Section 2.1: Linear Differential Equations (PREP WORK)

(Answer the following questions using the electronic submission in Canvas-under "Prep Assignments". You have unlimited submissions, so keep a record of your answers in case you decide to change any after our in-class discussion of the assignment. Prep Assignments will automatically be scored 10 out of 10, but will be spot-checked for legitimate attempted answers-failure to do so will result in a ZERO on the assignment!).

In this section we learn how to solve first order linear ODEs. Recall that they can be written as

$$P(t)y' + Q(t)y = G(t)$$

1. Suppose we wish to solve the ODE $t^2\dfrac{dy}{dt} + 2ty = \cos(t)$.

   (a) Write down $P(t)$ and $Q(t)$. What do you notice about Q as it relates to P?

   (b) Based on this, the left side of the equation is the derivative of what product?

   (c) Use this fact to solve the ODE. Just enter the solution in eCampus.

Suppose we do not have such a convenient left side. Start by dividing both sides of the equation by $P(t)$ to obtain the form

$$y' + p(t)y = g(t)$$

2. Our goal is to multiply both sides of the equation by some function $\mu(t)$ so that we obtain this convenient relationship on the left.

   (a) The result is $\mu(t)y' + \mu(t)p(t)y = \mu(t)g(t)$. What must be true to give us a product rule on the left?

   (b) Write a differential equation for μ and t which will make that happen (just use u instead of $\mu(t)$ when entering the ODE in Canvas).

   (c) Solve this equation for μ like we did in the first example of the 1.1-1.3 notes.

2.1: Linear Differential Equations and Integrating Factors

Prep Example: $t^2 \dfrac{dy}{dt} + 2ty = \cos(t)$

What if our linear ODE isn't so "nice"? Can we make it that way?

General strategy to solve $y' + p(t)y = g(t)$:

Examples:
Given the ODE $ty' + 2y = \sin(t)$, $t > 0$:
a) Plot a direction field. Based on the direction field, describe how solutions behave for large t.

b) Find the general solution.

c) Given the initial condition $y(\pi) = 0$, find the solution and plot it with the direction field.

Given the IVP $ty' + (t+1)y = 2te^{-t}$, $y(1) = a$, $t > 0$:

a) Plot a direction field. Based on the direction field, describe how solutions behave as $t \to 0$. Estimate the critical value of a for which a transition in this behavior occurs.

b) Solve the IVP.

c) Find the exact critical value from part a). What happens to the solution as $t \to 0$ when a is this critical value?