Quiz 1

- 5 minute individual quiz;
- Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
- Show and explain all work;
- Underline the answer of each steps;
- The use of books, personal notes, calculator, cellphone, laptop, and communication with others is forbidden;
- By taking this quiz, you agree to follow the university’s code of academic integrity.

**Exercise 1** 20%

For each differential equation, provide its order and say whether it is linear or not:

1. \( \frac{dy}{dx}(x) = \frac{y(x)(2 - 3x)}{x(1 - 3y(x))} \)

2. \( 3 \frac{d^2y}{dx^2}(x) + 4 \frac{dy}{dx}(x) + 9y(x) = 2 \cos(3x) \)

**Exercise 2** 80%

The quantity of a radiactive material \( Q(t) \) disintegrates according to the following ODE

\[ \frac{d}{dt} Q(t) = -rQ(t), \]

where \( r > 0 \).

- Find an expression of \( Q(t) \) at any time \( t \).
- Find the time required for \( Q(t) \) to decay to one-half its original amount.
Quiz 1: solutions

Exercise 1  20%

1. First order, nonlinear.
2. Second order, linear.

Exercise 2  80%

- The ODE is a first order linear ODE but is also separable. We can use either techniques seen in class. We go for the separable point of view and divide by \( Q(t) > 0 \):

\[
\frac{1}{Q} Q' = -r.
\]

Taking the anti-derivate and using the substitution rule yield

\[
\int_{Q=Q(t)} \frac{1}{Q} dq = -rt + C
\]

for any constant \( C \). This implies

\[
\ln |Q(t)| = -rt + C
\]

or

\[
|Q(t)| = Ce^{-rt}
\]

for any positive constant. Because \( Q(t) \) does not change sign (continuous) and we are interested in positive solutions, we deduce that

\[
Q(t) = Ce^{-rt}
\]

for any positive constant \( C \), which is the desired expression.

- Notice that if \( Q_0 \) denotes the original amount, we are looking for \( t^* \) such that

\[
Q_0 e^{-rt^*} = \frac{1}{2} Q_0
\]

or

\[
t^* = \ln 2/r.
\]