Fall 2004 MATH 171

Week in Review I

courtesy of David J. Manuel

Section 1.1 and 1.2

Section 1.1

1. Given vector $\mathbf{a}$ and scalars c and d, prove that $(cd)\mathbf{a} = c(d\mathbf{a})$.

2. Given vector $\mathbf{a}$ and scalar c, prove that $|c\mathbf{a}| = |c||\mathbf{a}|$

3. Use vectors to prove the midpoint formula: If C is the midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$, then C has coordinates $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

4. A quadrilateral has one pair of opposite parallel and of equal length. Use vectors to prove that the other pair of opposite sides is parallel and of equal length.

5. The *median* of a triangle is a segment from one vertex to the midpoint of the opposite side. Use vectors to prove that the medians of an equilateral triangle are congruent.

Section 1.2

6. Given vectors $\mathbf{a}$ and $\mathbf{b}$ and scalar c, prove that $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b})$.

7. Given $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors, prove that $\mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$ is orthogonal to $\mathbf{a}$.

8. Use the definition of the dot product to prove the *Cauchy-Schwarz Inequality*: $|\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}||\mathbf{b}|$.

9. Use the Cauchy-Schwarz Inequality above to prove the *Triangle Inequality*: $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$.

10. Given a rectangle $ABCD$, form quadrilateral $EFGH$ by joining the midpoints of consecutive sides (see figure below). Use vectors to prove that $EFGH$ is a rhombus.