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NOISE REDUCTION WITH MULTISCALE EDGE REPRESENTATION AND PERCEPTUAL CRITERIA

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ABSTRACT

The wavelet multiscale edge representation of signals developed by Mallat and Zhong provides a new tool for signal and image processing. We built up a tree structure of wavelet-transform(WT) maxima and developed metrics for analyzing the WT maxima tree. These metrics fall into two classes corresponding to two perceptual criteria, scaling and spatial stabilities, for discriminating features from background noise. Identified noisy branches are trimmed off the WT maxima tree. This technique of noise reduction preserves edges well while suppressing noise. We present experimental results of processing simulated and medical images with this technique.

1. INTRODUCTION

It has been well-known that the human visual system (HVS) performs "multichannel decompositions"[1] on input visual signals. This is done with retinal receptive fields of different sizes. There is physiological evidence that these receptive fields act like edge detectors of different sizes and with different spatial tunings. These findings led Marr[2][3] to conjecture that a multiscale edge representation of visual information might be complete. Recently, with the development of wavelet multiscale representations, there have been a number of investigations on this subject[4][5][6][7]. Results from these investigations show that a multiscale edge representation is precise and stable in terms of the quality of reconstruction[6], though it is not complete in general[7]. Such a representation is suitable for practical signal and image processing. Its applications have already been shown in compact coding[8] and noise reduction[9][10]. The multiscale edge representation resembles the functions of the HVS. Therefore, we were encouraged to study certain processing techniques that are believed to be functioning in the HVS.

In this paper, we report our continuing investigations on building perceptual-based metrics for analyzing a tree structure of WT maxima and discriminating features from noise. In [9] we obtained good results at low signal-to-noise ratio (SNR) with metrics related to one perceptual criterion, the scaling stability; however, we could not obtain much improvement in the regions where contrast-to-noise ratio (CNR) was low. Better results can be obtained by adding another criterion, the spatial stability, to the process of selecting signal maxima from noise. Metrics of both criteria for assessing the WT maxima trees are unified. We present experimental results of noise reduction in simulated and real medical images by pruning the WT maxima tree with these metrics. We recently discovered that one of our metrics of scaling stability is similar to one employed by Mallat and Hwang[10]. They developed the metric with a rigorous analyses of singularities and therefore gave a mathematical ground to our perceptual interpretations.

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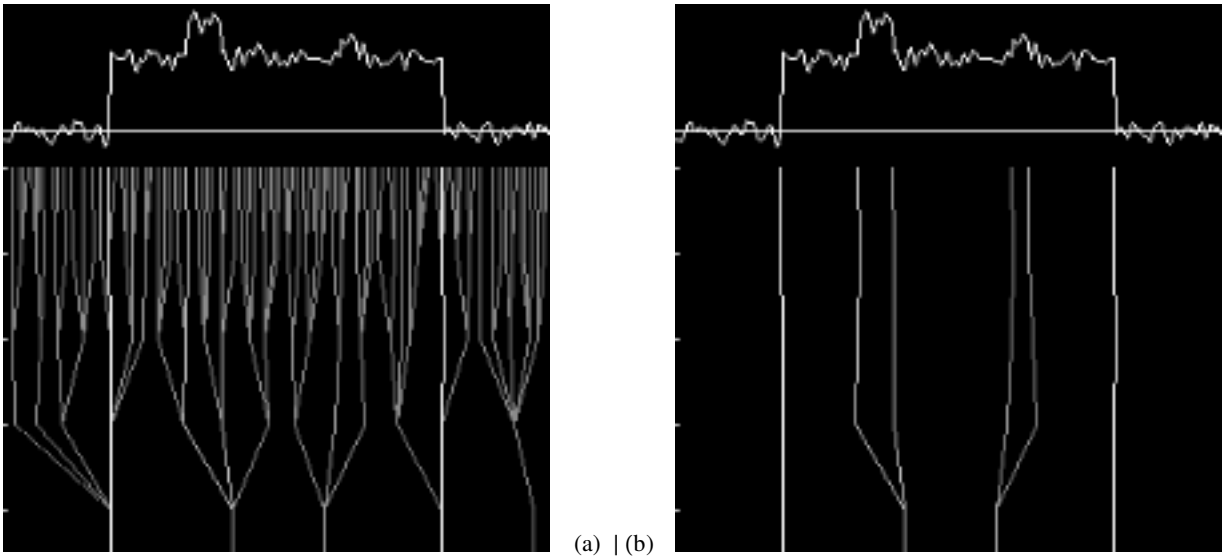
2. THE WT MAXIMA TREE AND ASSOCIATED METRICS

