Geodesics on Tensor Product Surfaces

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The calculation of a geodesic path on an arbitrary surface is a hard problem. It often involves solving a differential equation or a system of differential equations. These differential equations are often solved using numerical methods or approximation methods due to their difficulty. For this project, I plan to look specifically at the calculation of geodesics on tensor product surfaces. This is an important problem because tensor product surfaces have many applications in many different areas of study. These areas of study include, but are not limited to, engineering and the apparel industry as well as computer graphics and geometric modeling.

For this project, I plan to view the calculation of the geodesic as a minimization problem of the distance between a sequence of points on the tensor product surface. Because calculating distances on the surface is difficult, I plan to approximate the distance using the Euclidian distance, as the surface will be represented in 3D space. It is expected that the minimization process will result in a sequence of points that are not equally spaced along the approximate geodesic. To fix this, I plan to try a combination of increasing the number of points in the sequence and reparameterizing the curve after the minimization is completed.

As to the originality of this work, I was able to find previous work that addressed Bezier tensor product surfaces, but I wasn't able to find much about arbitrary tensor product surfaces. It is possible that work on this exists, and I simply am not aware of it. Most of the papers I found calculated geodesics using a numerical method for solving the differential equation(s), but I was also able to find some that used a minimization of a discrete path as well. Most of the papers that used the later method approximated the length of their curve using a spline rather than the straightline distance. So, there are aspects of this project that could be considered original, but they are smaller components.

List of Goals:

- Goals for first update deadline:
  - Determine how to represent the tensor product surface and paths in the code.
  - Be able to visualize a tensor product surface and paths along that surface.

  - Determine level of user-interactivity desired in project code.

- Goals for the second update deadline:
  - Define objective function(s) for the minimization problem.
  - Be able to run the minimization to get a shortest discrete path on the tensor product surface.

- Goals for the final deadline:
  - Adjust the output from the minimization if needed to generate a curve with points evenly spaced along the arc length.
  - Be able to visualize the intermediate steps of the minimization process.
  - Determine the minimization settings which give the best results.

- Stretch goals:
  - Utilize machine learning to speed up the calculation of these approximate geodesics.
  - There are several interesting strategies used in the discrete setting for calculating geodesics, I'd be interested in investigating if any of them could apply to the continuous case as well.