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# STABILITY ENHANCED MODELS OF CHEMICAL ENHANCED OIL RECOVERY PROCESSES

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## ABSTRACT

In this paper, we describe some simple multi-layer Hele-Shaw models of new flooding strategy of enhanced oil recovery processes and study their stability properties. This study is motivated by the need for design of more efficient flooding strategy. The new models are first studied based on simple fluids with interacting interfaces embedded in them with surface tension constant on each interfaces but can be different for different interfaces. A necessary condition for enhancing stability of two-layer flows by introducing another fluid in between is obtained. This necessary condition is new and has very practical relevance because it narrows the choice of middle-layer fluid by a very simple criterion. Importance of this condition which has been hitherto unknown is discussed.

## 1 Introduction

Quantitative results that can advance our understanding of interaction of many layers in fluid flows are too few. Such results are useful because these can be applied directly to harness their full potential in meaningful ways. For example, consider two-layer Hele-Shaw flows which have been well-studied for over a half-century now. There are many quantitative results related to these flows. One of these is the interfacial instability when a less viscous fluid (viscosity $\mu_l$) displaces a more viscous one (viscosity $\mu_r$): main driving force for the instability here is the viscosity ratio $\mu_r/\mu_l$ (see [1]). Such interfacial instabilities can be undesirable in many applications where instability driving mechanism is same as in Hele-Shaw flows. One such application happens to be secondary oil recovery: water displacing oil. Mitigating such instabilities by some sort of intervention is desirable in such applications. Though complete elimination has not been possible, partially taming the instability has become an alternative. One common practice in doing so in oil industry has been to use tertiary or enhanced oil recovery [2]. This, at a fundamental level, essentially amounts to turning a two-layer (e.g., water and oil) flow into a three-layer flow by having the viscosity $\mu$ of the middle-layer fluid in between, i.e., $\mu_l < \mu < \mu_r$, for this reduces the viscosity ratio at the leading interface and thereby reduces the growth rate of interfacial instability, at least so it appears from linear theory. However, this is only half the story because role of surface tension in the instability mechanism has been completely undermined in the selection of middle-layer fluid by considering only its viscosity.

It is also well known that increasing the surface tension $T$ between two fluids in two-layer Hele-Shaw flows also has a stabilizing effect on the interfacial instability. In three-layer flows, surface tensions $T_r$ and $T_l$ on leading and trailing interfaces respectively could be markedly different from $T$ since $T_r$ and $T_l$ depend on the middle-layer fluid as well (See Fig. 2). The viscosity of the middle-layer fluid may satisfy the condition $\mu_l < \mu < \mu_r$, and still it may not tame the instability of the leading interface unless the interfaces separating middle-layer fluid from other two extreme-layer fluids have *appropriate* surface tensions $T_r$ and $T_l$ in comparison to the surface tension $T$ of the interface in two-layer case. For example, if the surface tension $T_r << T$, it can offset any gain in stabilization due to any reduction in viscosity jump across the leading interface. Therefore, issue at hand, other than $\mu_l < \mu < \mu_r$, is quantification of selection criterion that takes into account surface tensions $T$, $T_r$ and $T_l$ for the purpose of stabilization. It would appear that most appropriate thing would be to choose $T_r \geq T$. But this is too restrictive and in fact, as we show in this paper, $T_r$ can be significantly less than $T$ and still significant enhancement in stability can be achieved. Thus the class of fluids that can be used as middle-layer fluid is larger than what it would otherwise be based on $T_r \geq T$ only. In this paper, we obtain such a criterion by very simple algebraic manipulation of a result we have reported earlier [2].

## 2 Background

In rectilinear flow of two immiscible fluids in a Hele-Shaw cell, the interface separating these fluids is hydro-dynamically unstable when a more viscous fluid is displaced by a less viscous fluid (assuming densities of both fluids are same). If $\mu_r$ is the viscosity of the displaced fluid, $\mu_l$ ($\mu_l < \mu_r$) is the viscosity of the displacing fluid, $U$ is the constant velocity, and the surface tension coefficient at the interface is $T$, then the celebrated Saffman-Taylor formula [1] for the

$$\sigma^*_{st} = \frac{2T}{(\mu_r + \mu_l)} \left( \frac{U(\mu_r - \mu_l)}{3T} \right)^{3/2}. \tag{1}$$

Now, consider an intermediate layer of another fluid having constant viscosity $\mu$ with $\mu_l < \mu < \mu_r$ so that we have a three-layer flow with positive viscosity jump in the direction of flow at each of the two interfaces (see Fig. fig2). In this case, each of the interfaces is individually unstable but the growth rates of unstable waves on each of these interfaces are modified due to the presence of the other interface even within linear theory. This aspect has been the subject of a recent paper [2] in which it has been shown that the absolute upper bound $\sigma^*_3$ is given by

$$\sigma^*_3 = \max \left\{ \frac{2T_r}{\mu_r} \left( \frac{U(\mu_r - \mu)}{3T_r} \right)^{3/2}, \frac{2T_l}{\mu_l} \left( \frac{U(\mu - \mu_l)}{3T_l} \right)^{3/2} \right\}, \tag{2}$$

where $T_r$ and $T_l$ are the surface tension coefficients at the leading and trailing interfaces respectively.

