a bid on the most preferred item of anyone who bids on the buyer’s item. Broad retaliation has the buyer bidding on all items if anyone bids on his most preferred item. A third concept, passive response involves the buyer bidding on his most preferred item to a level where no other buyer would value it.

⁶ Other differences between the auction institutions discussed in this paper and some of the theoretical literature is that this auction format features multiple items with simultaneous bids as opposed to a single unit auction. The greater number of items may make collusion easier. However, McCabe et al., (1990) suggest single unit results can be generalized to multi-unit auctions.

⁷ The buyer preferred allocation also has the property that it is joint payoff maximizing for all buyers and fully efficient. Additionally, each buyer is receiving roughly the same level of profit in the allocation, perhaps appealing to a sense of distributive justice. However these properties still remain in the descending price auction, where the buyer preferred allocation cannot be obtained.

The following theorems apply to the specific auction environments used in this paper.

Theorem 1. Under the SAPA, the competitive allocation (seller preferred allocation) can be supported as a Nash Equilibrium. This is defined as the competitive (seller preferred) equilibrium.

Proof: Intuitively, if each buyer bids his valuation on all items other than the buyer's own item and bids the second valuation plus epsilon on his most preferred item, there is neither incentive nor retaliatory reason to bid further. This is the standard result for ascending (and second price) auctions (Vickrey, 1961). A formal statement is found in Lemma 3 that provides one set of strategies that produce the seller preferred allocation as an equilibrium.⁸

By contrast to Theorem 1, the following Theorems 2 and 3 draw on retaliation possibilities. If all buyers follow retaliatory strategies, and if all buyers bid the minimum (or any other level) on their item and only their item no buyer has an incentive to change his behavior.

Theorem 2. Under the SAPA, the tacit collusive (buyer preferred) allocation can be supported as a Nash Equilibrium either with broad or narrow retaliation. This paper defines either allocation under either form of support as the tacit collusive (buyer preferred) equilibrium.

Proof: Lemma 1 proves the buyer preferred allocation can be sustained as an equilibrium when all buyers play broad retaliation strategies. Corollary 1 proves it can be sustained when all buyers play narrow retaliation strategies. Basically, if any buyer can obtain his most preferred item at the reservation price, and will face retaliation for bidding on another's item, it is a best-response to bid the reservation price on his item. Bidding on another's item will not win him that item (the buyer who prefers it most will take it back) and he will have to bid a higher price to obtain his most preferred item (because the retaliation featured a bid on his most preferred item).

⁸ See Appendix 1 for all Lemmas and Corollaries mentioned in this section.

Theorem 3. A continuum of equilibria exist between the competitive and the tacitly collusive equilibria.

Proof: Lemma 4 shows any allocation with prices on items between the reservation price and second highest valuation of that item can be sustained as an equilibrium with appropriate retaliatory bidding.

A dramatic difference exists between the equilibrium properties of the SDPA and the SAPA.⁹ It is made clear from the following theorems.

Theorem 4. The competitive (seller preferred) allocation can be supported as a Nash equilibrium under the SDPA. As before, this will be called the competitive (seller preferred) equilibrium.

Proof: Lemma 5 shows the seller-preferred allocation can be sustained as an equilibrium if all buyers bid on all non-preferred items that fall below their valuation, and epsilon above the second highest valuation of their most preferred item.

Theorem 5. Under the SDPA, the tacit collusive (buyer preferred) allocation cannot be supported as an equilibrium.

Proof: Follows directly from Lemma 6.

In a single stage auction,¹⁰ the SDPA does not have equilibria below the second valuation, compared to a continuum of equilibria in the SAPA. The seller-preferred equilibria

⁹ Comparisons between ascending and descending price auction have a rich history in economics literature. When valuations are independently drawn from the same distribution and unknown to other buyers - which does not apply in these environments (this auction has personal values that are known with certainty to all buyers before bidding) - auction theory predicts the ascending and descending auctions should give the same revenue to the seller of an item (Vickrey, 1961). Experiments contradict this prediction; descending auctions generally have lower prices in those cases (Cox et al 1982; Cox et al 1983; Kagel 1995).

¹⁰ Additional equilibrium concepts, which will not be explored here, can be derived from the fact that in these experiments, subjects participate in the SDPA over multiple rounds with an indefinite end. This results in far more possibilities for equilibria than the single stage equivalent. This paper does not model any of these multiple stage equilibria. It is generally believed repeated play will promote collusion (Fouraker and Siegel, 1963; Sherstyuk, forthcoming).

exists (Theorem 4) under both institutions. Notice intuitively that any outcome in which a buyer does not receive his item, either another buyer loses on the purchase, or he could do better by bidding higher. If any buyer receives his most preferred item at a level above its second highest valuation, he could do better by bidding at its second highest valuation and still acquire his item with certainty. If any buyer receives his most preferred item at a level below its second highest valuation, the buyer who values it second could do better by bidding epsilon above his bid. A formal proof of this argument is given in Lemma 6.

Theorem 5 relies on fact that in the SDPA each bid is final.¹¹ The price falls and once a buyer enters a bid for an item, he purchases it at that value. If a buyer has already purchased his most preferred item or purchases at the same time as another item, he has eliminated the possibility of retaliation by other buyers for that round. Even if retaliation were possible, the finality of bids creates a real cost. Each retaliation costs its deliverer a certain loss if the buyer bids more than his own value. In auctions in which bids are not final such as the SAPA, retaliation may be costless. A buyer may bid on an item at a potential loss to himself, but the buyer who values the item most may outbid him. In the SDPA, this cannot occur. The equilibria discussed in Theorems 2 and 3 in the SAPA relied on retaliation strategies.

In summary, the model suggests important properties of the SDPA: the finality of bids, punishment with certain costs, no single stage buyer preferred equilibria; are all characteristics the SAPA does not have. Such features lead to an understanding of the facts that are outlined in the next section that tacit collusion develops in the SAPA and is destroyed when the SDPA is imposed.¹²

¹¹ This property is analogous to the sealed-bid ascending auction, which is suggested to be more robust against collusion than the increasing ascending auction (Robinson, 1985).

¹² A similar theoretical result, Albano et al [1], [2] (2006) find that the Japanese ascending clock auction has fewer equilibria than the simultaneous ascending price auction, and is more robust against collusion.

6. RESULTS

Figures 1A-1F show average price levels over all rounds in each of the six experiments and illustrate the patterns of results. Under the simultaneous ascending price auction (SAPA), the prices evolved downward to the buyer preferred allocation. Each buyer receives his most preferred option at the lowest possible price. The institutional change to the simultaneous descending price auction (SDPA), indicated by the leftmost vertical bar, causes the prices to move from the buyer preferred to a region near the seller preferred. Further analysis is divided in two subsections of results. The first section addresses the SAPA, and the second section is focused on the SDPA.

Part 6.1: The Simultaneous Ascending Price Auction

Eight results are listed. Together they provide a precise analysis of the pattern of results together with an empirical explanation of why the patterns are seen. The first result states that an equilibrium is attained. The next seven results show how that equilibrium result is attained through a series of punishing and retaliatory strategies aimed at creating an incentive for all buyers to behave in a tacitly collusive manner.

Result 1-1: Within the collusion incubator environment, the SAPA reaches the buyer-preferred equilibrium and tends to stay at the buyer-preferred equilibrium once reached.

Support. The buyer-preferred equilibrium is the buyer-preferred allocation supported by equilibrium consistent behavior. Essentially, the strategy is for every buyer to bid the minimum possible bid on “his” item and to bid on no other item. Theorem 2 states that such a correspondence between allocation and strategy happens if and only if all buyers receive their

most preferred allocation at the minimum price. Thus, it need only be shown that the buyer-preferred allocation occurs and is sustained.

The buyer-preferred allocation is reached but not necessarily sustained in five of six experiments and in the sixth it is reached for all items but one. In three experiments, the equilibrium is sustained without movement for more than three periods and in the other three, the movement is easily explained. Figures 1A-1F show the average sale price in each round of each experiment. For all rounds, the buyer preferred equilibrium is 10. In experiment 2 rounds 9-11, experiment 3 rounds 9-11, experiment 4 rounds 8-11, experiment 5 round 15, and experiment 6 rounds 16-17, the average price is 10. In all cases, each buyer obtained his most preferred item at the reservation price. In experiment 1 round 10, all but one buyer obtained their most preferred item at that price. The average price for that round was 11.25.

In three experiments, time constraints (i.e. that the intuitions were switched if less than 40 minutes remained in the experiment) prevented the observation of subjects staying at the buyer-preferred equilibrium for three rounds. In experiment 1, the subjects were close to the buyer-preferred equilibrium before the auction institution switched. In experiment 5, subjects were under an average price of 20 in three other rounds before the switch. In experiment 6, round 18, the movement away from the buyer-preferred equilibrium can be explained by the bids of one subject, defined later in this paper as a “maverick”, a type of subject that plays an important role in the analysis.□

The data indicate the buyer-preferred allocation occurs and is sustained, supporting the idea of an equilibrium being reached. However, when each buyer obtains his most preferred item at the reservation price, it is not known what type of strategy he would use if another buyer had bid on his item. It is suspected buyers would retaliate against other buyers as in Theorem 2,

but other strategies such as passive response can support a weakly dominant Nash Equilibrium. Evidence of retaliatory bidding in the bid functions is studied in results 1-6, 1-7, and 1-8. The next result establishes the existence of a convergence process. Prices and allocations of items move to the buyer-preferred level over time. The nature of the movement and the dynamics of strategy changes are addressed in the Result 1-3.

Result 1-2: Within the collusion incubator environment, the SAPA is characterized by a convergence process across rounds detectable as movement from near the seller preferred allocation toward the buyer preferred equilibrium through price decreases, a decreasing number of bids and decreasing time duration of the auction.

Support. Figures 1A-1F show the average sale price in each round of each experiment. In all six figures, there is a clear downward trend before the switch to the SDPA. Figures 2A-2B group all experiments together and show the number of bids and duration of the SAPA for each round. Figures 2A-2B have a downward trend similar to figures 1A-1F, although not as profound. The visual impression of a downward trend is captured by the regressions shown in Table 4. Table 4 shows three regressions on average price, duration of auction and bid number over all rounds. Allowing for dummy variables for each experiment,¹³ all three dependent variables are negatively correlated with round (see row labeled “round” on table 4 for coefficients). Thus average price duration and number of bids all decrease as the round number increases. All three results are significant at the 0.001 level. Uncharacteristically, the average price went up in round 18 after staying at the buyer-preferred equilibrium for two rounds in

¹³ There is no dummy variable for experiment 1 because the regression contains a constant. With dummy variables for each experiment and a constant, the regressors would be linearly dependent. Removing a constant and adding a dummy variable increases the significance of each dummy variable and r-squared. Both regressions give the same coefficient and standard error of any other variable.

experiment 6. The regressions show the price increase in round 18 does not significantly weaken result 1-2. One subject is responsible for the price increase in round 18 (see result 1-5 for more detail). □

Central to the analysis that follows is a concept of cooperative behavior. For each round buyers will be classified as exhibiting cooperative or non-cooperative behavior. A buyer exhibits cooperative behavior in a round if all his bids are one of the following¹⁴

- 1) Bids on his most preferred item.
- 2) Bids on the most preferred item of another buyer are placed only if the other buyer has previously bid on his most preferred item in that round (i.e. retaliation).
- 3) Bids above his valuation on another buyer's most preferred item (defined later as punishment).

If all of a buyer's bids for a round do not satisfy the aforementioned definition, he is said to exhibit non-cooperative behavior for that round. Buyers that persistently exhibit non-cooperative behavior will be identified as "mavericks" (see Result 1-5).

Result 1-3: The dynamics of price movements can be described as a shift from non-cooperative behavior to cooperative behavior that is neither abrupt nor instantaneous.

- i. Subjects begin with non-cooperative behavior and switch from non-cooperative to cooperative behavior at different points.**
- ii. Subjects switch from non-cooperative to cooperative behavior at a relatively steady rate.**

¹⁴ There are a few key differences between the paper's definition of a cooperative behavior and Li and Plott's. First, Li and Plott allow a cooperator to bid on someone else's item that has remained at the reservation price for more than 60 seconds; this paper does not. Bidding after 60 seconds on another's item happened rarely in the experiments. In all cases, the item eventually went to the buyer with highest valuation, so the first bid raised prices above the buyer preferred level. Hence, bidding after 60 seconds on another's item appears to be non-cooperative behavior. Second, the third criterion considers "punishment bids" to be cooperative. Usually cooperative punishments are retaliatory and covered by the second criterion. However, a cooperative subject may try to punish a non-cooperative subject to enforce cooperation even when that subject has not bid on his item in the current round.

iii. Prices decrease at a steady rate, reflecting that subjects gradually switch from non-cooperative to cooperative behavior.

Support: (i) For each round, a buyer's bids are classified as cooperative or non-cooperative. Tables 5-10 show whether subjects are classified as cooperative (represented by a 0) or non-cooperative (represented by a 1) for each round in each experiment. The rightmost column of the table shows the total number of rounds each subject was non-cooperative. As the disparate totals indicate, some subjects were more cooperative in each experiment than others. Also, subjects began to first cooperate (indicated by the first 0 in each subject's row) at different points of the experiment. As Table 5 shows, some subjects began cooperatively (125, 126, 128), or began to cooperate after the first 1-2 rounds (124, 127). Others subjects were non-cooperative for a majority of the rounds (121, 122, 123). Each buyer did not exhibit the same type of behavior.

