Partitioning and Assembly of Gosper Sculptures

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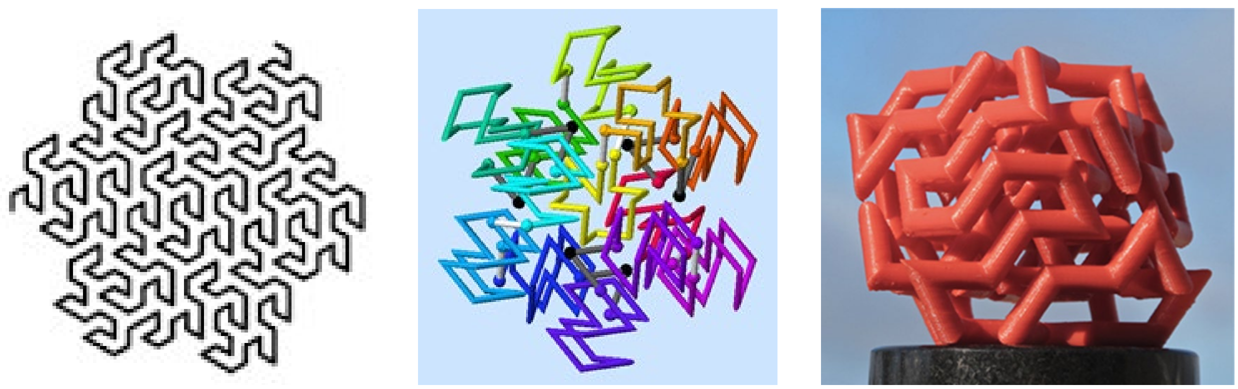
Abstract

This paper addresses the technical challenges of physically realizing some tubular geometrical sculptures, using two examples from opposite ends of a spectrum of possible Gosper sculptures. A first example concerns a large-scale metal sculpture of a *Gosper-Ball* with only 38 tubular segments, but which still exhibits the characteristics of a 3D Gosper curve quite nicely. The challenge is how to accurately assemble the 38 tubular pieces while realizing the required angles at all 38 joints. The paper presents a judicious partitioning of the overall sculpture and some simple jigs that help to put the different sub-assemblies together.

The second example concerns 3D-printing of desk-top models of complex *Gosper-Globes* with up to 756 segments, where a variant of a 2D plane-filling Gosper curve is constrained to a spherical shell. The lengthy tubular loop representing the Gosper curve requires a lot of support material during the additive fabrication phase. This support material is difficult to remove from the inside of the spherical shell. Partitioning the shell into a few smaller domains reduces this problem without unduly complicating the assembly of the overall sculpture.

Introduction and Previous Work

Taking a self-similar, 2D plane-filling curve, such as the Hilbert curve [10] or the Gosper curve (Fig.1a) into the third dimension offers interesting mathematical design challenges and can result in attractive geometrical sculptures [3]. For the last two years, I have been specifically interested in transforming variants of the Gosper curve [9][12] into attractive 3D geometrical sculptures [4] (Fig.1c). Making a 3D space-filling Gosper-like curve (Fig.1b) has its own geometrical and algorithmic challenges, which have been discussed elsewhere [8][4][5]. Physically realizing such sculptures, either in the form of a 3D-print with several hundred cylindrical segments, or as a human-scale sculpture made from metal tube segments, poses additional challenges. Some of these problems can be addressed by judiciously partitioning the overall sculpture into a few sub-assemblies that are relatively easy to fabricate, and which then are not too difficult to compose into the final sculpture. In this study, I use two examples from opposite ends of a spectrum of possible Gosper sculptures to discuss these fabrication challenges – in accordance with the stated interests of the SCULPT track.



(a) (b) (c)

Figure 1: *(a) 2D Gosper curve; (b) a 3D Gosper curve; (c) a small Gosper sculpture.*

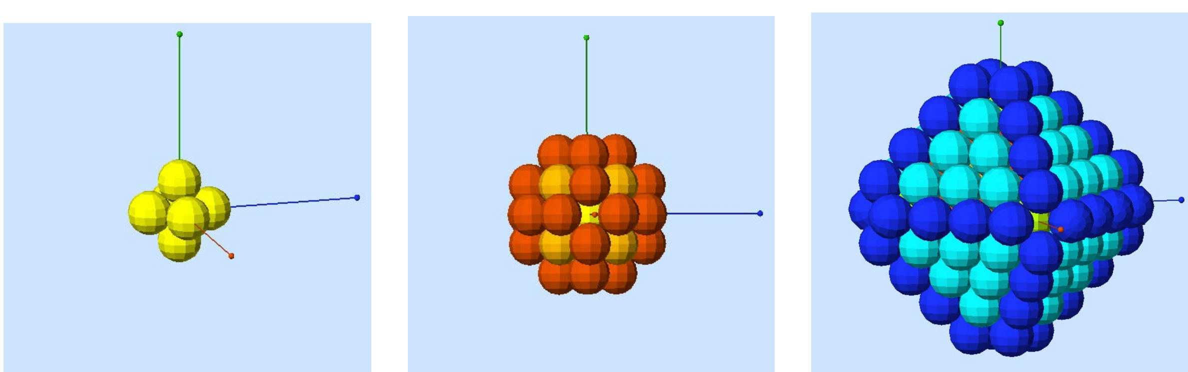
The first example concerns a *Gosper-Ball* of low complexity with only 38 segments made from metal tubes. Here the main problem is how to join the pre-cut metal tube segments while accurately realizing the required angles at all the joints, so that the overall sculpture will close nicely into a symmetrical loop. The first step is to pick the right sub-structures that can be assembled easily with good accuracy because they contain several co-planar segments. These sub-assemblies are then joined with simple, helpful jigs that assist in implementing proper angles and proper spacings.

The second example concerns 3D-printing of *Gosper-Globes*, where a variant of a 2D plane-filling Gosper curve is constrained to a spherical shell. I discuss making desk-top models of complex *Gosper-Globes* with up to 756 segments on low-end 3D-printers. The thin, lengthy tubular loop needs a lot of support material during the additive fabrication phase, which then is difficult to remove from the inside of the spherical shell. Partitioning the shell into a few smaller domains reduces this problem without unduly complicating the assembly of the overall sculpture.