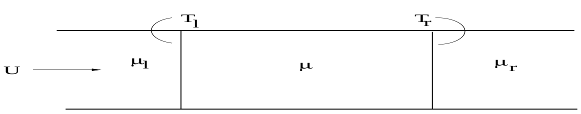


**FIGURE 1**. Three-layer Hele-Shaw flow: $\mu_l < \mu < \mu_r$. Surface tensions at leading and trailing interfaces are shown as $T_r$ and $T_l$ respectively.

## 3 A Necessary Condition for Stability Enhancement

Improvement in stability over two-layer case requires that absolute upper bound $\sigma^*_3$ for three-layer case defined in (2) is less than the absolute upper bound $\sigma^*_{st}$ defined in (1) for the two-layer case. This is guaranteed if each of the two terms in (2) is less than $\sigma^*_{st}$ given by (1). This leads to two inequalities:

$$\frac{2T_l}{\mu_l} \left( \frac{U(\mu - \mu_l)}{3T_l} \right)^{3/2} < \frac{2T}{(\mu_r + \mu_l)} \left( \frac{U(\mu_r - \mu_l)}{3T} \right)^{3/2}, \tag{3}$$

$$\frac{2T_r}{\mu_r} \left( \frac{U(\mu_r - \mu)}{3T_r} \right)^{3/2} < \frac{2T}{(\mu_r + \mu_l)} \left( \frac{U(\mu_r - \mu_l)}{3T} \right)^{3/2}. \tag{4}$$

The inequality (3) leads to

$$(\mu - \mu_l) < \left( \frac{T_l}{T} \right)^{1/3} (\mu_r - \mu_l) \left( \frac{\mu_l}{\mu_r + \mu_l} \right)^{2/3}. \tag{5}$$

Using the inequality $\mu_l/(\mu_r + \mu_l) < \mu_r/(\mu_r + \mu_l) < \mu_r/(2\mu_l)$, in inequality (5) gives

$$\mu < \mu_l + \left( \frac{T_l}{T} \right)^{1/3} (\mu_r - \mu_l) \left( \frac{\mu_r}{2\mu_l} \right)^{2/3}. \tag{6}$$

After similar manipulation with the inequality (4), we obtain

$$(\mu_r - \mu) < \left( \frac{T_r}{T} \right)^{1/3} (\mu_r - \mu_l) \left( \frac{\mu_r}{\mu_r + \mu_l} \right)^{2/3}. \tag{7}$$

Using the inequality $\mu_r/(\mu_r + \mu_l) < \mu_r/(2\mu_l)$ in (7), we obtain

$$\mu_r - \left( \frac{T_r}{T} \right)^{1/3} (\mu_r - \mu_l) \left( \frac{\mu_r}{2\mu_l} \right)^{2/3} < \mu. \tag{8}$$

The inequalities (6) and (8) when put together gives the following necessary condition after some manipulation.

$$\left( \frac{T_r}{T} \right)^{1/3} + \left( \frac{T_l}{T} \right)^{1/3} > \left( \frac{2\mu_l}{\mu_r} \right)^{2/3}. \tag{9}$$

With this necessary condition, it is appropriate here to recall the importance of the constraint $\mu_l < \mu_r$. Only when $\mu_l < \mu < \mu_r$, criterion (9) can be used to identify a class of fluids which are admissible as middle-layer fluids for the purpose of stabilization of at least one of the interfaces. Among those which are not admissible by this criterion, some may still enhance stability because the necessary criterion is based on an upper bound which may not be optimal. Important thing to note here is that it is not at all necessary for the surface tension $T_r$ to be more than $T$ and in fact, it can be very less than $T$ and still satisfy (9), a contention we alluded to earlier at the beginning of this paper. Finally, it is worth mentioning that a middle-layer fluid satisfying both $\mu_l < \mu < \mu_r$ and condition (9) may not exist for every choice of extreme-layer fluid viscosities.

## ACKNOWLEDGMENTS

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