(ii) Figure 3A shows the number of non-cooperative subjects decreases steadily for the first ten rounds over all the experiments. Notice that the trend line decreases at a fairly steady rate. Figure 3B shows the number of non-cooperative subjects over time by experiment for the first four experiments. For each experiment, the decline of non-cooperative subjects is not constant, but still occurs gradually. Figure 3C shows the same results for experiment 5 and 6, which are looked at separately since subjects in those experiments took considerably longer to reach the buyer-preferred equilibrium and hence featured more SAPA rounds. Table 4 shows a regression of round number on cooperative subjects with experiment as dummy variables. The table indicates (first row, fourth column) that moving to the next round increases the number of cooperative subjects by 0.41. The standard error on the coefficient of round is small (0.028), indicating that the coefficient is neither negative ($p < 0.001$) nor greater than one ($p < 0.001$).

Thus, the number of cooperative subjects increases after each round, but increases at a relatively small rate.

(iii) Figures 1A-1F show a gradual decrease in average price over all experiments. Comparing Figure 3B and 3C to Figures 1A-1F, it is clear that as the number of non-cooperative subjects decreases, the closer the price is to the buyer-preferred equilibrium. For instance, figure 3C shows there is one remaining non-cooperative subject after round 9 of experiment 5; figure 1E shows after round 9 average prices never exceed 300. When there were more uncooperative subjects (before round 9), average prices exceeded 300. □

Result 1-4: The changes from non-cooperative behavior to cooperative behavior are permanent and together yield a “regime shift” as opposed to an ephemeral deviation.

Support. Tables 5-10 show the shift in subject behavior from non-cooperative to cooperative is generally permanent – of all the rounds where a subject was cooperative they remained cooperative 93% (351 of 379 instances) of the time. If a subject does regress from cooperative to non-cooperative, it usually is for one period. Table 8 provides an example of these changes in experiment 4. All subjects changed from cooperative to non-cooperative at some point in the experiment, subject 121 and 126 became cooperative immediately. Subjects 121 and 127 regressed from cooperative to non-cooperative in rounds 3 and 6 respectively, but switched to cooperative permanently in the next round. All subjects were observed to have cooperative behavior for at least four periods, indicating their shift is permanent.

The regressions of Table 4 support the idea of a regime shift further. The coefficients on average price are negative and those on cooperative subjects are positive at any reasonable

significance level ($p < 0.001$). There is no sustained trend in the SAPA of an increase in price or decrease in cooperative behavior in any of the graphs and tables.

Table 11 provides a regression using the model of Noussair et al, (1995) to estimate convergence to a price level. It is written as

$$y_{it} = B_{11}D_1(1/t) + \dots + B_{1i}D_i(1/t) + \dots + B_{16}D_6(1/t) + B_2(1-t)/t + u.$$

The term i indicates the particular experiment number, t is the round number in the SAPA, D_i is a dummy variable that takes a value of 1 for values from experiment i and 0 otherwise, and B_{1i} is the origin of a possible convergence process for experiment i . If $t=1$ the value of the dependent variable is equal to B_{1i} for experiment i . B_2 is the asymptote of the dependent variable. The term u is the random error term distributed normally with mean zero. The predicted value of B_2 , the estimated average price that all ascending auctions should converge to, is 116.

Granted, this coefficient is unlikely to be 10 or less (p-value 0.00035), the buyer-preferred level, but it is far less likely to be greater than the seller-preferred level 766.125 (p-value 5.63×10^{-35}). If the seller and buyer-preferred level were the only possible price levels for convergence, the buyer-preferred level is considerably (5.63×10^{30} times) more likely. Hence, if the regression model holds and the experiment were run indefinitely, the decrease in price would converge to a level other than the seller-preferred. If convergence could only occur at the buyer-preferred or seller-preferred levels, it would occur at the buyer-preferred. The shift away from a competitive equilibrium is permanent. \square

An important type of buyer will be defined as a “maverick”, a buyer who acts non-cooperatively for three consecutive rounds after at least five (a majority) other subjects are acting cooperatively. Often the maverick bids up items that were not his highest valued for many

periods longer than other buyers. In some experiments, there appears to be more than one maverick.¹⁵

Result 1-5: All experiments contained at least one maverick.

Support. Tables 5-10 reveal that in all experiments there is always at least one non-cooperative hold-out that is defined as a maverick. For example, in experiment 5 subject 128 is the only person behaving non-cooperatively in rounds 9-14. Tables 5-10 show that the maverick(s) for experiment 1 is 122, for experiment 2 are 125 and 128, for experiment 3 are 121 and 122, and for experiment 4 is 128. In experiment 6, there appears to be 3 mavericks. Starting in round 9, subjects 121, 122 and 125 are the only subjects to be non-cooperative. All three are non-cooperative in rounds 9-11, 13-14 and at least one of them is non-cooperative during rounds 9-15 and round 18.□

Note that once the mavericks(s) exhibit cooperative behavior, the buyer-preferred equilibrium is reached. For example, in experiment 4 round 8, the buyer-preferred equilibrium is reached once number 128 cooperates.

“Non-cooperative bids” will be defined as bids placed by a buyer that are on items that are not the buyer’s highest valued item. Note that the definition of non-cooperative bids includes punishment and retaliatory bidding as special cases of non-cooperative. Thus, a buyer classified as cooperative in a round may have used non-cooperative bids if they were in response to non-

¹⁵ This definition of mavericks avoids a tautological argument. An easy way to identify mavericks would be to see how often they bid, how high they bid, etc. However, identifying mavericks based on how they drive up prices and then making the point that they drive up prices is circular reasoning. Notice that the definition of maverick used here is not based on prices. Instead, the definition is based on whether they bid on other people’s highest valued items without the other people bidding on their highest valued item first. Hence, the paper identifies the presence of a “maverick” that sustains uncooperative behavior longer than others, and can examine whether or not the maverick is important in driving up prices and can undertake the examination without using tautological reasoning. Of course, by definition, the presence of the maverick prohibits the buyer-preferred equilibrium from being reached.

cooperative bids by others. “Cooperative” as used here, is intended to be different from the intuitive notion of passive.

The next result identifies a type of selective response of non-mavericks. In particular, it involves the use of narrow (directed) retaliation strategies, bidding on the item of someone who bid on yours, as opposed to broad (undirected) retaliation strategies (retaliating against everyone if anyone bid on yours) that were defined in Section 5. Recall, the buyer-preferred equilibrium described in Theorem 2 can be sustained with either type of strategy.

Result 1-6: After a few rounds, mavericks make or receive (are tied to) all of the non-cooperative bids in each experiment. Non-mavericks respond to non-cooperative bids by the maverick on their most preferred items with non-cooperative bids on the maverick’s most preferred item. Non-mavericks do not respond to the mavericks bidding by retaliating against any other buyer.

Support. Figures 4A-4F show for each round and experiment the percent of all non-cooperative bids that are either bids by the maverick on the most preferred of someone else or bids by others on the most preferred of the maverick in retaliation. The proportion of non-cooperative bids tied to the bids of the maverick are shown at the total and the figure separates the percentage of these bids made by the maverick on the “items of others” and the bids of others on maverick’s most preferred in retaliation. The percentage of non-cooperative bids tied to the bids of the maverick begin as a low percentage during the first rounds but that percentage quickly increases to near 100%. This increase of the percentage is due to a reduction of non-cooperative bids by those other than the maverick except bids made as retaliation. If multiple mavericks were identified the

figures reflect the non-cooperative bids tied to all of them even though one maverick was usually far more responsible for the non-cooperative bids than the other.

The figures indicate that in all experiments the mavericks are usually responsible for the vast majority (if not all) and those not submitted by the maverick are tied to the bids of the maverick. This is especially true in later rounds of the experiment. For instance, Figure 4B shows two mavericks are tied to a majority of non-cooperative bids in rounds 2-7 and all non-cooperative bids in rounds 5-7. Thus, mavericks hold the most responsibility for the buyer-preferred equilibrium not being reached sooner. In experiment 6, one of the mavericks (121) is solely responsible for the buyer-preferred equilibrium not being sustained in round 18. The figures suggest that only a small number of retaliations (taken as a percentage of all non-cooperative bids) are needed to discourage the maverick from bidding non-cooperatively but Figure 4E round 14 illustrates that retaliations can be numerous. In that period, all non-cooperative bids were tied to the maverick who made about 75% of them and the other 25% of the non-cooperative bids were made by others on the most preferred of the maverick.□

Corollary 1-6: Narrow (directed) retaliation rather than broad retaliation or passive response is the most likely strategy associated with the buyer preferred equilibrium in result 1-1.

Support. Theorem 2 states a buyer preferred equilibrium would exist with either broad or narrow retaliation (shown in Corollary 1 and Lemma 1, respectively). Buyers do not retaliate when the buyer-preferred allocation is sustained because there is no need. But, result 1-6 shows that when buyers bid on items which are not their most preferred, they are met with narrow (directed) retaliation. Of course, after this deviation occurs, the state is not the buyer-preferred

equilibrium. It is very likely that if any buyer deviated at the buyer preferred allocation he would face narrow retaliation identical to the type observed in result 1-6. Thus, the evidence indicates buyers retaliate against the maverick for his bidding and not everyone else. Buyers are closer to playing the strategies described in Corollary 1 than Lemma 1.□

The next results concern punishment and/or spiteful behavior in the SAPA portion of the experiments. Both likely exist in the auction, but the paper will use restraint in classifying punishment. The paper defines punishment as any bid in the SAPA where the buyer exposes himself to a loss if the bid were final. Punishments may occur where a buyer is not bidding above his value but he is bidding up another buyer's price. Such actions are not classified as punishment bids because a buyer could be trying to obtain the item for profit rather than for punishment.

Punishments can be broken down into directed and undirected punishments. A buyer bidding above his value on another's most preferred item after the other buyer has bid on his most preferred item in that round is a directed punishment. An undirected punishment is any other bid above one's valuation.

Result 1-7: Punishment occurs in the auctions studied. Mavericks use both undirected and directed punishments. Non-mavericks use directed punishment against the maverick(s).

Support. Occasionally, but persistently, in the six experiments, buyers bid above their value on items that were not their most preferred. Bidding above one's valuation could be due to confusion, but the only reported cases of subject confusion occurred in round 1. Excluding the results of round 1 from the analysis, there were 282 bids that are over the buyer's value out of 5113 bids (5.5%). The number may seem small, but it is not expected to be great. Most bids in

the experiments were buyers bidding on their most preferred item or a maverick bidding on all items.

Of the 282 punishments, 144 were directed and 138 undirected. As Table 12 shows, mavericks provide a great amount (93%) of the undirected punishment observed. They also are responsible for a slight majority of directed punishment (57%). Mavericks are more likely to use undirected punishment (128/208) than directed (80/208).

There were 64 directed punishments that were not done by mavericks, and of these, 48 (75%) are directed toward mavericks. The other 25% often occur in early rounds as “bidding wars” between two buyers who share high values for the same two items. Adding punishments by mavericks (128 undirected and 80 directed) to punishments directed at mavericks (48) suggests that mavericks are responsible for 256 of 282 punishments (91%).

It is likely the case that punishments of mavericks are done to promote cooperative behavior as the model suggests. The punishments are immediate and reactive, not lasting; otherwise they would prevent the buyer-preferred allocation from being reached. After a round of punishment, it is even possible to move directly to the buyer-preferred allocation as in experiment 3 (which featured punishment in round 8).

The overwhelming majority of undirected punishments are done by mavericks. These actions imply that a maverick often exhibits a bit of spitefulness. In a few of the experiments, notably experiments 1 and 5 (two experiments with a clear single maverick), the maverick had a tendency to indiscriminately bid on all items the round after he received punishment.¹⁶ □

¹⁶ The maverick in experiment 5 was dismissed from the experiment room last so that the experimenter could ask him about his strategy. The maverick explained that he wanted to earn more in comparison to what everyone else was earning. Hence, he purposefully, and spitefully, drove up the prices of the items so that others would earn less compared to what he earned. He admitted that he did not think he was following the best strategy and he speculated (correctly) that he probably earned the least of the subjects.

Punishment can be classified further in a few special cases. Punishments where a buyer chose to bid over everyone's valuation is called "expressive punishment" because that buyer has paid a heavy cost to express his displeasure at the buyer who most prefers that item. There is also a "punishment defense" where a buyer chooses to let the buyer who has punished him take that item at a loss rather than overbidding and acquiring his most preferred item at a profit.

Result 1-8: Neither expressive punishment nor punishment defense are sustainable in the SAPA. Expressive punishment is always directed and may be used against a maverick. Punishment defense is most often used against a maverick's undirected punishments to prohibit that behavior.

Support. There are 4 instances where expressive punishment occurred or 1.6% of all punishments; all 4 were directed punishments but only 2 of the 4 were directed toward the maverick. The fact that 4 of these punishments are directed punishments suggests bidding above everyone's value is intended as a communication device.

Punishment defense occurred in 8 bids. In 6 of these 8 instances, the original punishment was undirected and in 5 of the 6 undirected punishments the original punishment is done by a maverick. Hence, the reverse punishment can serve to punish a maverick who is arbitrarily bidding above his valuation.

Neither expressive punishment nor punishment defense occurred frequently (1.6% and 3.3% of all punishments respectively). Expressive punishment and punishment defense likely could not be sustained because of their high costs. Expressive punishment requires buyers to take a real loss because expressive punishments are above everyone's valuation on an item.

Punishment defense requires buyers to endure an great opportunity cost as buyers do not acquire their most preferred item in a round.□

Part 6.2 The Simultaneous Descending Price Auction

After the buyer preferred allocation had been obtained and persisted for several rounds in the SAPA or where time constraints were reached, the institution was changed to the SDPA. It is important to notice that at the time of the institutional change a full tacit collusion was operating. Li and Plott demonstrate that many treatments suggested by theory, such as the removal of information, are not effective in changing the allocations from the buyer preferred level to the seller preferred. The next results show that the change in the institution has a dramatic effect on the allocation in favor of the seller and does so for understandable reasons.