Metal Sculpture: *Gosper-Onion_38*

Gosper-Balls fill a roughly spherical domain with a 3D polyline that tries to capture the basic characteristics of a 2D Gosper curve. In one approach, the Gosper strand visits the atomic sites of a face-centered cubic (FCC) lattice by making equal-size, nearest-neighbor steps. The FCC lattice is the lattice that allows densest sphere-packing. Figure 1c shows such a *Gosper-Ball* covering 140 FCC lattice sites.

Chris Ohler, who has created several abstract geometrical sculptures in metal [2], is interested in building a Gosper sculpture from metal tubing. When planning to make a first Gosper sculpture from individual metal tube segments, it is prudent to keep it rather simple. This raises the question: What is the minimal number of tube segments that can still present the characteristics of a 3D Gosper curve? One promising approach is to place some typical, hexagon-shaped "*Gosper-Claws"* in the outermost, peripheral layer of the sculpture, where these features can easily be seen, and to construct a small inner core that holds these outer elements in place. I call this the *Gosper-Onion* approach [4]. Working on the FCC lattice, I start with a core of six vertices on the corners of an octahedron (Fig.2a). I then surround this core with a second shell based on a truncated octahedron. This second shell has 24 atomic sites to define its convex hall and eight more atoms in the centers of the eight hexagonal faces (Fig.2b). This *Onion-Shell* approach can be continued. The third shell would have a total of 102 atomic sites (Fig.2c).



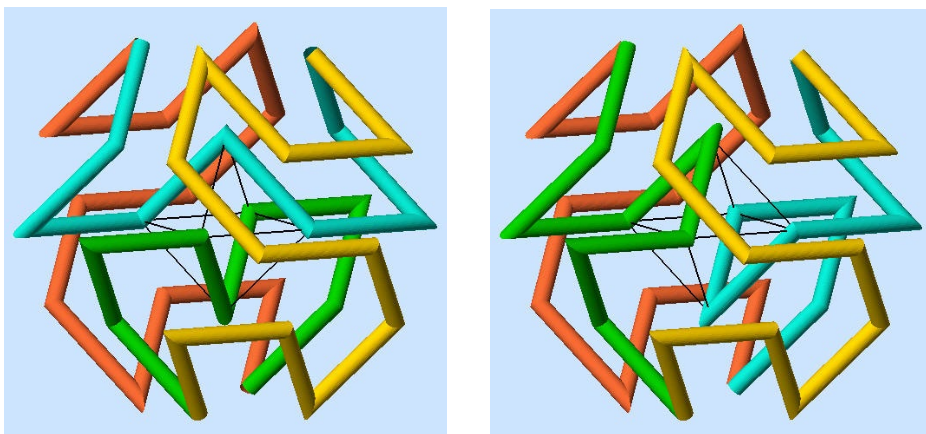
(a) (b) (c)

Figure 2: *(a) 6 atoms in shell 1; (b) 24+8=32 atoms in shell 2; (c) 6+48+48=102 atoms in shell 3.*

Various Design Options

The 38 sites on the FCC lattice can be connected in many different ways. To make a simple but convincing representation of a 3D Gosper curve, a *Level-2 Gosper-Onion* seems particularly well suited.

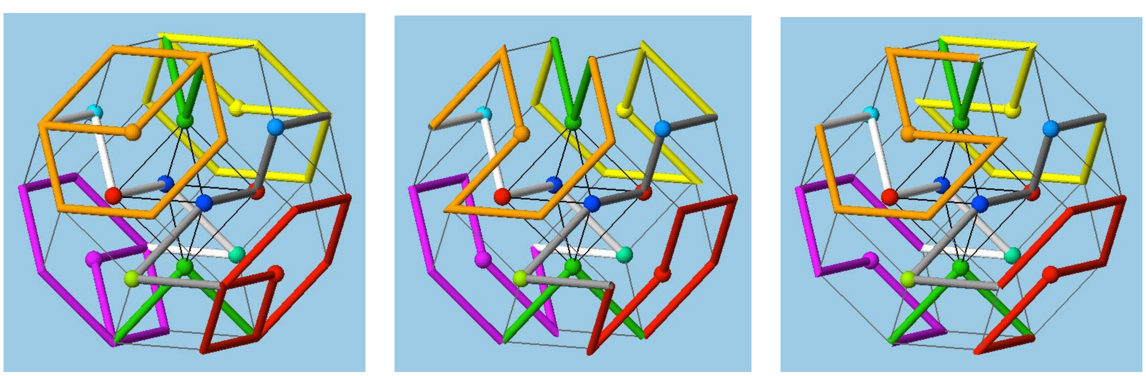
The seven sites in the hexagonal faces of shell 2 are perfectly suited to accommodate one typical Gosper-Claw. In a first approach, I place two such claws on adjacent faces and combine them into a "Double-Claw" (Fig.3). I can do this on two pairs of faces (yellow and orange), and this naturally splits the outer shell into two equal halves. The remaining sites in the other four hexagonal faces are used to connect the two outer half-shells to the inner octahedral sites. I found two different ways of doing this. In *Model A* (Fig.3a), the two Double-Claws are cross-connected with two inner struts (green and blue) that each use three vertices of the central octahedron. Each of the two internal connecting branches passes through two opposite vertices of the central octahedron and exhibits a right-angle bend in between. This structure exhibits 4-fold D_2-symmetry (with 3 mutually perpendicular C_2 rotation axes). Some purists may object to the 90° bend. In a second approach (*Model B*), the two connector branches each use three vertices associated with a single triangular face of the central octahedron. This structure employs only 60° and 120° bends, like the 2D Gosper curve, but it exhibits only 2-fold C_2-symmetry (Fig.3b).



(a) (b)

Figure 3: *Gosper-Onion_38 paths: (a) Model A with D_2 symmetry, (b) Model B with C_2 symmetry.*

As an alternative solution, the four hexagon-shaped Gosper-Claws can be placed on four non-adjacent hexagonal faces of the truncated octahedron – instead of forming two Double-Claws on two pairs of adjacent faces. This approach offers additional possibilities, since the four claws can be rotated or reflected individually, whereas in the construction of the Double-Claws, the orientation of the individual claws is prescribed. On the other hand, it is quite a challenging puzzle to find good ways to connect the four individual claws into a single loop, while avoiding, as much as possible, bending angles of 90° or 0°, and while maintaining at least C_2-symmetry.



