Spring 2020 Math 152

Week in Review XII

courtesy: David J. Manuel

(covering 11.10, 11.11, and Exam III Review)

1 Section 11.10

1. Use the definition to find the Taylor series of the following functions:

   (a) $f(x) = e^{-x}$ centered at $a = -1$

   (b) $f(x) = \sin(x)$ centered at $a = \frac{\pi}{2}$

   (c) $f(x) = \ln(x)$ centered at $a = 2$

2. Use known Maclaurin Series to find the Maclaurin series of the following:

   (a) $f(x) = x\sin(2x)$

   (b) $f(x) = \int_0^x e^{-t^2}\,dt$

3. Find the coefficient of x^2 in the Maclaurin series for $f(x) = \frac{1}{1-x} \cdot e^x$.

2 Section 11.11

1. Approximate $f(x) = \sec x$ by a Taylor polynomial of degree 2 at $x = 0$.

2. Approximate $f(x) = \sqrt{x}$ by a Taylor polynomial of degree 3 at $x = 4$.

3 Exam III Review

1. Determine if the following series are absolutely convergent, convergent but not absolutely, or divergent. Explain which test you use and show clearly that all conditions are met.

   (a) $\sum_{n=1}^{\infty} \frac{n^3}{n^5 + 5n^4 + 7}$

   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+3}$

2. At least how many terms are needed to sum the series in #1(b) to within $\frac{1}{10000}$?

3. Find the radius and interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n (x-2)^n}{(n+1)2^{2n}}$.

4. Write $f(x) = \ln(1+x^3)$ as a power series centered at $a = 0$ and find its radius of convergence. (HINT: start with a power series for $f'(x)$)