Recall that Theorem 5 demonstrates that the buyer preferred allocation cannot be supported as an equilibrium under the SDPA. None of the supporting strategies that are found in the SAPA are available for use in the SDPA. Result 2-1 makes clear that the institution does have the impact that the theory suggests.

Result 2-1: An institutional change from the SAPA to SDPA destroys the buyer-preferred equilibrium and moves the resulting allocation toward the seller preferred allocation.

Support. Figures 1A-1F show the average price in each experiment by round. The switch from the SAPA to SDPA is indicated by the leftmost vertical line through each graph. The titles of each chart as well as Table 3 also indicate when the auction was switched to SDPA. The graphs show an immediate jump in prices away from the buyer-preferred allocation of 10. Figure 1D shows the most dramatic jump in experiment 4 as average price moved from 10 to 592 from

period 11 to 12. The sale price of items jumped on average 387 units after the shift. There is no evidence from the graphs of prices ever again coming near an average price of 10. Table 13 shows the average price of items sold in the SDPA auction is 610.85 (first row, second column). The number 610.85 is also the average of the average prices in all 54 descending price auctions. The standard deviation of the average prices is 141.78. The buyer-preferred average price of 10 can be rejected from the distribution of SDPA average prices at any reasonable level of significance ($p < 0.001$). Table 13 indicates that the Pareto efficient allocation occurred in only 3 out of 54 rounds of the SDPA (row 7, column 2). A Pareto efficient allocation must occur if every buyer was cooperative, so at most in three instances every buyer was cooperative. The data suggest that even if substantially more experiments were run, it is unlikely the buyer-preferred equilibrium would be reached.

Theorem 5 demonstrates that a buyer-preferred allocation cannot be supported as an equilibrium in the SDPA. As the results indicate, the data are not near the buyer preferred allocation. Appendix 1, Lemma 6 also indicates no other equilibrium exists other than the seller-preferred equilibrium in the single period model of the auction. Full convergence to the seller-preferred equilibrium is not observed. The repeated nature of the experiment may have been responsible for the deviations from that level. (See result 2-4.)

Figures 1A-1F show that price levels do not linger or return to the buyer preferred equilibrium after a change of institution. In Figure 1F, the average price never reaches 440 units of the buyer-preferred level. That is, there is no evidence of a hysteresis at the buyer-preferred allocation after an institutional change, except perhaps in experiment 2 round 12 (Figure 1B). The lag in round 12 only lasted one period. By round 13, the price had moved away from the

buyer preferred equilibrium. Prices are closer to the seller-preferred equilibrium than in the SAPA.□

Result 2-2: The choice of institution (SPDA or SAPA) has an effect on prices, allocations and efficiencies. The SAPA provides lower prices and more efficiency than the SDPA.

Support. Table 13 shows the overall average price. That price is considerably higher in the SPDA than SAPA (row 1).

The SAPA has more efficient outcomes. Table 13 shows the SAPA is roughly 4% more efficient than the SDPA (row 6). The difference is better expressed when one notes that the Pareto efficient allocation, the outcome where each buyer receives his most preferred good, occurred in only 3 of 54 rounds¹⁷ in the SDPA (row 7, column 2). The SAPA achieved the Pareto efficient allocation 63 out of 78 times. The SDPA does not achieve the Pareto optimal allocation often. The model did not predict the Pareto efficient allocation would occur so rarely. The lack of Pareto efficient allocations is understandable given the properties of the auction, specifically, that there are limited responses to each bid. The buyer who values an item the most cannot respond to any other bids on that item.

There appears to be a price floor in the SDPA. Of the combined 432 items sold in the SDPA, only 18 items (4.2%) were sold below their third valuation, the level of exposure to a third buyer. Conversely, 42.6% of goods in the SAPA were sold below the average third valuation. Trades that occur below the third valuation level rely on cooperation among more than two buyers. Recall that collusion beyond more than two buyers is not often found in the theoretical literature (see section 3.1).□

¹⁷ Two of these three cases occurred with hidden IDs (see result 2-4).

Result 2-3: Punishments in the SDPA were costly and rarely occurred. Punishments in the SDPA were not a credible threat and could not enforce cooperative behavior.

Support. As mentioned earlier, punishments in the SAPA often preceded the buyer-preferred equilibrium. That institution was characterized by relatively costless punishment. The SDPA did not have costless punishments. An average punishment cost subjects 267.08 units (\$1.34) per punishment.

In the 54 rounds of the SDPA in the experiment, there were 85 steals. Steals, a clear instance of non-cooperative behavior, occur when a buyer wins another's most preferred item at a profit. In only two instances were these steals met with an immediate punishment in that round or the next round. Of the 35 subjects (of 48) that stole an item at some point in the SDPA, only 15 ever encountered a punishment after their steal(s), and it is unclear whether that punishment was a response to uncooperative behavior (see Table 14 for other explanations of punishment). Most likely, the high cost of punishment caused buyers not to use punishment to enforce cooperative behavior. If buyers would not use punishment, then punishment can no longer be an effective deterrent of non-cooperative behavior.

In total, there were only 26 punishments in the six experiments, less than one punishment every two rounds. Table 14 shows the suspected cause of these punishments.

Quick trigger punishments (Table 14, row 3) are those when a buyer takes a slight loss by acquiring another's item too early. Indiscriminate frustration punishments are those punishments in which a buyer took another's item at a loss when the other had done nothing to him (row 2). Both quick-trigger and indiscriminate frustration punishments can be classified as undirected. They account for 18 punishments. Table 14 shows of the eight directed punishments, half are

responses to other punishments (row 4, column 2), thus punishing others may have a greater cost than just the loss one takes on acquiring items. Punishment to enforce cooperation (row 1) accounted for less than one fourth ($4/26$) of all punishment. Thus, in 54 rounds only 4 times was punishment used to enforce cooperation.□

Curiously, the seller preferred equilibrium was not observed even though it is the unique equilibrium of the one shot game model of the auction. Given the previously successful tacit collusion among buyers, it was suspected that reputations might play a role. Even though the buyers were involved in the SDPA, they still had full information about prices and thus knew who had the “second highest” and “third highest” valuations of their items. The next result demonstrates that elements of “trust” remained in the sense that buyers expected certain other buyers to continue to follow a “cooperative strategy”. The treatment was to remove the identification of participants so residual reputations could not play a role.

Result 2-4: Removing IDs from subjects in the SDPA increases average price and efficiency. There is an immediate movement to the seller-preferred allocation. Prices do not drop after multiple rounds indicating the existence of an equilibrium.

Support. In two experiments after subjects had experienced a few rounds of the SDPA, subject ID information from purchases was removed. Rounds 21-27 of experiment 4 and rounds 23-25 of experiment 6 featured no ID information. Figures 1D and 1F indicate the removal of subject IDs in the SDPA by the rightmost vertical bar through the graph. The average prices to the right of the rightmost vertical bar are all very near the average second valuation line, the seller-preferred allocation average price (the top horizontal line in the graph). Figures 1D and 1F show no evidence of a hysteresis when IDs are removed – the movement to the average second

valuation is immediate. The lack of hysteresis is indicated by the movement from round 20 to 21 in figure 1D and round 22 to 23 in figure 1F. Prices do not drop after the seller-preferred allocation is reached. They remain at that level.

Table 15 gives a comparison between IDs and no IDs in each SDPA. Notice that the average price is higher with hidden IDs (row 1). The auction also has more efficient outcomes (row 6), probably caused by individuals acquiring their most preferred item before that item is valuable to someone else. In more than half of all observations (45/80), an item is purchased at a price greater than its second highest valuation (row 9, column 2). In those cases, the item is usually (38/45) acquired within 10 francs (5 seconds) of the second highest valuation (row 11, column 2 minus row 9, column 2). The movement to the seller-preferred equilibrium can be most supported by the last two columns of Table 15 which show over 76% (61/80) of items sold are within 10 francs of seller-preferred allocation value. Theorem 4 proves that seller-preferred equilibrium exists and is unique in a one shot game; it appears that removing IDs can make the repeated SDPA in this experiment more like a single-shot game.□

In all rounds that are close to the seller preferred allocation, it is impossible to observe the strategy buyers are using with items which are not their most preferred. Buyers bid on their most preferred item before it is exposed to other buyers. Thus, one cannot conclude what strategy is being played and if it sustains an equilibrium. It is very likely strategies that sustain Theorem 4 are being used because buyers are repeatedly buying their most preferred item before exposure to a second buyer. They would only do this if (i) the second buyer will likely bid on their item or (ii) they are making a mistake about the intentions of the second buyer. Since these buyers have bid with each other for over an hour, they probably have decent intuition into the strategy of other buyers, and likely are not making a mistake. Most likely the strategies that

support the seller preferred allocation as an equilibrium are being used. There is some direct evidence that buyers will take their non-most preferred item when it is exposed to them. Of the 80 items that were sold with no IDs in the SDPA, only 12 were bought under ten points below the second highest buyer valuation (Table 15: Row 3 – Row 11 – Row 13). In those 12 trades, 6 were taken by the buyer with the second highest valuation, a rate of non-cooperative bidding that is much higher than in other conditions of this experiment.

The power of the removal of the IDs and thus the associated reputation effects is of particular interest. Li and Plott found removal of IDs alone did not terminate the collusive equilibrium in the SAPA auction. Since cooperation in the SDPA is not fully collusive - buyers only cooperate with one other buyer - it is likely reputation effects are more important than in a fully collusive equilibrium. For that reason, removal of IDs matter here, but have no effect in the SAPA once the buyer preferred equilibrium is sustained in Li and Plott's experiment. Once the buyer-preferred equilibrium is achieved, everyone must be trusted so specific IDs are not important.

7. Summary of Conclusions

A very special economic environment developed by Li and Plott can be viewed as a type of “collusion incubator” in the sense when the institution is the simultaneous ascending price auction, tacit collusion develops quickly to the disadvantage of the seller. Previous research demonstrated that such collusions are indeed hard to “break up” and once established the normal remedies suggested by game theory and associated industrial organization literature simply do not work or certainly are not as powerful as one might like. The tacit collusions remain. The experiments reported here explored the sources of the strength of such collusive patterns of

behavior together with institutional changes that would undermine those sources of strength and thereby break the collusive patterns.

The experiments reported here found strong evidence for tacit collusion in the SAPA, consistent with Li and Plott. Over the first rounds of the experiment, the SAPA auction's prices decreased greatly. Their range moved from those in the seller-preferred allocation (which can be sustained as an equilibrium) to the buyer-preferred allocation (which can be sustained and indeed was sustained as an equilibrium). The study of the evolution of behavior revealed a type of transition of behavior at the individual level. The movement of prices from the seller-preferred allocation to buyer-preferred equilibrium is represented by a buyer's transition from non-cooperative to cooperative behavior, where cooperative behavior is defined independent of prices in the auction to avoid tautological reasoning. These transitions of behavior are facilitated by strategies of retaliation and punishment until all eight buyers are cooperative. Of particular interest in this regard, are buyers who are characterized as "mavericks", who resist conforming to the cooperative strategies of others. One non-cooperative holdout can remove the allocation from the buyer-preferred equilibrium. Often punishment is used to force this maverick to realize that he is not following a best response strategy to the bids of the seven other cooperators. Once the buyer-preferred equilibrium is achieved, it persists. It is observed to last up to five periods in the SAPA.

Once the auction is switched to a SDPA, the buyer-preferred equilibrium is destroyed and the data respond substantially as predicted by the model. The response is immediate and dramatic but the prices do not move all of the way to the unique equilibrium, which is the value of the second highest valuation. The prices stay above the third valuation in almost all cases in this auction, but rarely reach the second highest valuation, the level predicted by the model.

It is not fully clear why prices do not reach the seller-preferred levels in the SDPA. The data support a conjecture that the multiple periods cause subjects to learn the cooperative natures of their partners and consequently subjects choose to not engage in non-cooperative behavior because of such reputation effects. The models address only the behavior of a single period but tested the conjecture that more complex behavior was involved by eliminating subject IDs in two of the experiments. In those rounds, prices quickly converged to those in the seller-preferred equilibria. The information associated with subject IDs together with established reputations for cooperation could be necessary to sustain prices below the second price level in the SDPA.

The question remains why prices would not fall below the third price level in the presence of subject IDs in the SDPA. A possible explanation is that a buyer may trust his partner and the buyer who values his item third, but may believe his partner does not trust the buyer that values his item third. In that case, a buyer suspects that if the item falls below its third valuation, his partner will take it to prevent it from being taken by the buyer who values the item third most. If a buyer believes his partner would take an item after it falls below the third price level, then the buyer would take it right before that price. Alternatively, it may be too risky for most individuals to trust more than one person. The theoretical properties of collusion models that only apply to two buyers may also be involved. Whatever the reason, very few bids occur below the third price level in the SDPA.

Not surprisingly, revenue equivalence does not occur in ascending and descending price auctions with these types of valuations. Sellers would do much better with a SDPA because other equilibria more favorable to the buyer do not exist. On average, sellers in these markets made over three times the revenue in a SDPA auction than a SAPA. The seller would do even

better if they were able to have the auction be anonymous without anyone knowing the bids of other buyers.

While it contains very unique preferences, the collusion incubator in this experiment is important as a test bed for studies that focus on methods of creating competition under circumstances where tacit collusion is thought to exist. Institutional changes have great power in this context, especially when they remove the capacity for facilitating behaviors. This feature is well illustrated by the power of a descending auction in breaking up collusion. The quick manner in which a non-cooperative buyer can steal goods as well as the high cost of punishment is the source of the institution's power.