(a) (b) (c)

Figure 4: *(a) Claw faces shown as hexagonal rings; (b) rings replaced with claws to form Model C; (c) rings replaced with reversed claws to form Model D.*

To tackle this design challenge, I present the four hex-faces that were supposed to carry the four individual claws as simple hex-rings (Fig.4a). I can then focus on finding four connector paths that connect the four claws into a single loop. This requires that the two connecting points to each of the four hex-rings are placed 120° apart with respect to the center of the hex-faces. The individual claws can then be rotated to have their two endpoints (which lie 120° apart) line up with the two path connection points.

After some amount of searching through possible solutions, I found a nice configuration that exhibits C_2-symmetry around the (vertical) z-axis (Fig.4b). This design also keeps the four hex-faces that do not carry an individual Gosper-Claw relatively "flat", without the pointy spikes produced by 120°-bends that jut out in the centers of these faces, as is the case in both *Model A* and *Model B*. This model still has two of the square faces of the cuboctahedron spanned by four spiky 120°-bends, which may serve as the "legs" of an actual sculpture – to give it a rather "airy" look (Figs.19b,c). However, this model does have *six* bends of 90°. More recently I found that by reversing the orientation of the four Gosper-Claws I could get rid of four of the 90° bends (Fig.4c). By mixing and matching the orientation of the four Gosper-Claws, there are other interesting possibilities to create different internal connecter paths.

Fabrication Issues

Chris Ohler [2], has committed himself to construct such a low-complexity Gosper sculpture based on *Model A*, using metal tubing with a diameter of 133mm. He has worked out the relevant cutting angles for all 38 tubular segments. For each tube segment, one needs to know the mitre angle at which the ends of the tube must be cut, as well as the torsion angle between the two cuts. For a poly-line path that lies in a plane, these torsion angles are 0° as long as the bending continues in the same CW- or CCW-sense, and they are 180° wherever there is an inflection point. For an arbitrary polyline in 3D space, many possible torsion angles may occur. It is interesting to note that for these Gosper curves that connect nearest neighbours on the FCC lattice, only four non-planar torsion angles appear: 54.74°, 70.53°, 109.47°, and 125.26°. Figure 5a shows Ohler's list of cutting angles α and torsion angles β for one quarter of *Model A* discussed above. The remainder of the model follows the same list – or it's reversal – because of the symmetry of the sculpture. Figures 5b and 5c show some of the pre-cut metal tubes and the formation of a properly mitred turning angle of 120°.

(a) (b) (c)

Figure 5: *(a) Cutting angles α and torsion angles β for one quarter of all the tube segments. (b) Pre-cut metal tube segments. (c) A mitered 120° tubular junction. (Images by C. Ohler).*

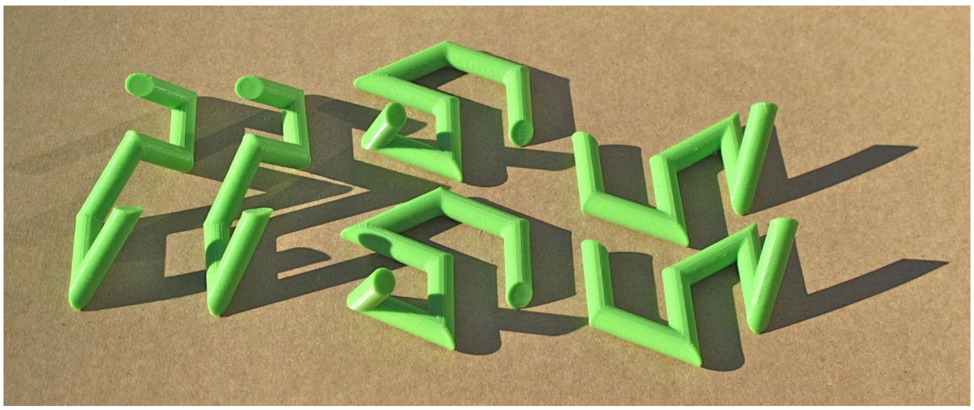
Assembly Issues

The next issues to be addressed are: In what sequence should the individual tube segments be assembled? And: What kinds of jigs are required to hold individual tube segments and partial subassemblies in exactly the right positions, so that in the end the Gosper-loop closes nicely on itself? It would not be accurate enough to just match the elliptical cross sections of two consecutive tube segments to obtain a clean realization of the whole sculpture. It is advantageous to have additional help from some suitably designed jigs to hold small pre-assembled chains of a few consecutive tubular segment in their proper positions. In the following I present a proposed assembly scheme, verified on a small 3D-print model.

In each Gosper-Claw there are six consecutive segments that lie in the same common plane. It makes a lot of sense to preassemble those tube-chains, since this is mostly a 2D layout and we need not worry about obtaining the proper torsion angles. It may also be practical to add one or two additional tubular segments at the ends of such a Gosper-Claw. In the following tests and evaluations of the proposed assembly sequences, these parts with five to eight tube segments were 3D-printed as cohesive units. The large, planar regions of the Gosper-Claws then permit the insertion of some judiciously chosen "spacer slabs," which allow additional parts to be added in suitably offset, parallel neighbor-planes.