The original form of the WT maxima representation developed by Mallat and Zhong[6] is a set containing the local maxima of the wavelet transform modulus. Mallat and Zhong constructed a family of spline wavelets; the low order ones, quadratic and cubic, are most commonly used[6][11]. These wavelets are similar to the derivative of a Gaussian and the associated wavelet transform acts like an edge detector at multiple scales. Therefore, a collection of module local maxima of the WT is really a set of multiscale edgels (edge elements). In a WT maxima representation, each maximum (edgel) in the set has two attributes: its position in the scale-space and its amplitude. These WT maxima representations are sufficient for coding purposes, but too loosely structured for applications of signal processing and analysis. We need a representation that characterizes scale-spatial relations between WT maxima explicitly.

2.1. The WT Maxima Tree of 1-D Signals

A WT maxima tree ("tree", hereafter) is a hierarchical data structure that organizes maxima (edgels) across scales. Each node at each level on the tree represents a WT maximum point at a scale. A branch on the tree connects certain nodes according to the split/merge relations between WT maxima. The tree can be constructed by the coarse-to-fine tracking[12] of the WT maxima. If a continuous WT is used, the evolution of WT maxima can be traced precisely by slowly varying the scale parameter[13]. Uncertainties exist in the tree construction from the discrete WT maxima. Starting at the coarsest scale, we track the WT maxima from one scale to another inside a window determined by the size of the wavelet reproducing kernel (correlation function) between the two scales. Connections between maxima at two scales are made as follows.

Assume a maximum at scale 2^i, denoted by m^i, falls into the tracking windows of N maxima at the immediate coarser scale 2^{i+1}, denoted by $\{m_j^{i+1}, j=1,2,...,N\}$. Let $\{d_j, j=1,2,...,N\}$ be spatial distance from m_j^{i+1} to m^i, Then m^i will be connected to a maximum (called the father node) at scale 2^{i+1} that gives,



(a) | (b)

Fig. 1. A demonstration of the WT maxima tree and associated metrics. a) a noisy signal and its WT maxima tree. The grayness of a branch is proportional to the average amplitude of maxima on that branch. b) Branches in a) having largest P_{sc} values. They correspond to meaningful features in the signal.

$$\text{Max}\{\, | a_j^{i+1} R(d_j) |,\ j=1,2,...,N\}$$

where a_j^{i+1} is the amplitude of m_j^{i+1} and $R(x)$ is the wavelet reproducing kernel between scale 2^i and 2^{i+1}.

We find that this method of tree construction works well in general, but may lead to erroneous connections in the region where WT maxima are dense. More sophisticated methods such as probabilistic decision-making can be used to refine the tree construction in these cases.

When a WT maximum at a coarse scale splits into several maxima at the next finer scale, it is important to check which children maximum correspond to the same feature as its father. A child maximum corresponds to the same feature as its father if they have: identical signs, positional proximity and amplitude similarity. Maxima corresponding to the same feature at different scales form the path of an edge in scale-space, which is a unique branch on the tree. Fig. 1a shows a simulated signal of three boxcars embedded in a noise background (on top) and its WT maxima tree (at bottom).

2.2. The WT Maxima Tree of 2-D Images

The WT maxima tree can be generalized to represent 2-D images. In two or higher dimensions, individual WT maxima or edgels are often not independent features; they belong to certain lines or curves. A more semantic primitive would be these lines or curves instead of their elements. Mallat and Zhong[6][11] have discovered that thresholding and coding WT maxima contours are more efficient and robust than doing the same with individual maxima. We generalize the WT maxima tree to two dimensional images by assigning maxima contours to nodes on the tree.

Contouring edges at a fixed scale is a well-studied problem. Three measures are most often used to decide if two edgels should be linked. They are: geometric compatibility, positional proximity, and amplitude similarity. For contouring WT maxima, it is possible to exploit information at multiple scales. Again we follow a coarse-to-fine strategy. At the coarsest scale, we construct WT maxima contours by the above three measures. As we move down to the fine scales, the positional proximity requirement is relaxed to include proximity to the contours already constructed at the immediate coarser scale. For example, if two edgels are separated by a gap, they would still be linked if there is a contour going through these two positions at the immediate coarser scale. This algorithm reflects the idea of using features at the coarse scale as context for fine scales, which was suggested by Rosenfeld[14] nearly a decade ago.

Once WT maxima are contoured at each scale, the generalized WT maxima tree is constructed by assigning each maxima contour to a node in a tree and following a procedure similar to that described in §2.1.