References:

- Albano, G. L., F. Germano, and S. Lovo [1] (2006). "Ascending auctions for multiple objects: The case for the Japanese design", *Economic Theory*, 28 (2):331-355.
- Albano, G. L., F. Germano, and S. Lovo [2] (2006). 'Retaliatory equilibria in a Japanese ascending auction for multiple objects', *Review of Economic Design*, 10 (1):1-8.
- Ausubel, L. M. and J. A. Schwartz (1999). The Ascending Auction Paradox, University of Maryland. Available at www.market-design.com/files/ausubel-schwartz-ascending-auction-paradox.pdf.
- Benoit, J. and V. Krishna (2001). "Multiple-Object Auctions with Budget Constrained Bidders." *Review of Economic Studies* 68(1):155-179.
- Brusco, S. and G. Lopomo (2002). "Collusion via Signaling in Simultaneous Ascending Bid Auctions with Heterogeneous Objects, with and without Complementarities." *Review of Economic Studies* 69(2):407-436.
- Clouser, L. and C. R. Plott (1993). "On the Anatomy of the 'Nonfacilitating' Features of the Double Auction Institution in Conspiratorial Markets." *The Double Auction Market: Institutions, Theories and Laboratory Evidence*. D. Friedman and J. Rust (eds), Addison-Wesley.
- Cox, J. C., R. Bruce, and V. L. Smith (1982). "Theory and Behavior of Single Object Auctions." *Research in Experimental Economics*. V. L. Smith. Greenwich, CN, JAI Press.
- Cox, J. C., V. L. Smith, and J. M. Walker (1983). "A test that discriminates between two models of the Dutch-first auction non-isomorphism." *Journal of Economic Behavior & Organization* 4(2-3):205-219.
- Engelbrecht-Wiggans, R. and C. M. Kahn (2005). "Low-revenue equilibria in simultaneous ascending-bid auctions." *Management Science* 51(3):508-518.

- Fouraker, L. E. and S. Siegel (1963). *Bargaining Behavior*. New York, McGraw-Hill.
- Grimm, V. and D. Engelmann (2006). "Bidding Behavior in Multi-Unit Auctions-An Experimental Investigation and some Theoretical Insights." Unpublished working paper. University of London. Available at <http://eprints.rhul.ac.uk/456/1/derex-EJ.pdf>.
- Isaac, R. M. and C. R. Plott (1981). "The opportunity for conspiracy in restraint of trade : An experimental study." *Journal of Economic Behavior & Organization* 2(1):1-30.
- Isaac, R. M. and J. M. Walker (1985). "Information and conspiracy in sealed bid auctions." *Journal of Economic Behavior & Organization* 6(2):139-159.
- Kagel, J. H. (1995). Auctions: A Survey of Experimental Research. *The Handbook of Experimental Economics*. J. H. Kagel and A. E. Roth. Princeton, Princeton University Press: 501-585.
- Kwasnica, A. M. (2000). "The choice of cooperative strategies in sealed bid auctions." *Journal of Economic Behavior & Organization* 42(3):323-346.
- Kwasnica, A. M. and K. Sherstyuk (2007). "Collusion and Equilibrium Selection in Auctions." *The Economic Journal* 177(1): 120-145.
- Li, J. and C. R. Plott (2005). "Tacit Collusion in Auctions and Conditions for Its Facilitation and Prevention: Equilibrium Selection in Laboratory Experimental Markets." Unpublished working paper. Massachusetts Institute of Technology.
- Lind, B. and C. R. Plott (1991). "The Winner's Curse: Experiments with Buyers and with Sellers." *The American Economic Review* 81(1): 335-346.
- McCabe, K. A., S. J. Rassenti, and V. L. Smith. (1990). "Auction Institutional Design: Theory and Behavior of Simultaneous Multiple-Unit Generalizations of the Dutch and English Auctions." *The American Economic Review* 80(5):1276-1283.

- Milgrom, P. (2000). "Putting Auction Theory to Work: The Simultaneous Ascending Auction." *Journal of Political Economy* 108(2):245-272.
- Noussair, C. N., C. R. Plott, and Raymond G. Riezman (1995). "An Experimental Investigation of the Patterns of International Trade." *The American Economic Review* 85(3):462-491.
- Pesendorfer, M. (2000). "A Study of Collusion in First-Price Auctions." *Review of Economic Studies* 67(3): 381-411.
- Phillips, W. R., D. J. Menkhaus, and K. T. Coatney. (2003). "Collusive practices in repeated English auctions: Experimental evidence on bidding rings." *American Economic Review* 93(3): 965-979.
- Robinson, M. S. (1985). "Collusion and the Choice of Auction." *The Rand Journal of Economics* 16(1):141-145.
- Sherstyuk, K. (1999). "Collusion Without Conspiracy: An Experimental Study of One-Sided Auctions." *Experimental Economics* 2(1):59-75.
- Sherstyuk, K. (2002). "Collusion in private value ascending price auctions." *Journal of Economic Behavior & Organization* 48(2):177-195.
- Sherstyuk, K. (forthcoming). Some results on anti-competitive behavior in multi-unit ascending price auctions. *The Handbook of Experimental Economics Results*. C. R. Plott and V. L. Smith. Amsterdam, Elsevier Press.
- Van Huyck, J. B., R. C. Battalio, and R. O. Beil (1990). "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure." *The American Economic Review* 80(1):234-248.
- Vickrey, W. (1961). "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance* 16(1):8-37.

Table 1: An example of subject valuations of items

		Item #							
		1	2	3	4	5	6	7	8
Subject ID #	121	833	212	706	101	290	180	317	94
	122	787	164	893	69	325	223	266	146
	123	327	121	284	214	782	76	808	187
	124	252	55	303	158	856	105	738	241
	125	238	844	194	343	81	745	106	277
	126	159	788	218	276	122	841	75	340
	127	143	303	52	848	157	280	235	796
	128	81	266	116	795	215	342	181	827

Table 2: Procedural differences in SAPA and SDPA auctions

SAPA	SDPA
Buyers can place bids on all items at start (simultaneity).	Buyers can place bids on all items at start (simultaneity).
A bid on an item purchases it if no other buyer bids over that bid.	A bid on an item purchases it.
Buyers must place a bid higher than the last bid (if applicable) on any item.	The price buyers may bid on an item decreases following a fixed rate unless that item has been purchased already.
The auction ends when no buyer takes any action for a preset time interval.	The auction ends when all items have been purchased, or the price reaches zero.

Table 3: Design of Experiments 1-6

Experiment	1	2	3	4	5	6
date	2005/01/19	2005/01/26	2005/02/07	2005/04/27	2005/05/02	2005/05/11
total rounds	15	21	23	28	21	25
SAPA rounds	1-10	1-11	1-12	1-11	1-16	1-18
SDPA rounds	11-15	12-21	13-23	12-20	17-21	19-22
blank ID rounds	NA	NA	NA	22-28	NA	23-25
duration (in minutes)	87	111	114	109	115	128

Table 4: Regressions of round number on Average Price, Duration, Bid Number and Cooperative Bidders (SAPA rounds only)

	average price	duration	number of bids	cooperative bidders
round	-50.63*** (3.80)	-12.88*** (1.38)	-24.88*** (4.45)	0.41*** (0.028)
exp 2	87.43 (59.19)	26.08 (21.57)	26.64 (69.35)	0.59 0.43
exp 3	150.97* (58.10)	-2.95 (21.17)	-79.34 (68.07)	-0.19 0.42
exp 4	-450.64*** (53.50)	-107.20*** (19.49)	-238.77*** (62.68)	3.06*** (0.39)
exp 5	177.65** (55.76)	44.40* (20.32)	60.65 (65.33)	-0.75 0.41
exp 6	447.87*** (55.52)	71.37*** (20.23)	80.77 (65.05)	-2.29 0.41
cons	481.49*** (47.64)	150.86*** (17.36)	386.64 (55.82)	2.94*** (0.35)
r ²	0.7543	0.5867	0.3784	0.7753

*p-value<0.05

**p-value<0.01

***p-value<0.001

Table 5: Cooperation in SAPA by Subject and Round in Experiment 1 (0 indicates cooperative behavior)

subj\rd	1	2	3	4	5	6	7	8	9	10	Total
121	1	1	1	1	0	0	0	1	1	0	6
122	1	1	1	1	1	1	1	1	1	1	10
123	1	1	1	1	0	1	1	0	0	0	6
124	1	1	0	0	0	0	0	0	0	0	2
125	0	0	0	0	0	0	0	0	0	0	0
126	0	0	0	0	0	1	0	0	0	0	1
127	1	0	0	0	0	0	0	0	0	0	1
128	0	0	0	0	1	0	1	0	0	0	2
Total	5	4	3	3	2	3	3	2	2	1	28

Table 6: Cooperation in SAPA by Subject and Round in Experiment 2 (0 indicates cooperative behavior)

subj\rd	1	2	3	4	5	6	7	8	9	10	11	Total
121	1	0	0	0	0	0	0	0	0	0	0	1
122	1	0	0	0	0	0	0	0	0	0	0	1
123	1	1	1	0	1	0	0	0	0	0	0	4
124	0	0	0	0	0	0	0	0	0	0	0	0
125	1	1	1	1	1	1	1	0	0	0	0	7
126	1	1	0	0	0	0	0	0	0	0	0	2
127	1	0	0	0	0	0	0	0	0	0	0	1
128	1	1	1	1	1	0	1	0	0	0	0	6
Total	7	4	3	2	3	1	2	0	0	0	0	22

Table 7: Cooperation in SAPA by Subject and Round in Experiment 3 (0 indicates cooperative behavior)

subj\rd	1	2	3	4	5	6	7	8	9	10	11	12	Total
121	1	1	1	1	0	1	1	1	0	0	0	0	7
122	1	1	1	1	1	1	1	1	0	0	0	0	8
123	0	1	1	1	0	0	0	0	0	0	0	0	3
124	1	0	0	1	0	0	0	0	0	0	0	0	2
125	1	1	0	0	0	0	0	0	0	0	0	0	2
126	1	1	1	1	1	0	0	0	0	0	0	0	5
127	0	0	0	0	0	0	0	0	0	0	0	0	0
128	1	1	1	0	1	0	0	0	0	0	0	0	4
Total	6	6	5	5	3	2	2	2	0	0	0	0	31

Table 8: Cooperation in SAPA by Subject and Round in Experiment 4 (0 indicates cooperative behavior)

subj\rd	1	2	3	4	5	6	7	8	9	10	11	Total
121	0	0	1	0	0	0	0	0	0	0	0	1
122	1	1	1	0	0	0	0	0	0	0	0	3
123	1	1	1	0	0	0	0	0	0	0	0	3
124	1	0	0	0	0	0	0	0	0	0	0	1
125	1	0	0	0	0	0	0	0	0	0	0	1
126	0	0	0	0	0	0	0	0	0	0	0	0
127	1	1	1	0	0	1	0	0	0	0	0	4
128	1	1	1	1	1	1	1	0	0	0	0	7
Total	6	4	5	1	1	2	1	0	0	0	0	20

Table 9: Cooperation in SAPA by Subject and Round in Experiment 5 (0 indicates cooperative behavior)

subj\rd	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
121	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	5
122	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
123	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	6
124	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
125	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	6
126	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
127	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	5
128	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	13
Total	6	4	5	4	2	2	3	4	1	1	1	1	1	1	0	1	37

Table 10: Cooperation in SAPA by Subject and Round in Experiment 6 (0 indicates cooperative behavior)

subj\rd	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	total
121	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	16
122	1	1	1	0	1	1	1	1	1	1	1	0	1	1	0	0	0	0	12
123	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
124	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	6
125	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	0	13
126	1	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	6
127	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	5
128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	7	5	5	5	5	6	5	4	3	3	3	2	3	3	2	0	0	1	62

Table 11: Regressions estimating price convergence

	value (std error)
B_{11}	481.16** (154.98)
B_{12}	831.15*** (154.84)
B_{13}	799.94*** (154.76)
B_{14}	-614.57** (214.32)
B_{15}	743.51*** (154.79)
B_{16}	1275.33*** (154.92)
B_2	116.37*** (30.15)
r^2	0.7797

**p-value<0.01

***p-value<0.001

Table 12: Types of punishment used by mavericks and non-mavericks

	Directed Punishments	Undirected Punishments
Done by the Maverick(s)	80 (57%)	128 (93%)
Not Done by the Maverick(s)	64 (43%)	10 (7%)

Table 13: Comparison of prices, allocations, and efficiencies between SAPA and SDPA rounds

	SAPA	SDPA
Avg Price	285.12	610.85
Rounds	78	54
Goods Sold	624	432
Total Redemption Value of Sales	519,703	346,235
Total Possible Redemption Value	526,500	364,500
Pct Efficient	98.7%	95.0%
Pareto Efficient Allocations	63	3
Pct of Rounds	80.77%	5.55%
Trades under 3rd value	266	18
Pct of Trades	42.6%	4.2%