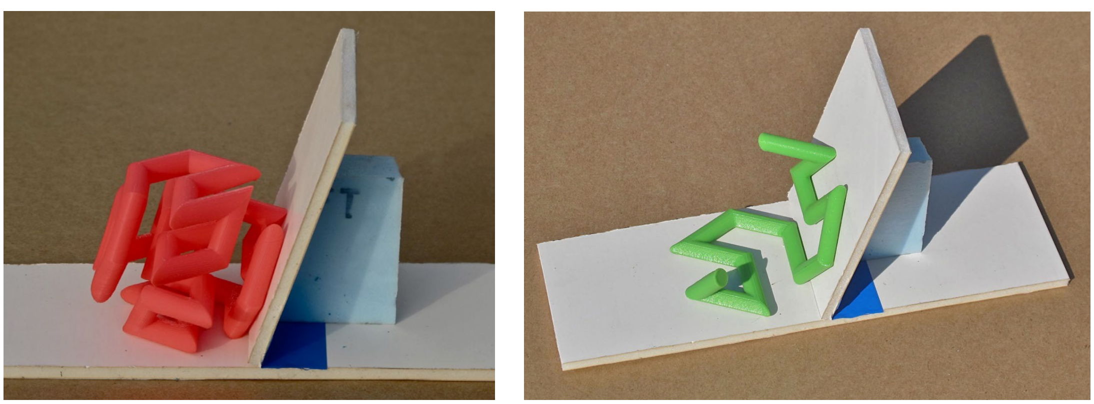
An Initial Assembly Step

Figure 6 shows the six tubular chains that form the sculptural sub-assemblies of *Model A*: two internal cross-connector branches (Fig.6a), two complete Gosper-Claws (Fig.6b), and two partial Gosper-Claws that constitute the remainder of a Double-Claw (Fig.6c).



(a) (b) (c)

Figure 6: *Sub-assembly tube chains: (a) internal connectors, (b,c) peripheral Gosper claws.*

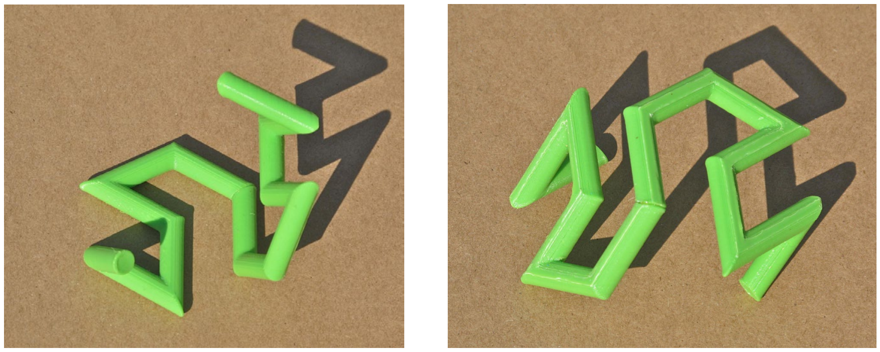


(a) (b)

Figure 7: *(a) Jig with proper dihedral angle. (b) Two simple claws fused into a Double-Claw.*

The two Gosper-Claws making up a Double-Claw occupy two adjacent faces of the outer *Onion* shell and form a dihedral angle of 109.47° between them. A jig with a wall properly slanted at this angle (Fig.7a) makes it easy to put two such parts together (Fig.7b) to form one of the two Double-Claws (Fig.8a,b). This is also the first step for assembling *Model B*.

Next, I needed to find a suitable jig for lining up the two internal connectors. The two different sculpture models, *Model A* and *Model B* (Fig.3a,b), require slightly different approaches.



(a) (b)

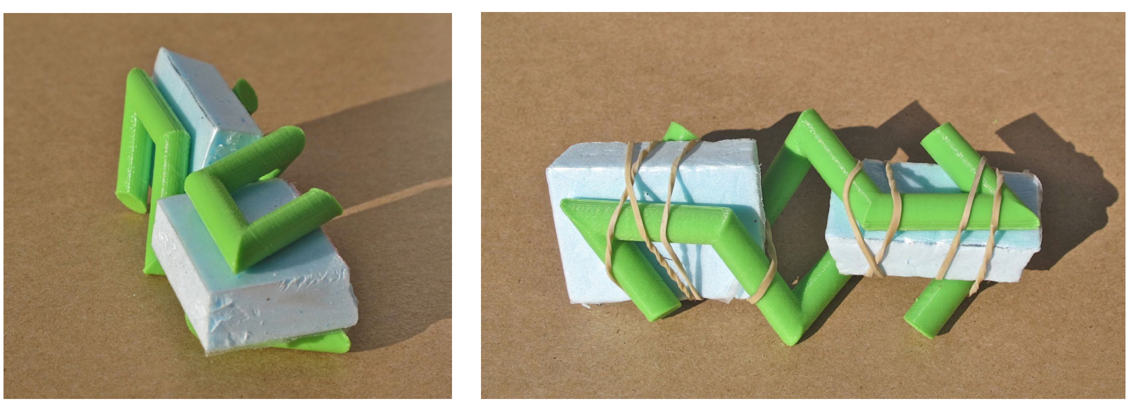
Figure 8: *The resulting peripheral Double-Claws: (a) inside view, (b) outside view.*

Assembling Model A

In *Model A* (Fig.3a) the internal connectors form some "crooked" struts that cross each other in the central octahedron. Figure 9 shows how the two inner struts relate to one another. Pairs of some tubular segments in one strut happen to be collinear with the corresponding segments in the other strut. Furthermore, the 3-segment, planar end-chains in each strut are parallel to the corresponding end-chains in the other strut. Thus, two *spacer-slabs* of proper thickness can be used to keep the two struts in proper position with respect to one another. The thickness of this spacer-slab is calculated as the basic path-segment separation, *ss*, minus the given tube diameter, *td*. In 2D, the distance between nearest parallel path segments is: $ul * \cos(30°) = 0.866\ ul$, where *ul* is the unit-length step of the Gosper path. In the 3D sculpture, the critical separation of a whole group of coplanar segments from another group in an adjacent parallel plane is further reduced by: $\sin(109.47°) = 0.9428$, (the dihedral angle of the octahedron). Thus:

$ss = 0.8165\ ul$, and thus the spacer-slab thickness, *st*, should be: $st = 0.8165\ ul - td$.

