Homework 8

P 2.1 – counts as 2 problems

M 10-4 see below
[Λ is the thermal de Broglie wavelength, p is the pressure.]

M 10-27 see next page
[10-26 is only for background.]

M 10-29 see next page
[Interpretation for us: Get E from our grand partition function for the radiation field, then use this and energy flux = (1/4)(E/V)c to get the Stefan Boltzmann law, evaluating σ_B.]

10–4. Derive Eqs. (10–11) and (10–12) from Eqs. (10–9) and (10–10).

$$N = 2\pi\left(\frac{2m}{h^2}\right)^{3/2} V \int_0^\infty \frac{\lambda\varepsilon^{1/2}e^{-\beta\varepsilon}\,d\varepsilon}{1+\lambda e^{-\beta\varepsilon}} \tag{10–9}$$

$$pV = 2\pi kT\left(\frac{2m}{h^2}\right)^{3/2} V \int_0^\infty \varepsilon^{1/2}\ln(1+\lambda e^{-\beta\varepsilon})\,d\varepsilon \tag{10–10}$$

$$\rho = \frac{1}{\Lambda^3}\sum_{l=1}^{\infty}\frac{(-1)^{l+1}\lambda^l}{l^{3/2}} \tag{10–11}$$

$$\frac{p}{kT} = \frac{1}{\Lambda^3}\sum_{l=1}^{\infty}\frac{(-1)^{l+1}\lambda^l}{l^{5/2}} \tag{10–12}$$

[next page]

10–26. Show that the Planck blackbody distribution can be written in terms of wavelengths λ rather than frequency:

$$\rho(\lambda, T)\, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

where $\rho(\lambda, T)\, d\lambda$ is the amount of energy between wavelength λ and $\lambda + d\lambda$.

10–27. If ω_{max} is the frequency at which $\rho(\omega, T)$ is a maximum, illustrate by maximizing $\ln \rho(\omega, T)$ that ω_{max} is given by

$$\frac{\hbar\omega_{max}}{kT} = 3(1 - e^{-\hbar\omega_{max}/kT})$$

and so

$$\frac{\hbar\omega_{max}}{kT} = 2.82$$

Similarly show that

$$\lambda_{max} T = 0.290 \text{ cm-deg}$$

10–29. In Problem 7–24, it was shown that the number of molecules striking a surface per unit area per unit time is $\rho\bar{v}/4$. By a similar approach, show that the total energy flux radiated by a blackbody is

$$e(T) = \frac{c}{4}\frac{E}{V} = \sigma T^4$$

where $\sigma = 2\pi^5 k^4/15h^3c^3$. This result is known as the Stefan-Boltzmann law, and σ is the Stefan-Boltzmann constant. Verify that σ, a universal constant, equals 5.669×10^{-5} erg/cm^2-deg^4-sec.