Homework 7

Math 469 (section 500), Spring 2016

This homework is due on Thursday, March 3. For full credit, show your work.

0. *(This problem is not to be turned in.)* Explore `opentreeoflife.org`

1. Consider the following two DNA sequences of length ten:

$$\sigma_1 = \text{ACGGCTTAGG} \quad \text{and} \quad \sigma_2 = \text{CGAGTCTATG} \,.$$

How many alignments do these two sequences have? Find all alignments of these two sequences that maximize the number of matches.

2. The following 5×5-matrix D has two unknown parameters x and y:

$$D = \begin{pmatrix} 0 & 4 & 10 & 8 & 7 \\ 4 & 0 & 12 & 10 & 9 \\ 10 & 12 & 0 & x & 7 \\ 8 & 10 & x & 0 & y \\ 7 & 9 & 7 & y & 0 \end{pmatrix} \,.$$

Draw the set of all points (x, y) in the plane for which D is a metric. In your diagram, mark all points (x, y) for which D is a tree metric.

3. Let d be the metric which gives the pairwise distances (in miles) among the four cities College Station, Dallas, Austin, and Houston. Build a phylogenetic tree on these "taxa" by applying the Neighbor-Joining Algorithm to d.

4. (a) Is there is a tree with 6 leaves for which both 14|2356 and 1234|56 are splits? If so, find one. If not, explain.

   (b) Give an example of a split that is *not* compatible with 123|456.

5. (This part of your homework pertains to your final project) *You may write these together with your project partner – if so, only one of you needs to turn in this part, but state clearly on both homeworks that you are doing this.*

   (a) Pick one main mathematical result (for instance, a theorem) from the paper you are reading. State it (mathematically).

   (b) Interpret (in words) the result you stated in (a), and describe (in several sentences) its scientific and/or mathematical significance.