2.3. Metrics on the WT Maxima Tree and Perceptual Criteria

Three metrics are designed for a WT maxima tree of 1-D signals. They are: 1) length of a tree branch; 2) average maxima amplitude over a branch; 3) rate of change of maxima amplitude along a branch.

These three metrics represent one perceptual criterion to measure the stability of an edge over scale changes, or *scaling stability* for short. Witkin has observed in a Gaussian scale-space that meaningful features which "survive over a broad range of scales tend to leap out at the eye, while the most ephemeral are not perceived at all"[12]. He then defined a measure of scaling stability as the *lifespan* of a scale-space feature along the scale dimension. Later, Bischof and Caelli justified the use of such a measure by showing that the measure is a function of cross-scale correlation and will have a weak and randomly distributed response to white noise[15]. Our first metric is similar to Witkin's in the Gaussian scale-space. The second is important in discriminating isolated noise in smooth regions that may survive over a fairly large range of scales but are of relatively small maxima amplitudes in average[16]. The third delineates different types of edges. Mallat and Hwang have shown that upon appropriate scaling of WT maxima amplitudes, the third measure is an estimate of the Lipschitz regularity of singularities. A noise process has almost all singularities of negative Lipschitz exponent, implying that a

maxima branch that has an increased amplitude as scale decreases is most likely to be noise driven. In Fig. 1a, the grayness of each tree branch is proportional to the average maxima amplitude over that branch. We pick out tree branches that have largest length-amplitude products in Fig. 1b. It can be seen that those branches correspond to the meaningful features we hope to keep. We now aggregate the first two metrics by defining

P_{sc} = *(length)*(average amplitude) of a tree branch.*

Another three metrics are employed for WT maxima contours. They are: 1) length of the maxima contour; 2) average amplitude of the maxima contour; 3) closure of the maxima contour.

These three metrics represent another perceptual criterion called *spatial stability*. In contouring WT maxima, we have used two important principles of perceptual groupings[17] known as proximity and (amplitude) similarity. A large group often indicates spatially stable features that catch our eyes. Indeed, noisy maxima contours are often short, of small average amplitude and do not self-close. The product of the first two metrics have been used by Mallat and Zhong for compact image coding[8][11]. We define this product by

P_{sp} = *(length)*(average amplitude) of a maxima contour.*

It is easy to see that P_{sc} and P_{sp} are similar in that the former measures across scales around a certain position in an image while the latter measures across the image plane at a fixed scale. The advantage of using a multiscale representation is that we can benefit from employing both criteria. Finally, for a generalized WT maxima tree of images we define a unified metric,

$$P = P_{sc}\ P_{sp}$$

A high value of P indicates that the measured feature has significant content in scale-space.

3. EXPERIMENTAL RESULTS

We have implemented algorithms of noise reduction by the method described above for both 1-D signals and 2-D images. P_{sc} and P are used in 1-D and 2-D algorithms, respectively, for assessing the WT maxima tree. Branches having low values of P_{sc} or P are trimmed off the WT maxima tree. The processed signal can then be reconstructed from the trimmed WT maxima tree by a method developed by Mallat and Zhong[6].

A threshold must be set up for both P_{sc} or P in the program as in many other techniques. It is hoped that a good metric should make threshold selection easy, and this selection should not be arbitrary. Fig. 2 is a histogram of P_{sc} values of all branches in the WT maxima tree of Fig. 1a. It can be seen that noisy branches form a big cluster of low P_{sc} values. This is a typical P_{sc} histogram of a noisy signal. Without any knowledge about the noise, a reasonable choice of a threshold would be around the valley down the big bump. We have suggested [9] that in the low CNR regions, P_{sc} is not a very effective metric. This is because when a signal maximum has a similar strength to that of noisy maxima, The P_{sc} value of the signal maximum falls into the P_{sc} cluster of noisy maxima's. In two dimensions, it is hoped that the use of P_{sp} , as combined in P, can help pull out small features from the noise cluster.

Figs. 3a shows a simulated phantom images with the addition of Gaussian white noise. The image was processed by the 1-D algorithm along rows and columns separately with P_{sc} thresholdings. The result is shown in Fig. 3b. It appears

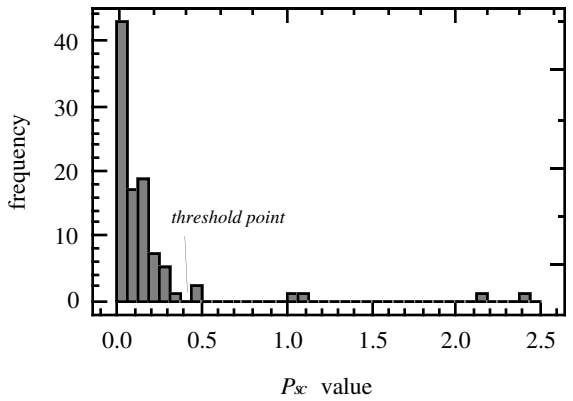


Fig. 2. A histogram of P_{sc} values of all branches in Fig. 1a.