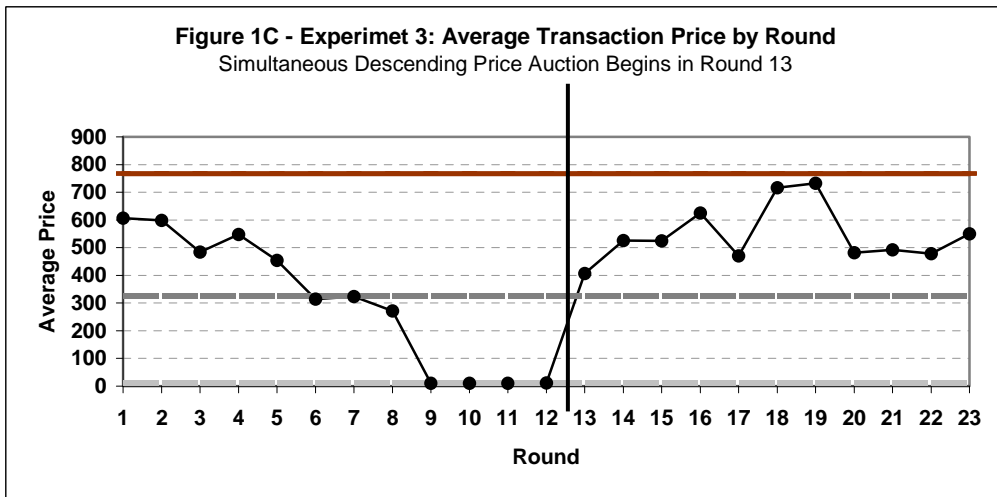
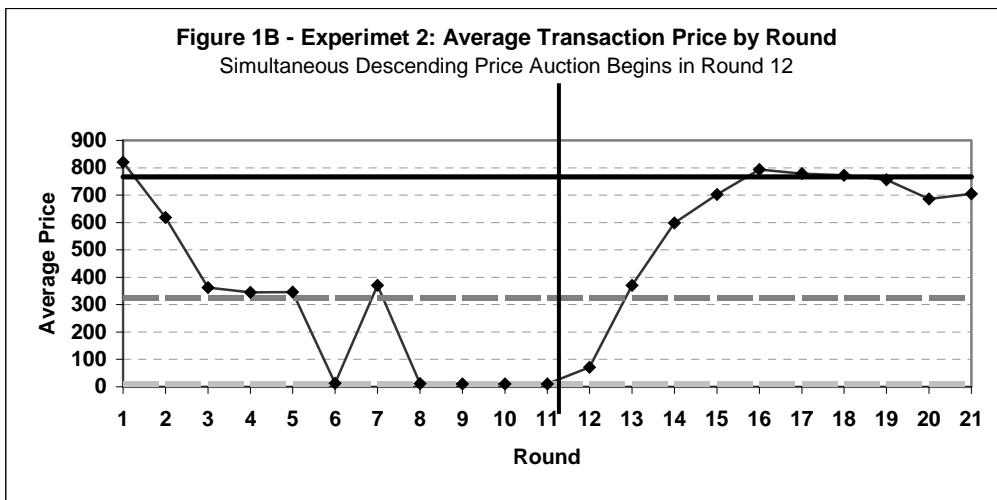
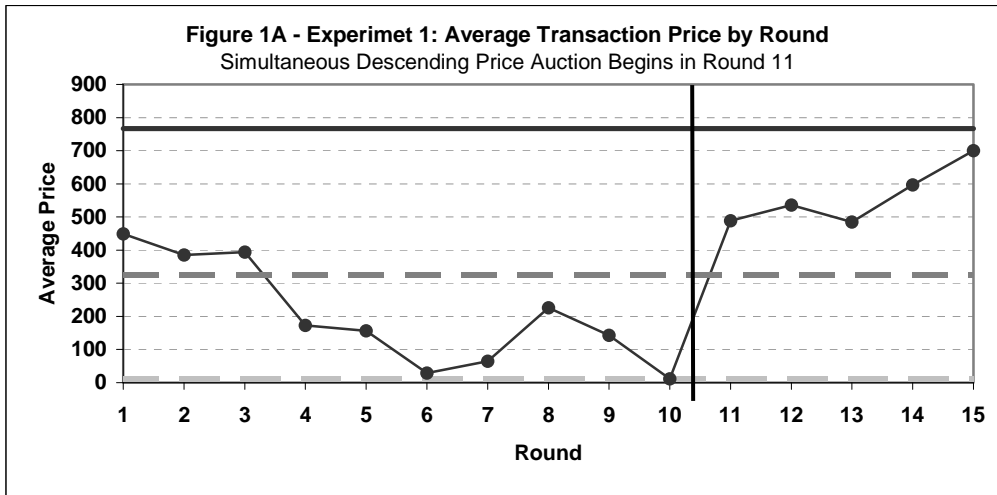
Table 14: Types of Punishments used by buyers in SDPA

	Number	Pct
Enforcing cooperation	4	15.4%
Indiscriminate frustration	9	34.6%
Quick trigger	9	34.6%
Response to punishment	4	15.4%
Total	26	100%

Table 15: Comparison of prices, allocations, and efficiencies between SDPA rounds with visible and hidden subject IDs

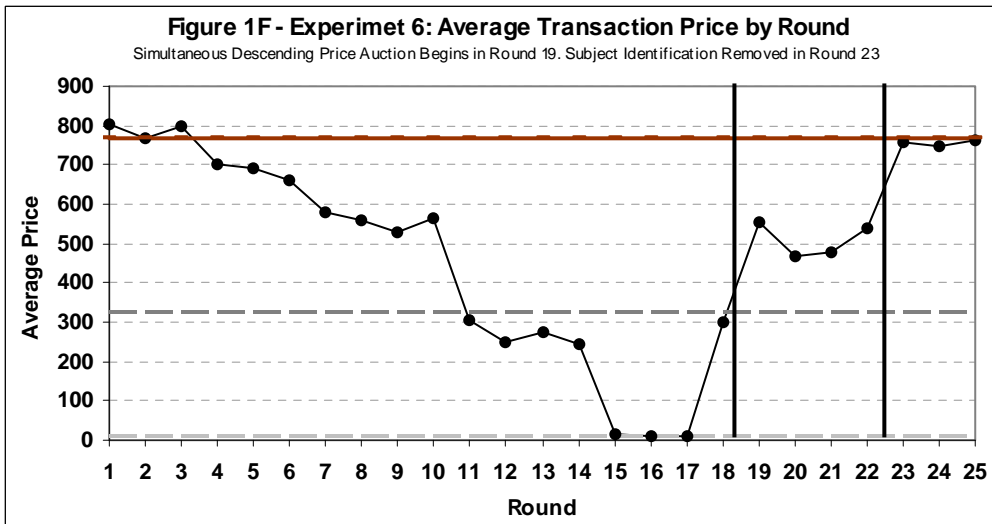
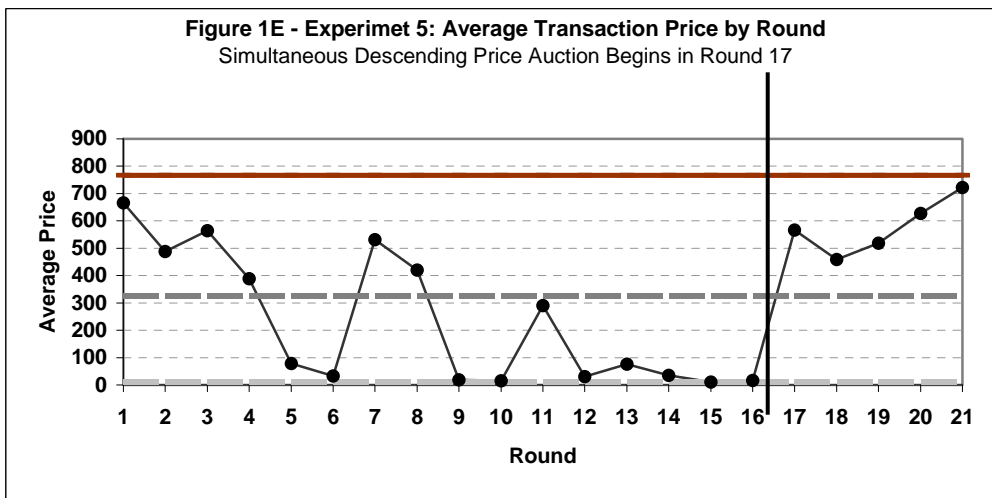
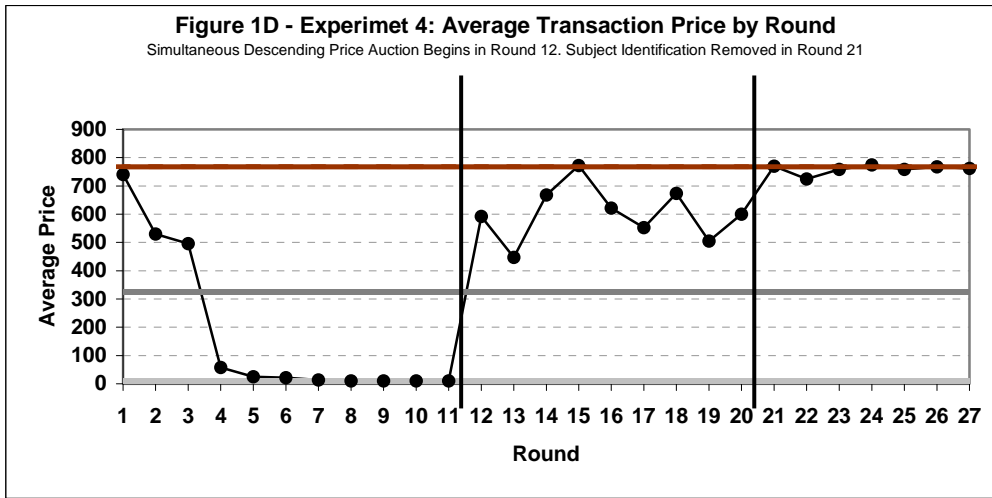
	ID	no ID
Avg Price	577.41	758.00
Total Rounds	44	10
Goods Sold	352	80
Total Value of Sales	279,480	66,755
Total Possible Value	297,000	67,500
Pct Efficient	94.1%	98.9%
Trades under 3rd value	18	0
Pct of Trades	5.1%	0%
Over 2 nd valuation	72	45
Pct	20.5%	56.2%
Over 2 nd by more than 10	35	7
Pct	9.9%	8.8%
within 10 of 2 nd	74	61
Pct	21.0%	76.2%

Figures 1A-1C*:



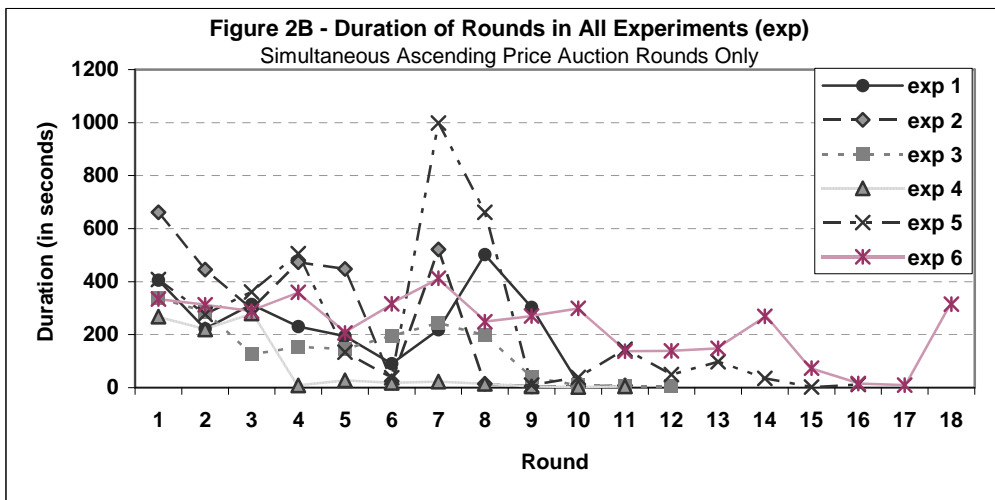
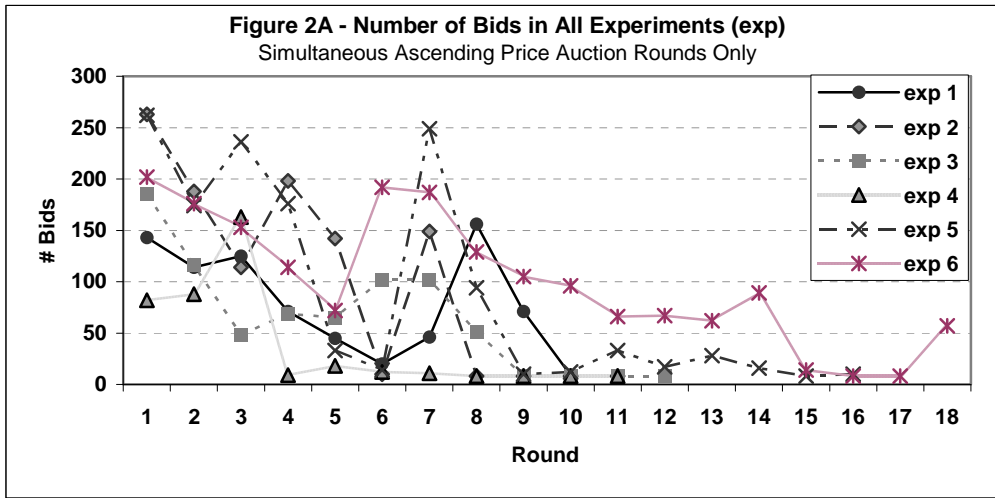
*Top horizontal line represents both the seller-preferred equilibrium and exposure to the 2nd buyer, the middle horizontal line represents exposure to the third buyer and the bottom horizontal line (at average price = 10) represents the buyer-preferred equilibrium. The vertical lines demarcate when a change occurs, from SAPA to SDPA or from SDPA to blank IDs.

