

# Homework 6

Math 469 (section 500), Spring 2019

This homework is due on Thursday, February 21.

0. Read Section 2.7

1. This problem concerns the relationship between *locally stable* equilibria (page 39) and *sensitive dependence on initial condition*<sup>1</sup> (page 60). For this problem, assume that  $\bar{x}$  is an equilibrium of a difference equation  $x_{t+1} = f(x_t)$ , where  $f : I \rightarrow I$  is a function (for some interval  $I$ ).

(a) Complete the following sentence (by negating the definition): *The function  $f$  does **not** have sensitive dependence on initial condition  $x_0$  if . . . .*

(b) Prove or disprove: *If  $\bar{x}$  is locally stable, then  $f$  does **not** have sensitive dependence on initial condition  $x_0 = \bar{x}$ .*

(c) Prove or disprove: *If  $f$  does **not** have sensitive dependence on initial condition  $x_0 = \bar{x}$ , then  $\bar{x}$  is locally stable.*

2. Section 2.12 #16–18, 20

3. Is anything in Sections 2.1–2.7 related to the paper you are reading for the final project? Explain briefly.

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<sup>1</sup>In Definition 2.7 on page 60, “an integer  $k$ ” should be “a nonnegative integer  $k$ ”.