that the 1-D algorithm is excellent in boosting SNRs and preserving edges. However, the small disks in the top row of Fig. 3b where the CNR is low appear distorted, indicating the algorithm's vulnerability at low CNRs. Fig. 4a is a noisy MR image. It was processed by the 2-D algorithm with P thresholding. The processed image is shown in Fig. 4b. It can be seen that low contrast features are preserved very well.

4. CONCLUSIONS

A tree structured WT maxima representation characterizes relations between features in the wavelet scale-space. Such representation is suitable for signal analysis and processing. We have designed metrics on the WT maxima tree that correspond to some perceptual criteria for discriminating features from background noise. This technique has been experimentally used for noise reduction on simulated and real medical images. When a unified metric for both scaling and spatial stabilities are used, the algorithm preserves edges well while suppressing noise at low SNRs and low CNRs.

REFERENCES

[1] M. D. Levine, *Vision in Man and Machine*, New York: McGraw-Hill, 1985.

[2] D. Marr and S. Ullman, "Bandpass channels, zero-crossings, and early visual information processing," *J. Opt. Soc. Am.*, vol. 69, No. 6, June 1979.

[3] D. Marr, *Vision*, San Francisco, CA: W. H. Freeman, 1982.

[4] S. G. Mallat, "Multifrequency channel decomposition of images and wavelet models," *IEEE Trans Acoust. Speech and Signal Proc.*, vol. ASSP-37, pp. 2091-2110, Dec. 1989.

[5] S. G. Mallat, "Zero-crossings of a wavelet transform," *IEEE Trans. Inform. Theory*, vol. 37, pp. 1019-1033, July 1991.

[6] S. G. Mallat and S. Zhong, "Characterization of signals from multiscale edges," NYU Technical Report No. 592, Nov. 1991.

[7] Y. Meyer, "Un contre-example a la conjecture de Marr et a celle de S. Mallat," *preprint*, 1991.

[8] S. G. Mallat and S. Zhong, "Compact image coding from edges," in *Proc. Int. Conf. Acoust. Speech and Signal Proc.,* Toronto, May 1991.

[9] J. Lu, Y. Xu, J.B. Weaver and D.M. Healy, Jr. "Noise reduction by constrained reconstructions in the

wavelet-transform domain," presented on *the 7th Multi-dimensional Signal Processing Workshop*, IEEE Signal Processing Society, Lake Placid, NY, Sept. 1991.

[10] S. G. Mallat and W. L. Hwang, "Singularity detection and processing with wavelets," *IEEE Trans. Inform. Theory*, vol. 38, pp.617-643, March 1992.

[11] S. Zhong, "Edge representation from wavelet transform maxima," Ph.D. Dissertation, New York University, September, 1990.

[12] A. P. Witkin, "Scale space filtering," in *Proc. Int. Joint Conf. Artificial Intell.*, 1983.

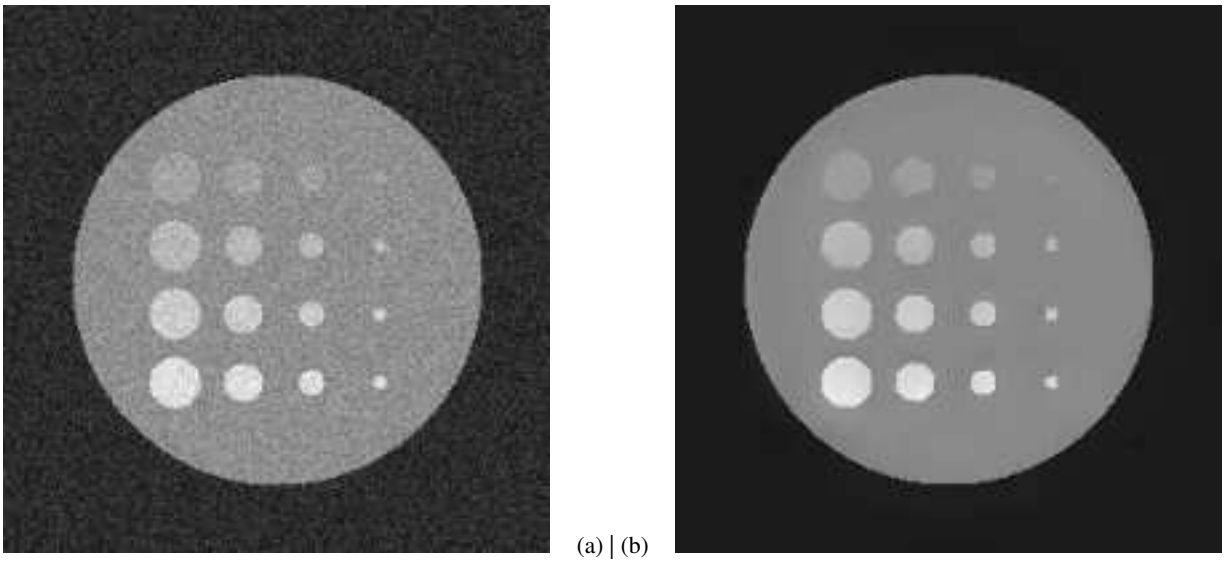
[13] F. Bergholm, "Edge focusing," *IEEE Trans. Pattern Anal. and Machine Intell.,* vol. PAMI-9, pp. 726-741, Nov. 1987.