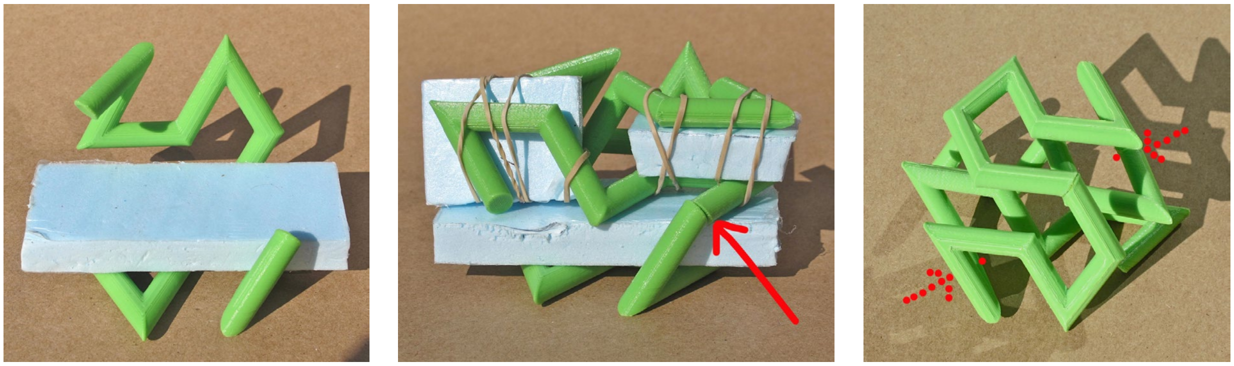
With the two spacer slabs in place (Fig.9a), the collinearity mentioned above can be checked and fine-adjusted, and the two crossing struts can be bundled into a single rigid assembly (Fig.9b).



(a) (b)

Figure 9: *(a) Alignment of the two inner cross-connectors: (a) end-view, (b) side-view.*

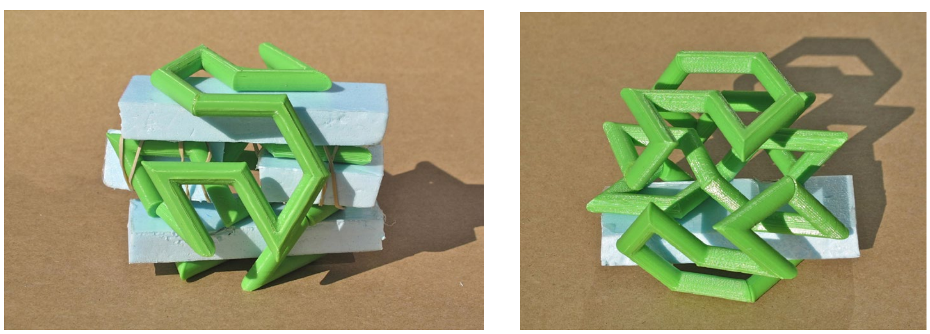
The same thickness spacer-slab is also used to define the proper distancing of that central bundle from the outer claws. Thus, a spacer-slab can be placed inside one of the Double-Claws (Fig.10a) and the central bundle is then placed on top of this slab (Fig.10b). This brings two connection points into close proximity and allows them to be fused with the proper orientation and alignment (see red arrows in Figures 10b and 10c).



(a) (b) (c)

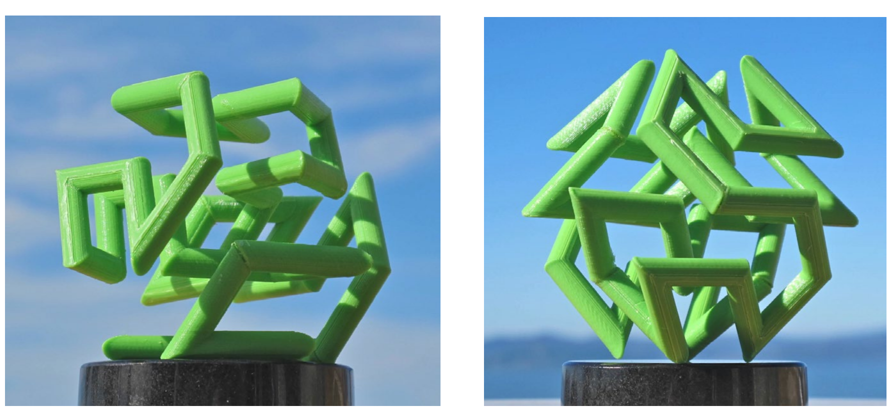
Figure 10: *Inserting cross connectors: (a) spacer-slab in a Double-Claw; (b) connector bundle placed; (c) the result with two connections made (shown upside down).*

Because of the symmetry of this sculpture, the same type of spacer-slab also controls the spacing of the central cross-connector bundle to the second Double-Claw. A spacer-slab is placed onto the central bundle, and the second Double-Claw is then draped across this assembly (Fig.11a). This would allow me to make the final two connections to this upper Double-Claw. However, in reality, it is better to just make the connections between the bottom Double-Claw and the connector bundle, and then remove the spacer slab. This results in Figure 10c. This structure, turned upside down, can then be placed on a second configuration that looks like Figure 10a, and the final two connections can be made (Fig.11b). Eventually the last spacer slab is also removed, revealing the completed *Model A* (Fig.12).



(a) (b)

Figure 11: *(a) All four parts properly spaced. (b) Pre-connected part (Fig.10c) on top of spacer slab.*

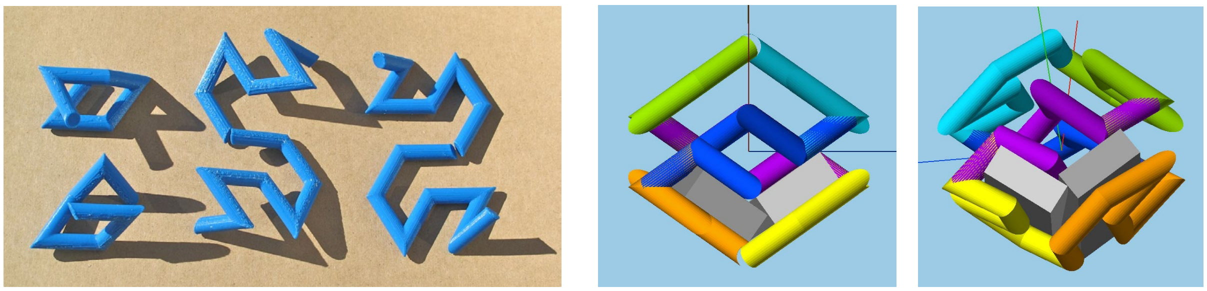


(a) (b)