Figures 1D-1F*:

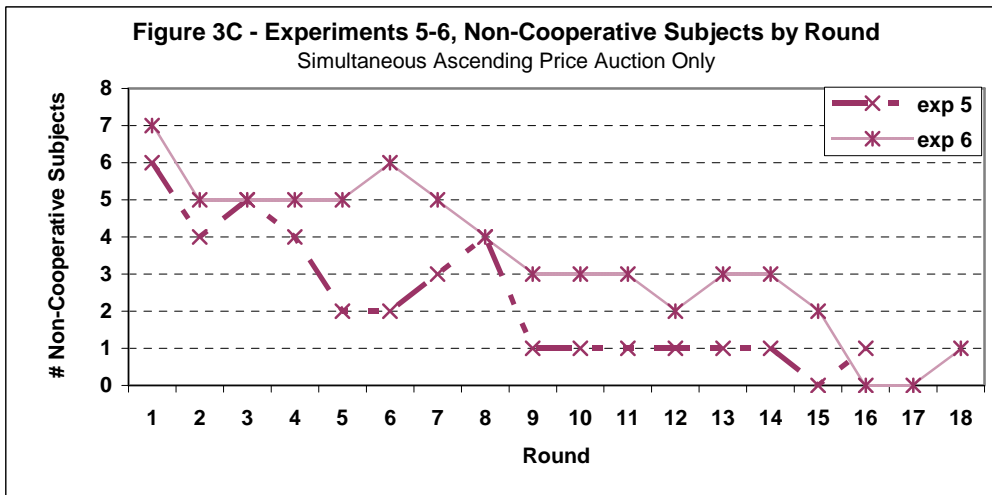
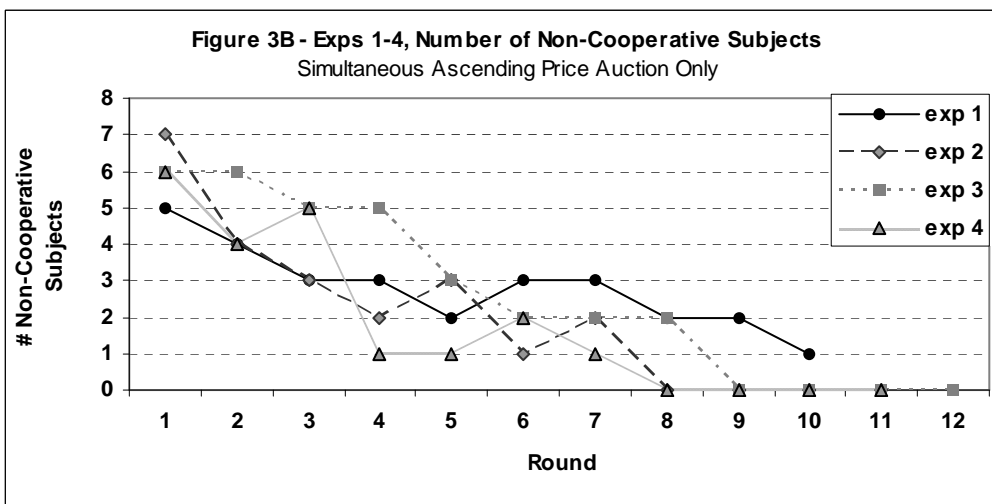
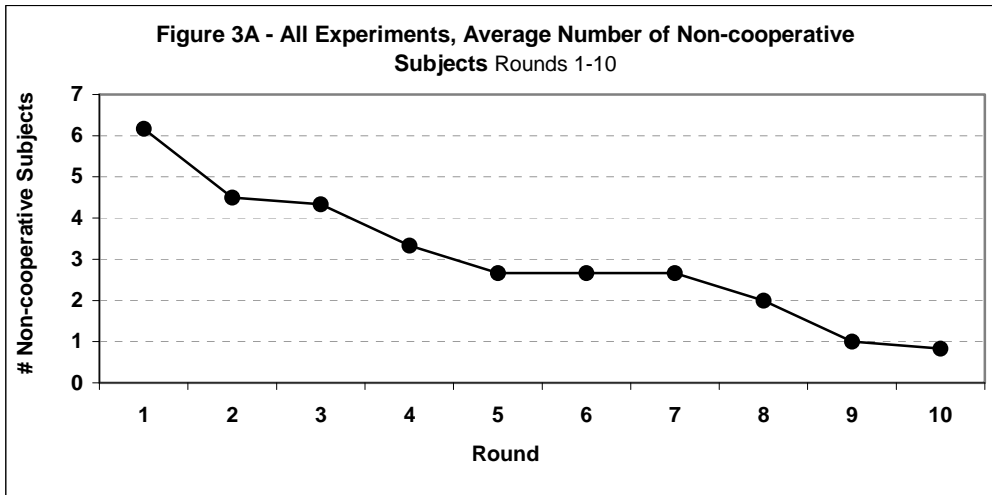


*Top horizontal line represents both the seller-preferred equilibrium and exposure to the 2nd buyer, the middle horizontal line represents exposure to the third buyer and the bottom horizontal line (at average price = 10) represents the buyer-preferred equilibrium. The vertical lines demarcate when a change occurs, from SAPA to SDPA or from SDPA to blank IDs.

Figures 2A-2B:



Figures 3A-3C



Figures 4A-4F:

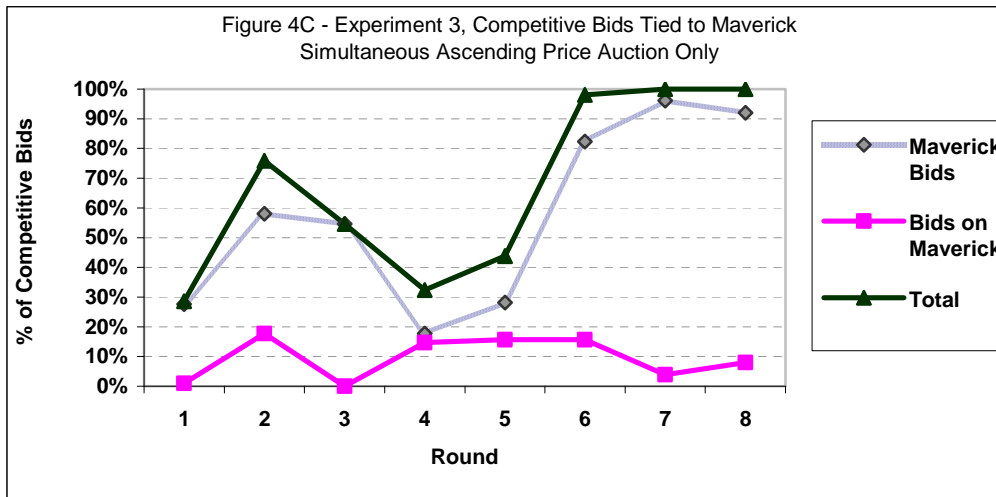
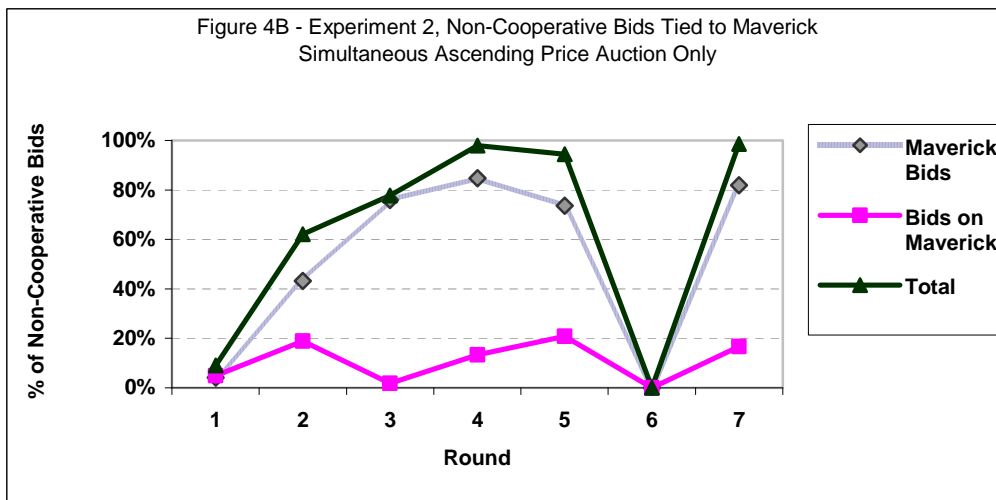
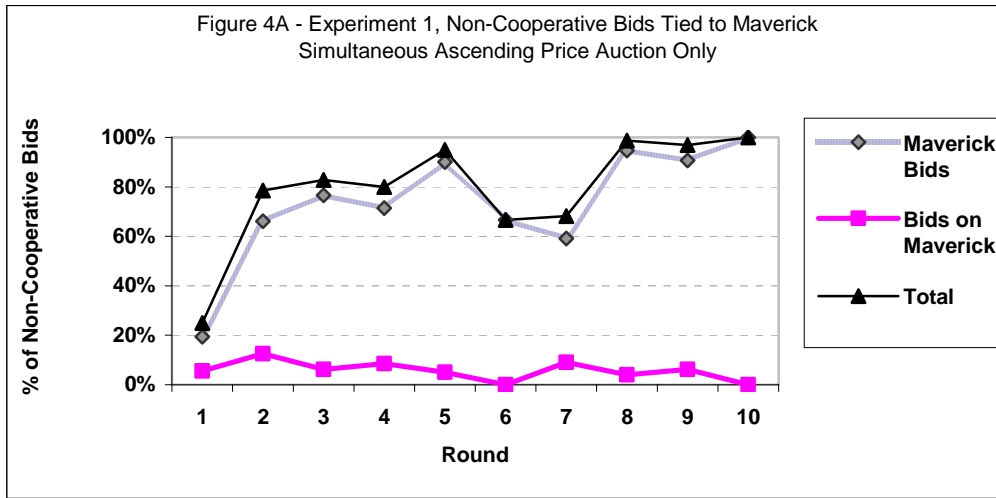


Figure 4D - Experiment 4, Non-Cooperative Bids Tied to Maverick
Simultaneous Ascending Price Auction Only

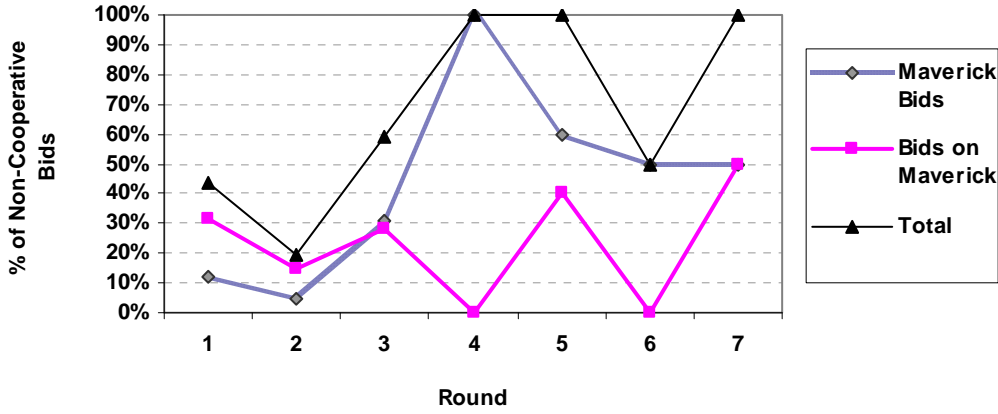


Figure 4E - Experiment 5, Non-Cooperative Bids Tied to Maverick
Simultaneous Ascending Price Auction Only

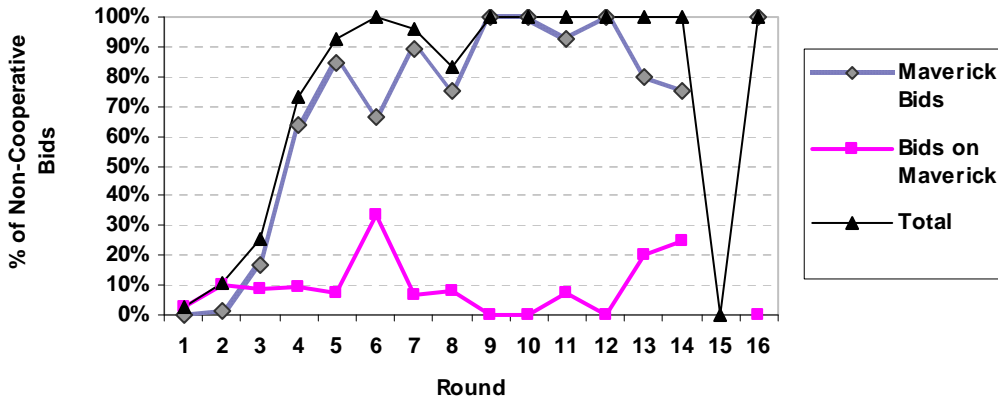
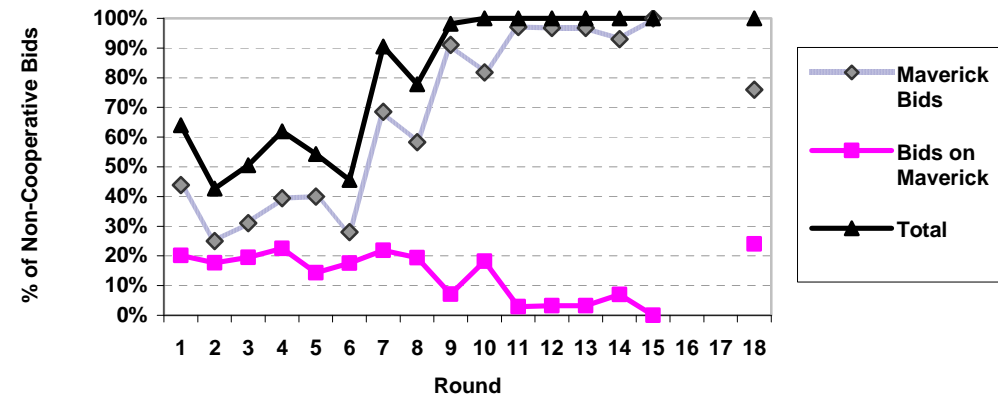


Figure 4F - Exp 6, Non-Cooperative Bids Tied to Maverick
Simultaneous Ascending Price Auction Only



Appendix 1: The Formal Model of the Two Simultaneous Auctions

We will begin the description of our model by defining the state of the auctions. There will be eight buyers and items. The buyers will be represented by i and items by j .