[14] A. Rosenfeld, "Some useful properties of pyramids," In *Multiresolution Image Processing and Analysis*, A. Rosenfeld, ed., Berlin: Springer-Verlag, 1984.

[15] W. F. Bischof and T. Caelli, "Parsing scale-space and spatial stability analysis," *Computer Vision, Graphics, and Image Processing*, vol. 42, pp. 192-205, 1988.

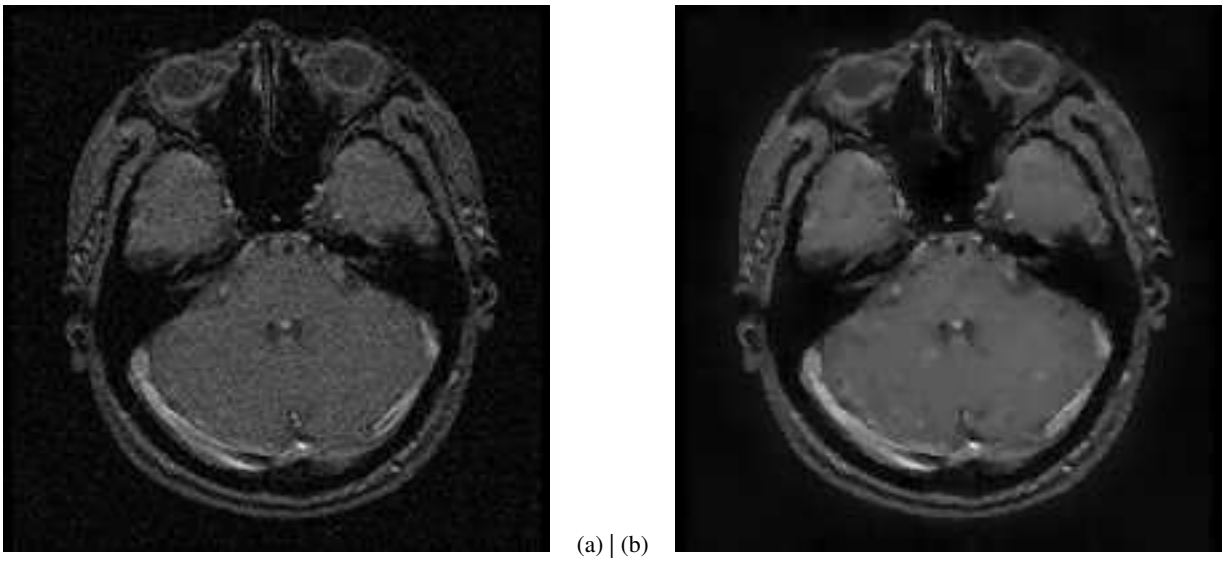
[16] T. Lindeberg and J. -O. Eklundh, "On the computation of a scale-space primal sketch," *J. Visual Comm. Image Proc.*, vol. 2, No. 2, pp. 55-78, 1991.

[17] M. Wertheimer, "Principles of perceptual organization," in *Readings in Perception*, D.C. Beardslee and M. Wertheimer, Eds., Princeton, NJ: Van Nostrand, 1958.



(a) | (b)

Fig. 3. Experiments on simulated images by the 1-D algorithm with P_{sc} thresholding. a) noisy: SNR=12db. b) processed by the 1-D algorithm. SNR=26.4db.



(a) | (b)

Fig. 4. Experiments on real MR images by the 2-D algorithm with P thresholding. a) noisy. b) processed by the 2-D algorithm.