Figure 12: *Resulting sculptural maquette: (a) resting on one Gosper-Claw, (b) standing on four "legs."*

Assembling Model B

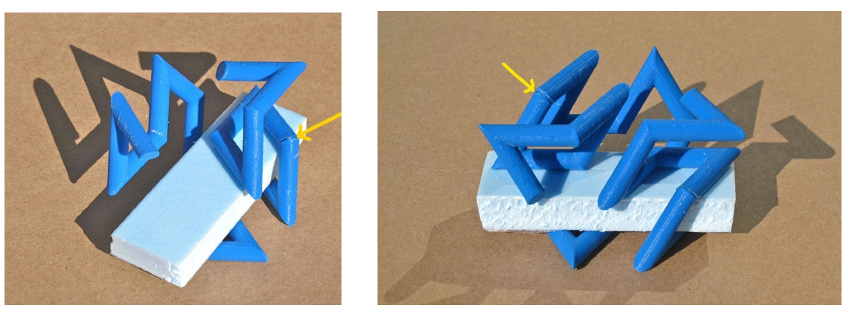
Figure 13 shows the six sub-assemblies for *Model B*. The first step is the same as for *Model A*: The two Double-Claws are formed by re-using the jig shown in Figure 7. In *Model B* the internal connectors are more compact and do not cross each other. Figures 13b,c show that in this model, too, there are open slots that can accommodate appropriate spacer-slabs.



(a) (b) (c)

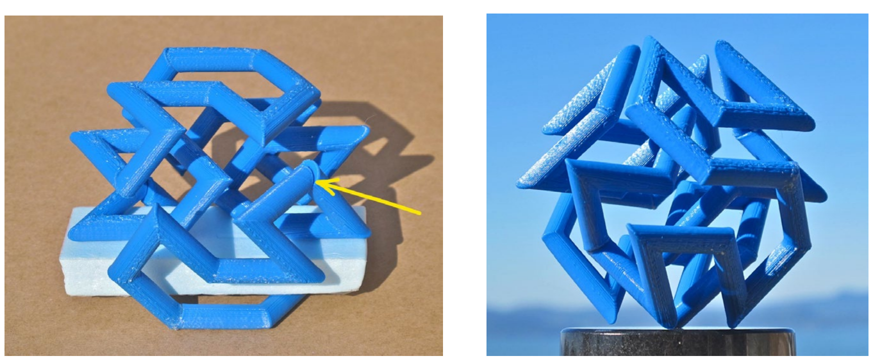
Figure 13: *(a) Sub-assemblies for Model B. (b,c) Open slots for spacer-slabs in Model B.*

First, the two connector parts are aligned and fused against the bottom Double-Claw. To do this, I form a configuration with one spacer-slab placed inside one Double-Claw, as seen in Figure 10a. On this slab I place one of the connector pieces and fuse it with one end to the underlying Double-Claw (yellow arrow in Fig.14a). Then the spacer slab is removed and re-inserted flush against the other half of the same Double-Claw. In this configuration, I add the second connector unit and fuse it to the other end of the Double-Claw (yellow arrow in Fig.14b). Next, I form a new cradle (like Fig.10a) with the 2nd Double-Claw. Into this cradle, I drop the assembly described above and fuse both ends of the Double-Claw to the two connector units (Fig.15a). Removal of the spacer slab reveals the completed *Model B* (Fig.15b).



(a) (b)

Figure 14: *(a) Placing and fusing a first connector; (b) switching spacer slab to place 2nd connector.*



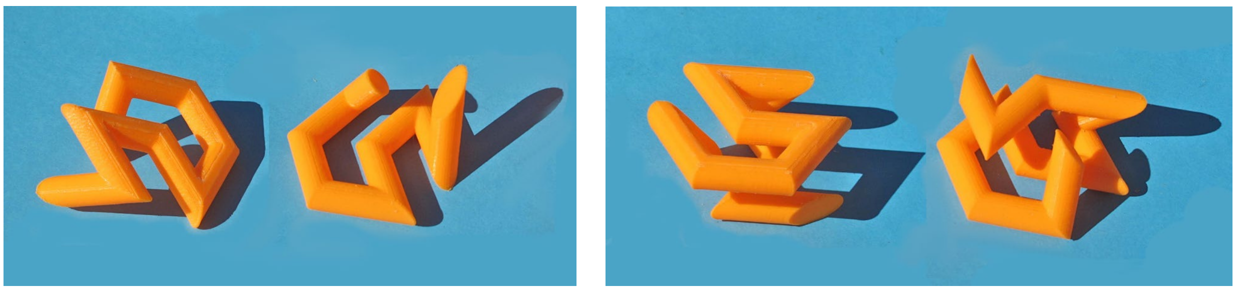
(a) (b)

Figure 15: *(a) One Double-Claw plus two connectors placed on 2nd Double-Claw plus spacer-slab.*
(b) Resulting sculptural maquette, Model B, standing on four "legs."

Assembling Model C

Model C can also be partitioned into parts that start with a planar, 6-segment Gosper-Claw. However, in this model there are no clearly identifiable internal connector parts. Instead, the Gosper-Claws are enhanced in two different ways. In the *simpler part* (Fig.16a), one extra segment is added to each end of the Gosper-Claw. In the more *complex part* (Fig.16b), one of the ends of the Gosper-Claw carries a chain of *four* tubular segments that form a "zig-zag" path that lies in a plane parallel to the Gosper-Claw (Fig.16b). Figure 16 shows the four 3D-printed subunits, composed of 8 and 11 segments, respectively.