Definition 1 (Auction State) Let $s \in S$ where $S = \mathbb{R}_{++}^8 \times \{0, 1, 2, 3, 4, 5, 6, 7, 8\}^8$. Then s is a 2×8 matrix. The first row of s will denote the current bids where s_{1j} is the current bid on item j . The second row will denote the buyer who posted the bid. The value s_{2j} is the buyer that gave the last bid on item j . A column of $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ denotes an item that has yet to have a bid. We can write s as

$$s = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} & s_{18} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} & s_{28} \end{bmatrix}$$

The initial state, a matrix of all zeros, will be denoted as s_0 .

Definition 2 (SAPA bidding function) For all $i, s \in S$ we will define buyer i 's strategy as a bidding function $B_i : S \rightarrow \mathbb{R}_{++}^8$. The function $B_i(s')$ is a 1×8 matrix that represents buyer i 's bids in state s' . The value $B_i(s')_m$ is the value buyer i bids on item m in state s' .

Definition 3 (Valuation Matrix) The 8×8 valuation matrix V has elements v_{ij} which is buyer i 's valuation of item j . Let \vec{V} be an item sorted valuation matrix. In this case \vec{v}_{ij} is the i th highest valuation of item j .

Definition 4 Let k_{ij} be i 's j th preferred item.

We will now define the SAPA mechanism Γ .

Definition 5 (SAPA mechanism) Let $I(a, b) = \begin{cases} 0 & \text{if } a \neq b \\ 1 & \text{if } a = b \end{cases}$

and $\gamma_n = \max \left\{ s_{1n}, \max_i \{B_i(s)_n\} \right\}$

Define $\Gamma'(s, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8) =$

$$\left[I(\gamma_1, s_{11}) s_{11} + (1 - I(\gamma_1, s_{11})) \arg \min_{i \in \arg \max_i \{B_i(s)_1\}} \{v_{i1}\} \dots \right]. \text{ Then}$$

$$\Gamma(B) = \lim_{n \rightarrow \infty} \Gamma'^{\text{on}}(s, B).$$

We have assumed ties will go to the bidder with the lower valuation arbitrarily. This assumption is not essential for our findings, but is a way to resolve ties in bidding. We also will assume that the bids are given so that the auction can end. That is, $\Gamma(B)$ is defined and $\Gamma(B), \Gamma'(B) \in S$.

Definition 6 We will define buyer i 's profit function as

$$\Pi_i(B) = \sum_{j=1}^8 (v_{ij} - \Gamma(B)_{1j}) I(\Gamma(B)_{2j}, i).$$

$$\Pi_{ij}(B) = (v_{ij} - \Gamma(B)_{1j}) I(\Gamma(B)_{2j}, i).$$

We can now find bidding functions that produce the buyer-preferred equilibria. We will classify bidder strategies into a few types. The first features broad retaliation for non-collusive bids.

Definition 7 (Buyer Preferred Strategy with Broad Retaliation) *A buyer preferred strategy with broad retaliation involves a buyer who bids the reservation price on his most preferred item, and will defect to bidding his valuation on all non-most-preferred items and a price higher than any other valuation of his most preferred item if anyone bids on his most preferred item. Formally, let $\tilde{B}_i =$*

$$\left\{ \begin{array}{l} [0, 0, 0, 0, 0, 0, 0, 0] \text{ if } s_{2k_{i1}} = i \\ \left\{ \begin{array}{l} \tilde{B}_{ik_{i1}} = 10 \\ \tilde{B}_{il} = 0, l \neq k_{i1} \end{array} \right. \text{ if } s_{2k_{i1}} = 0 \\ \left\{ \begin{array}{l} \tilde{B}_{ik_{i1}} = \vec{v}_{2k_{i1}} + \epsilon \\ \tilde{B}_{il} = v_{il}, l \neq k_{i1} \end{array} \right. \text{ if } s_{2k_{i1}} \neq i \text{ and } s_{2k_{i1}} \neq 0 \end{array} \right.$$

It should be apparent that if all bidders play \tilde{B}_i , we will have an outcome where all bidders receive their most preferred item at price 10. Lemma 1 shows this strategy profile is a Nash equilibrium.

Lemma 1 *For any buyer i facing \tilde{B}_{-i}*

(i) *He cannot make a profit on his non-most-preferred items. For all $j \neq k_{i1}$, $\Pi_{ij}(\tilde{B}_i, \tilde{B}_{-i}) = 0 \geq \Pi_{ij}(B_i, \tilde{B}_{-i}) \forall B_i \Rightarrow \tilde{B}_i \in \arg \max_{B_i} \Pi_{ij}(B_i, \tilde{B}_{-i})$*

(ii) *He cannot make a greater profit on his most-preferred item than from strategy \tilde{B}_i . For $j = k_{i1}$, $\Pi_{ij}(\tilde{B}_i, \tilde{B}_{-i}) = v_{ik_{i1}} - 10 \geq \Pi_{ij}(B_i, \tilde{B}_{-i}) \forall B_i \Rightarrow \tilde{B}_i \in \arg \max_{B_i} \Pi_{ij}(B_i, \tilde{B}_{-i})$*

(iii) *The strategy profile $\tilde{B} = (\tilde{B}_i, \tilde{B}_{-i})$ is a Nash equilibrium.*

Proof. Choose any i .

(i) For any $j \neq k_{i1}$, $\Pi_{ij}(\tilde{B}) = 0$, and there does not exist a B'_i s.t. $\Pi_{ij}(B'_i, \tilde{B}_{-i}) > 0$. Suppose not, that $\exists j', B'_i$ s.t. $\Pi_{ij'}(B'_i, \tilde{B}_{-i}) > 0$. Then $\Gamma(B'_i, \tilde{B}_{-i})_{2j'} = i$. Given \tilde{B}_{-i} , we must have $\Gamma(B'_i, \tilde{B}_{-i})_{1j'} > \vec{v}_{2j'} + \epsilon$. Since $\Pi_{ij'}(B'_i, \tilde{B}_{-i}) > 0$, $v_{ij'} - \vec{v}_{2j'} - \epsilon > 0$. But $v_{ij'} > \vec{v}_{2j'} \Rightarrow j' = k_{i1}$ which contradicts our given. Hence $\tilde{B}_i \in \arg \max_{B_i} \Pi_{ij}(B_i, \tilde{B}_{-i}) \forall j \neq k_{i1}$

(ii) Now for k_{i1} , i 's most preferred item, $\Pi_{ik_{i1}}(\tilde{B}) = v_{ik_{i1}} - \Gamma(\tilde{B})_{1k_{i1}} = v_{ik_{i1}} - 10$ and $\Gamma(\tilde{B})_{1k_{i1}} = i$. Clearly, $\forall B_i$ s.t. $\Gamma(B_i, \tilde{B}_{-i})_{1k_{i1}} > \Gamma(\tilde{B})_{1k_{i1}}$, $\Pi_{ik_{i1}}(B_i, \tilde{B}_{-i}) < \Pi_{ik_{i1}}(\tilde{B})$. For all B_i s.t. $\Gamma(B_i, \tilde{B}_{-i})_{1k_{i1}} = \Gamma(\tilde{B})_{1k_{i1}}$, $\Pi_{ik_{i1}}(B_i, \tilde{B}_{-i}) = \Pi_{ik_{i1}}(\tilde{B})$. For all B_i s.t. $\Gamma(B_i, \tilde{B}_{-i})_{1k_{i1}} < \Gamma(\tilde{B})_{1k_{i1}}$,

$\Gamma(B_i, \tilde{B}_{-i})_{1k_{i1}} = 0$, since 10 is the minimum bid. Then $\Gamma(B_i, \tilde{B}_{-i})_{2k_{i1}} \neq i$ and $\Pi_{ik_{i1}}(B_i, \tilde{B}_{-i}) = 0 < \Pi_{ik_{i1}}(\tilde{B})$. Hence $\tilde{B}_i \in \arg \max_{B_i} \Pi_{ik_{i1}}(B_i, \tilde{B}_{-i})$.

(iii) From (i) and (ii), $\tilde{B}_i \in \arg \max_{B_i} \Pi_{ij}(B_i, \tilde{B}_{-i}) \forall j$ which implies $\tilde{B}_i \in \arg \max_{B_i} \left\{ \sum_{j=1}^8 \Pi_{ij}(B_i, \tilde{B}_{-i}) \right\} = \arg \max_{B_i} \Pi_i(B_i, \tilde{B}_{-i})$. Then \tilde{B}_i is a best response to \tilde{B}_{-i} . Since i was chosen arbitrarily, the profile \tilde{B} is a Nash equilibrium. ■

We will now define a similar bidding strategy with narrow retaliation.

Definition 8 (Buyer Preferred Strategy with Narrow Retaliation) *A buyer preferred strategy with narrow retaliation involves a buyer who will bid his valuation on another bidder's most preferred item when that bidder bids on his most preferred item. He will not bid on any other bidder's most-preferred items.*

Formally, let $\overleftarrow{B}_i =$

$$\begin{cases} [0, 0, 0, 0, 0, 0, 0, 0] & \text{if } s_{2k_{i1}} = i \\ \begin{cases} \overleftarrow{B}_{ik_{i1}} = 10 \\ \overleftarrow{B}_{il} = 0, l \neq k_{i1} \end{cases} & \text{if } s_{1k_{i1}} = 0 \\ \begin{cases} \overleftarrow{B}_{ik_{i1}} = \vec{v}_{2k_{i1}} + \epsilon \\ \overleftarrow{B}_{ik(s_{2k_{i1}})_1} = v_{ik(s_{2k_{i1}})_1} \\ \overleftarrow{B}_{il} = 0 \text{ for } l \neq k(s_{2k_{i1}})_1, k_{i1} \end{cases} & \text{if } s_{2k_{i1}} \neq i \text{ and } s_{1k_{i1}} \neq 0 \end{cases}$$

It should be apparent that if all bidders play \overleftarrow{B}_i , or any combination \overleftarrow{B}_i and \tilde{B}_i , we will have an outcome where all bidders receive their most preferred item at price 10. Corollary 1 shows the strategy profile \overleftarrow{B} is a Nash equilibrium.

Corollary 1 *For any buyer i facing \overleftarrow{B}_{-i}*

(i) *He cannot make a profit on his non-most-preferred items. For all $j \neq k_{i1}$, $\Pi_{ij}(\overleftarrow{B}_i, \overleftarrow{B}_{-i}) = 0 \geq \Pi_{ij}(B_i, \overleftarrow{B}_{-i}) \forall B_i \Rightarrow \overleftarrow{B}_i \in \arg \max_{B_i} \Pi_{ij}(B_i, \overleftarrow{B}_{-i})$*

(ii) *He cannot make a greater profit on his most-preferred item than from strategy \overleftarrow{B}_i . For $j = k_{i1}$, $\Pi_{ij}(\overleftarrow{B}_i, \overleftarrow{B}_{-i}) = v_{ik_{i1}} - 10 \geq \Pi_{ij}(B_i, \overleftarrow{B}_{-i}) \forall B_i \Rightarrow \overleftarrow{B}_i \in \arg \max_{B_i} \Pi_{ij}(B_i, \overleftarrow{B}_{-i})$*

(iii) *The strategy profile $\overleftarrow{B} = (\overleftarrow{B}_i, \overleftarrow{B}_{-i})$ is a Nash equilibrium.*

Proof. Identical to the proof of Lemma 1 with \tilde{B} replaced by \overleftarrow{B} . ■

Both \overleftarrow{B} and \tilde{B} feature negative reciprocity against anyone who bids on a bidder's most preferred item. While this property is not necessary to prove that \overleftarrow{B} and \tilde{B} are Nash equilibria—the proof of Lemma 1 only requires bidders

retake their most preferred item when it is bid on—it does lead to an interesting corollary.

Lemma 2 Facing $\overleftrightarrow{B}_{-i}$ or \tilde{B}_{-i} , it is never a best response for i to bid on items other than his most preferred when no one has bid on his. He should not initiate competitive bidding. Formally, if $j \neq k_{i1}$, $s_{2k_{i1}} \in \{0, i\}$ then $\forall B_i$ s.t. $B_{ij'}(s) > 0$, $\Pi_i(B_i, \tilde{B}_{-i}) < \Pi_i(\tilde{B})$ and $\Pi_i(B_i, \overleftrightarrow{B}_{-i}) < \Pi_i(\overleftrightarrow{B})$.

