7–32. Consider a two-dimensional harmonic oscillator with Hamiltonian

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{k}{2}(x^2 + y^2)$$

According to the principle of equipartition of energy, the average energy will be $2kT$. Now transform this Hamiltonian to plane polar coordinates to get

$$H = \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) + \frac{k}{2}r^2$$

What would you predict for the average energy now? Show by direct integration in plane polar coordinates that $\bar{\varepsilon} = 2kT$. Is anything wrong here? Why not?

8–15. The classical rotational kinetic energy of a symmetric top molecule is

$$K = \frac{p_\theta^2}{2I_A} + \frac{(p_\phi - p_\psi \cos\theta)^2}{2I_A \sin^2\theta} + \frac{p_\psi^2}{2I_C}$$

where I_A, I_A, and I_C are the principal moments of inertia, and θ, ϕ, and ψ are the three Euler angles. Derive the classical limit of the rotational partition function for a symmetric top molecule. Hint: Recall that the Euler angles have the ranges:

$$0 \le \theta \le \pi$$
$$0 \le \phi \le 2\pi$$
$$0 \le \psi \le 2\pi$$

8–16. The classical Hamiltonian for an asymmetric top molecule with principal moments of inertia I_A, I_B, and I_C is given by

$$H = \frac{1}{2I_A \sin^2\theta}\{(p_\phi - p_\psi \cos\theta)\cos\psi - p_\theta \sin\theta \sin\psi\}^2$$
$$+ \frac{1}{2I_B \sin^2\theta}\{(p_\phi - p_\psi \cos\theta)\sin\psi + p_\theta \sin\theta \cos\psi\}^2 + \frac{1}{2I_C}p_\psi^2$$

Derive the classical limit of the rotational partition function for an asymmetric top molecule. Hint: It may help to rearrange the Hamiltonian and integrate over p_θ, p_ϕ, p_ψ in that order.