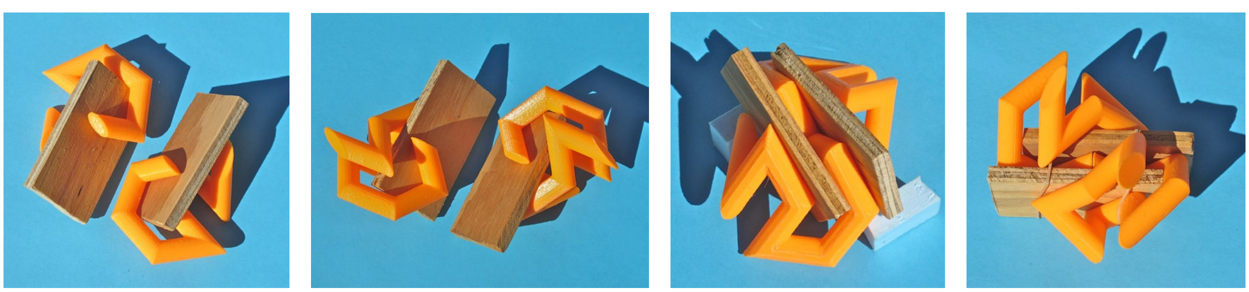
For a metal tube construction, the 11-segment part may be further partitioned into another *simple* part with 8 segments plus a separate planar branch of three segments, which can be added in a plane offset from the Gosper-Claw by using one of the spacer slabs described below.



(a) (b)

Figure 16: *Model C: (a) the two simple parts, (b) the two complex parts. – (different views).*

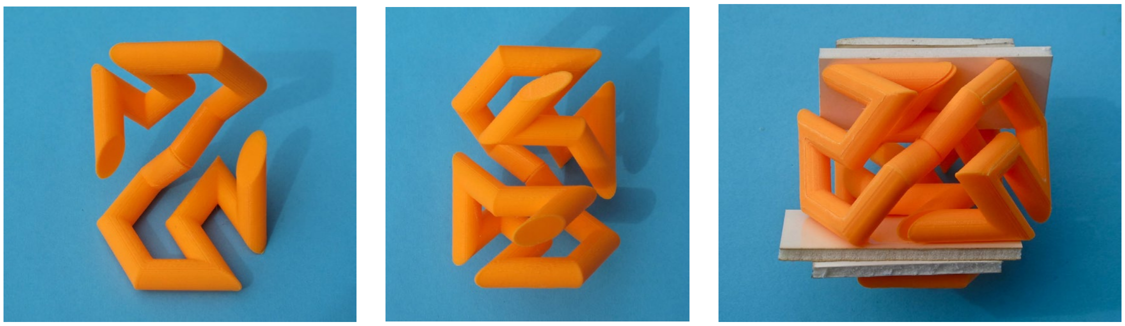
To assemble these parts, spacer-slabs again come in handy! Figures 17a,b show that there are spacer-slabs that fit snuggly into both the simple and the complex part. Note that the orientation of these slabs is perpendicular to the dominant coordinate axes of the octahedron, and the thickness of these spacer-slabs thus is only: $ss = 0.7071 * ul - td$. Pairs of identical parts with their inserted slabs then fit together nicely, so that the two slabs become parallel to each other. Between them, they squeeze the zig-zag branch that needs to be joined, and they guarantee that this branch remains planar (Fig.17c,d).



(a) (b) (c) (d)

Figure 17: *Spacer-slabs: (a) in the simple part; (b) in the complex part; (c) joining two simple parts; (d) joining two complex parts.*

Figures 18a,b show the resulting joined pairs of identical parts. To assemble these into the final model, two spacer-slabs are used simultaneously (Fig.18c). These slabs are again parallel to the claw-faces, and thus their thickness is again: $st = 0.8165\ ul - td$, as discussed above for *Model A*. After removal of the spacer-slabs, the sculpture *Model C* emerges (Fig.19).



(a) (b) (c)

Figure 18: *The two glued double-parts: (a) simple; (b) complex; (c) aligned with proper spacer-slabs.*

(a) (b) (c)

Figure 19: *Final sculpture Model C: (a) placed on one Gosper-Claw, (b,c) standing on four "legs."*

Similar sub-assemblies and compositions with the help of corresponding spacer-slabs also apply in the construction of *Model D* – and probably for most other variants of this *Gosper-Onion_38* with four separate Gosper-Claws in different orientations and different rotations.

Scaling Issues

In our SLIDE [6] or JIPCAD [1] programs for designing and visualizing these sculptural models, I can readily adjust the tube diameter used in the sweep along the Gosper polyline to find the best level of "transparency" into the inner parts of the sculpture. I like a value that leaves a little more than a tube diameter of empty space between adjacent parallel tube segments, say, a tube diameter of: $td = 0.4\ ul$.

When considering a physical sculpture made from metal tube segments, the diameter of the tubes used is typically given, and thus the unit step length, *ul*, that results in the desired separation between nearest parallel tube segments would then be calculated as: $ul = 2.5\ td$. Thus, if the sculpture is constructed from tubes with a diameter of 133mm, the Gosper polyline would be scaled so that the unit-step-length, *ul*, is about 33cm. The overall *Gosper-Onion_38* sculpture would then have a diameter of about: $3\ ul + td$, or a little more than 1 meter.

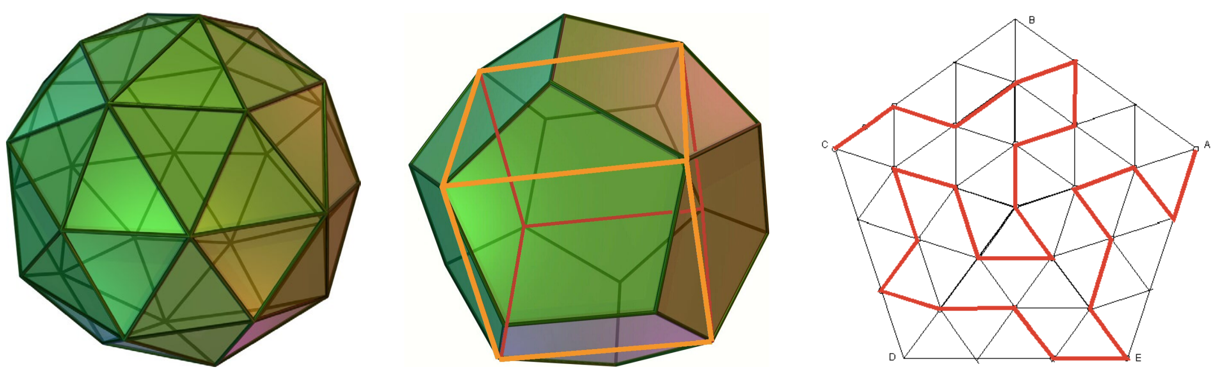
3D-Printed *Gosper-Globes*

In my *Gosper-Globes* the Gosper strand is confined to lie in a thin spherical shell. The sites where the strand makes a turn lie on a sphere. But these sites do not form a completely regular array, and the step-length between nearest neighbors will vary somewhat. In the examples discussed in this section, I approximate the sphere with a pentakis-dodecahedron (Fig.20a) [11]. I then cover each of the 60 triangles with an array of subdivision sites ranging from a subdivision level of 3 (Fig.20c) for the smaller globe (Fig.24) to a subdivision level of 5 in the final sculpture (Fig.28).