Proof. By Lemma 1 (i), $\forall B_i$, $\Pi_{ij}(B_i, \tilde{B}_{-i}) \leq 0 \forall j \neq k_{i1}$. By Lemma 1 (ii), $\forall B_i$, $\Pi_{ik_{i1}}(B_i, \tilde{B}_{-i}) \leq v_{ik_{i1}} - 10$. If $\Gamma(B_i, \tilde{B}_{-i})_{2k_{i1}} \neq i$, $\Pi_i(B_i, \tilde{B}_{-i}) \leq 0 < \Pi_i(\tilde{B})$. If $\Gamma(B_i, \tilde{B}_{-i})_{2k_{i1}} = i$, by our given, $\exists s', j' \neq k_{i1}$ on the equilibrium path s.t. $\Gamma(s', B_i, \tilde{B}_{-i})_{2j'} = i$. Following \tilde{B}_{-i} , the buyer who values j' most will bid his valuation on k_{i1} . Thus $\Gamma(B_i, \tilde{B}_{-i})_{2k_{i1}} > 10 \implies \Pi_{ik_{i1}}(B_i, \tilde{B}_{-i}) < \Pi(\tilde{B})$. The proof can be repeated with \tilde{B} replaced by \overleftrightarrow{B} to show $\Pi_i(B_i, \overleftrightarrow{B}_{-i}) < \Pi(\overleftrightarrow{B})$. ■

Let us define a new bidding strategy that will produce seller-preferred equilibria.

Definition 9 (Myopic Strategy) A myopic strategy involves a buyer who bids his valuation on all items except his most preferred. On his most preferred he bids the second highest valuation of the item plus a minimal amount.

Formally, let $\hat{B}_i =$

$$\begin{cases} \hat{B}_{ik_{i1}} = \vec{v}_{2k_{i1}} + \epsilon \\ \hat{B}_{il} = v_{il}, l \neq k_{i1} \end{cases}$$

Lemma 3 The strategy profile $\hat{B} = (\hat{B}_1, \hat{B}_2, \hat{B}_3, \hat{B}_4, \hat{B}_5, \hat{B}_6, \hat{B}_7, \hat{B}_8)$ is a Nash equilibrium.

Proof. Choose any i . An identical argument as the proof of Lemma 1 (i) with \tilde{B} replaced with \hat{B} reveals $\hat{B}_i \in \arg \max_{B_i} \left\{ \Pi_{ij}(B_i, \hat{B}_{-i}) \right\} \forall j \neq k_{i1}$.

Now $\Pi_{ik_{i1}}(\hat{B}_i, \hat{B}_{-i}) = v_{ik_{i1}} - \Gamma(\hat{B})_{1k_{i1}} = v_{ik_{i1}} - \vec{v}_{2k_{i1}} - \epsilon > 0$. Clearly, for any B_i s.t. $\Gamma(B_i, \hat{B}_{-i})_{1k_{i1}} > \Gamma(\hat{B})_{1k_{i1}}$ implies $\Pi_{ik_{i1}}(B_i, \hat{B}_{-i}) < \Pi_{ik_{i1}}(\hat{B})$. If $\Gamma(B_i, \hat{B}_{-i})_{1k_{i1}} = \Gamma(\hat{B})_{1k_{i1}}$ then $\Pi_{ik_{i1}}(B_i, \hat{B}_{-i}) = \Pi_{ik_{i1}}(\hat{B})$. If $\Gamma(B_i, \hat{B}_{-i})_{1k_{i1}} < \Gamma(\hat{B})_{1k_{i1}}$, we know $\Gamma(B_i, \hat{B}_{-i})_{2k_{i1}} \neq i$ and $\Pi_{ik_{i1}}(B_i, \hat{B}_{-i}) = 0$, because another buyer who values the item less has bid $\vec{v}_{2k_{i1}}$. Thus $\hat{B}_i \in \arg \max_{B_i} \left\{ \Pi_{ik_{i1}}(B_i, \hat{B}_{-i}) \right\}$.

It follows that $\hat{B}_i \in \arg \max_{B_i} \left\{ \Pi_i(B_i, \hat{B}_{-i}) \right\}$. Hence \hat{B} is a Nash equilibrium. ■

There is a continuum of equilibria between the buyer-preferred and seller-preferred equilibrium. It features buyers with retaliatory bidding beyond a certain price level.

Definition 10 (Generalized Strategy with Broad Retaliation) *A generalized strategy with broad retaliation involves a buyer that bids between the reservation price and the second highest valuation on all items. He will defect to bidding his valuation on all non-most-preferred items and a price higher than any other valuation of his most preferred item if anyone bids above his initial bid on his most-preferred item. Formally, $\forall i, j$ let $10 < p_j < \bar{v}_{2j}$ and $q_{ij} = \min \{p_j, v_{ij}\}$. Then $\bar{B}_i =$*

$$\left\{ \begin{array}{l} [0, 0, 0, 0, 0, 0, 0, 0] \text{ if } s_{2k_{i1}} = i \\ \left\{ \begin{array}{l} \bar{B}_{ik_{i1}} = p_{ik_{i1}} + \epsilon \\ \bar{B}_{il} = q_{il}, l \neq k_{i1} \end{array} \right. \text{ if } s_{2k_{i1}} = 0 \text{ or } s_{2k_{i1}} \leq p_{k_{i1}} \\ \left\{ \begin{array}{l} \bar{B}_{ik_{i1}} = \bar{v}_{2k_{i1}} + \epsilon \\ \bar{B}_{il} = v_{il}, l \neq k_{i1} \end{array} \right. \text{ if } s_{2k_{i1}} \neq i \text{ and } s_{2k_{i1}} > p_{k_{i1}} \end{array} \right.$$

Lemma 4 *The strategy $\bar{B} = (\bar{B}_1, \bar{B}_2, \bar{B}_3, \bar{B}_4, \bar{B}_5, \bar{B}_6, \bar{B}_7, \bar{B}_8)$ is a Nash equilibrium.*

Proof. Choose any i . An identical argument to the proof of Lemma 1 (i) with \tilde{B} replaced by \bar{B} reveals $\bar{B}_i \in \arg \max_{B_i} \{\Pi_{ij}(B_i, \bar{B}_{-i})\} \forall j \neq k_{i1}$.

Now $\Pi_{ik_{i1}}(B_i, \bar{B}_{-i}) = v_{ik_{i1}} - \Gamma(\bar{B})_{1k_{i1}} = v_{ik_{i1}} - p_{k_{i1}} - \epsilon > 0$. An identical argument as in the proof Lemma 3 with \tilde{B} replaced by \bar{B} and $\bar{v}_{2k_{i1}}$ replaced by $p_{k_{i1}}$ reveals $\bar{B}_i \in \arg \max_{B_i} \{\Pi_{ik_{i1}}(B_i, \bar{B}_{-i})\}$. Thus $\bar{B}_i \in \arg \max_{B_i} \{\Pi_{ij}(B_i, \bar{B}_{-i})\}$

$\forall j \implies \bar{B}_i \in \arg \max_{B_i} \{\Pi_i(B_i, \bar{B}_{-i})\}$. Hence \bar{B} is a Nash equilibrium. ■

We now turn our attention to the simultaneous descending price auction.

Definition 11 (SDPA bidding function) *Let $T = \{n : n \in \mathbb{N}, 1 \leq n \leq 900\}$. For all $i, s \in S, t \in T$ we will define buyer i 's strategy as a bidding function $\beta_i : S \times T \rightarrow \{0, 1\}^8$. The function $\beta_i(t, s)$ is a 1×8 matrix that represents buyer i 's bid on all eight items in state s at time t .*

Note that time will be discrete in this model for simplicity, but the general properties of the SDPA will remain with continuous time.

Definition 12 (SDPA mechanism) *Define $\tilde{\Gamma}'(t, s, \beta) =$*

$$\left[\begin{array}{l} s_{11} + tI(s_{11}, 0) \max \{\beta_i(t, s)_1\} \dots \\ s_{21} + I(s_{21}, 0) \arg \min_{\{i: \beta_i(s)_1=1\}} \{v_{i1}\} \dots \end{array} \right].$$
 Then $\tilde{\Gamma}(s, \beta) = \tilde{\Gamma}'(0, \dots, \tilde{\Gamma}'(900, s, \beta), \dots, \beta)$.

The SDPA mechanism assigns ties to the bidder with lower valuation of the item. This assumption makes the next proofs easier to generalize, but is not essential to show the existence and non-existence of certain equilibrium. We can describe the most common types of bids in a manner similar to a sealed bid auction.

Definition 13 A general class of bidding functions will denote those with finite cut-off points. They will be written as $\xi_i = (\xi_{i1}, \xi_{i2}, \xi_{i3}, \xi_{i4}, \xi_{i5}, \xi_{i6}, \xi_{i7}, \xi_{i8}) = \beta_i(s, t)$ s.t. $\forall s \in S$

$$\beta_i(s, t) = \left[\begin{array}{cccccccc} \begin{matrix} 0 & \text{if } t > \xi_{i1} \\ 1 & \text{if } t \leq \xi_{i1} \end{matrix} & \begin{matrix} 0 & \text{if } t > \xi_{i2} \\ 1 & \text{if } t \leq \xi_{i2} \end{matrix} & \begin{matrix} 0 & \text{if } t > \xi_{i3} \\ 1 & \text{if } t \leq \xi_{i3} \end{matrix} & \begin{matrix} 0 & \text{if } t > \xi_{i4} \\ 1 & \text{if } t \leq \xi_{i4} \end{matrix} & \begin{matrix} 0 & \text{if } t > \xi_{i5} \\ 1 & \text{if } t \leq \xi_{i5} \end{matrix} & \begin{matrix} 0 & \text{if } t > \xi_{i6} \\ 1 & \text{if } t \leq \xi_{i6} \end{matrix} & \begin{matrix} 0 & \text{if } t > \xi_{i7} \\ 1 & \text{if } t \leq \xi_{i7} \end{matrix} & \begin{matrix} 0 & \text{if } t > \xi_{i8} \\ 1 & \text{if } t \leq \xi_{i8} \end{matrix} \end{array} \right]$$

A seller-preferred equilibrium exists.

Definition 14 (Descending Myopic Strategy) A myopic strategy in the SDPA involves a buyer that bids on all non-preferred items at all levels below his valuation. He bids on his most preferred item at all levels below the minimum value

necessary to acquire it. Formally let $\hat{\xi}_i = \begin{cases} \hat{\xi}_{ik_{i1}} = \vec{v}_{2k_{i1}} + \epsilon \\ \hat{\xi}_{il} = v_{il}, l \neq k_{i1} \end{cases}$.

Lemma 5 The strategy profile $\hat{\beta} = (\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3, \hat{\xi}_4, \hat{\xi}_5, \hat{\xi}_6, \hat{\xi}_7, \hat{\xi}_8)$ is a Nash equilibrium.

Proof. Identical to Lemma 3 with \hat{B} replaced by $\hat{\beta}$ and Γ replaced by $\tilde{\Gamma}$. ■

However, there is not an analogous continuum of equilibria nor a buyer-preferred equilibrium in the SDPA.

Lemma 6 There does not exist another Nash equilibrium of the form $\beta = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7, \xi_8)$ where $\exists \tilde{\Gamma}(\beta)_{1j'} < \vec{v}_{2j'}$ for any j' .

Proof. Suppose not, that there exists such $\bar{\beta}$. Then $\exists j'$ s.t. $\tilde{\Gamma}(\bar{\beta})_{1j'} < \vec{v}_{2j'}$. Let $i = \tilde{\Gamma}(\bar{\beta})_{2j'}$. There exists an $i' \neq i$ s.t. $v_{i'j'} \geq \vec{v}_{2j'}$. Let $\xi'_{i'm} = \bar{\xi}_{i'm} \forall m \neq j'$ and $\xi'_{i'j'} = \vec{v}_{2j'} - \epsilon > \tilde{\Gamma}(\bar{\beta})_{1j'}$. Then $\Pi_{i'j'}(\xi'_{i'}, \bar{\beta}_{-i'}) > \Pi_{i'j'}(\xi'_{i'}, \bar{\beta}_{-i'})$ and $\Pi_{i'm'}(\xi'_{i'}, \bar{\beta}_{-i'}) = \Pi_{i'm'}(\xi'_{i'}, \bar{\beta}_{-i'}) \forall m \neq j'$ imply $\Pi_{i'}(\xi'_{i'}, \bar{\beta}_{-i'}) > \Pi_{i'}(\bar{\beta}_{i'}, \bar{\beta}_{-i'})$. Thus $\bar{\beta}_{i'}$ is not a best response to $\bar{\beta}_{-i'}$ and $\bar{\beta}$ is not a Nash equilibrium. ■

Experiment Instructions

Today you will be participating in a series of auctions with multiple items for sale. Before each auction, you will receive a sheet of paper, which contains your and other subjects' valuations of the items. A sample valuation sheet looks like the following:

		Items			
		1	2	3	4
Subjects	121	128	207	5	434
	122	318	372	25	94
	123	357	295	325	168
	124	422	197	152	780

The unit of valuation is in francs. In the table above, for example, subject 123 values Item 2 at 295 francs (in bold; intersection of highlighted row and column). Subject 123 also values items 1, 3, and 4 at values of 357, 325, and 168 francs respectively. In this experiment, two francs are equivalent to one cent.

In each round, a subject's earning is the sum of his profits from all the items he acquires. The profit that a subject makes on each item equals his valuation minus his final bid. For example, if subject 4 acquires item 1 and 4 with price 322 and 600, then his profit equals $(422-322)+(780-600)=280$ francs.

A subject's total earning in this experiment equals the sum of his earnings in each round. The subjects will be paid in cash when the experiment ends.

Recording Items Acquired:

In this example, in some round, let's say 5, subject 4 has acquired items 1 and 4 at price 322 and 600. He should record these transactions in his record sheet as follows:

Round	Item			
	Purchased	Price	Value	Gain
5	1	322	422	100
	4	600	780	180

Notice that subject 4 has recorded his valuation from his first table into this table under "value". "Gain" is the difference of value over price.

A description of how items are acquired is explained on a separate diagram.