Stability Issues

To stay true to the 2D Gosper curve, a Gosper sculpture should consist of just one (possibly closed-loop) strand. However, if the Gosper strand has more than a few hundred segments and no intermediate support, it would result in a rather flimsy sculpture, where parts of it would sag under the influence of gravity. In order to make more complex *Gosper-Globes*, I allow the Gosper strand to assume a more complex graph structure. In the following, this graph has the topology of the wire-frame of a cube. Thus, there will be eight 3-way branching points. The "edges" between two nearest branching points then "fractalized," i.e., deformed into Gosper-like polylines that together will cover the surface of a sphere as uniformly as possible.

To define the best set of "lattice" points on the sphere surface, I want to start with as "round" a polyhedron as possible, – not with a cube. Good candidates are the dodecahedron, or better yet, the pentakis-dodecahedron (Fig.20a). The 60 triangles can then readily yield an array of subdivision points. When these points are projected onto the circum-sphere, they yield a nice pattern of almost uniformly spaced points. It is also easy to integrate the desired wire-frame cube with this dodecahedral geometry. Each pentagonal region carries one of the 12 cube edges, and these edges connect two non-adjacent vertices in each pentagon (Fig.20b). These 12 wire segments are then "fractalized" to cover the whole pentagonal region. In Figure 20c, one "fractalized cube edge" runs from vertex A to vertex C.

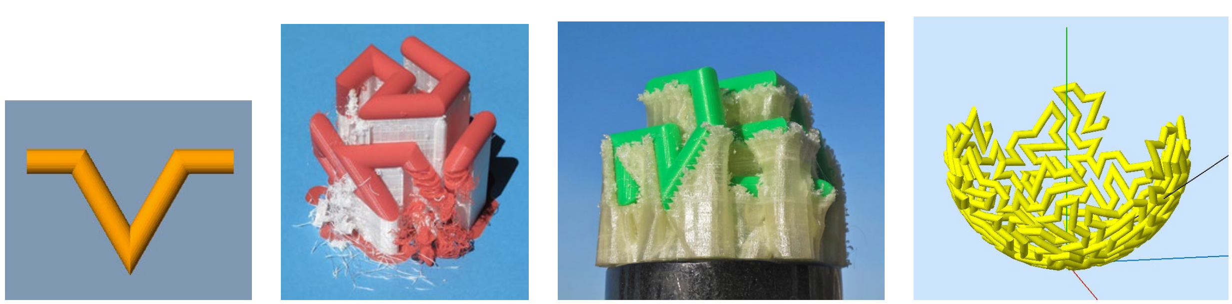


(a) (b) (c)

Figure 20: *(a) Pentakis-dodecahedron. (b) Cube-frame embedded in a dodecahedron. (c) Level-3 fractalized cube edge to cover a pentagonal region.*

Fabrication Issues

Complex spherical balls or shells (Figs. 1c, 22a) formed by an intricate web of tubular pathways, such as 3D polylines following a Hilbert curve [10] or mimicking a Gosper curve [9][12], are difficult to print on most 3D printers that do not provide an all-around, support structure as is the case in SLS (Selective Laser Sintering) or in Z-Corp printers [13].



(a) (b) (c) (d)

Figure 21: *(a) Down-spike in the Gosper curve, (b) collapsed because of insufficient support structure, (c) fabricated with a copious amount of support material. (d) Half of a spherical Gosper-Globe.*

One reason is that it is difficult to remove the support material from the inside regions of the sphere. A second difficulty, more specific to the Gosper sculptures that I am currently interested in, is related to any downward-pointing spikes (Fig.21a). These spiky points may not get an adequate support structure (Fig.21b) that would allow them to be anchored in a sturdy enough way, so that they can then support any lengthy, complex tubular pathways that connect to this down-spike [4]. This problem can be mitigated by using a very generous amount of scaffolding material to support every model facet with a downward-pointing face normal (Fig.21c). However, this creates an enormous amount of scaffolding, which may fuse into a rather solid block that is near-impossible to remove from the inner parts of the whole structure – unless the support material is soluble.

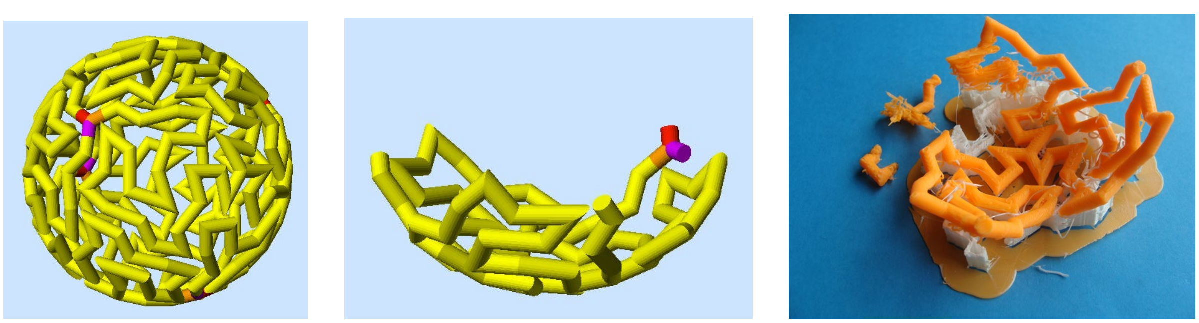
One idea to work around this problem is to split the spherical shell into two hemispheres and print each in the orientation of a hemispherical bowl (Fig.21d). This places the support material on the outside of the bowl and instantiates it radially only as far until the slope of the bowl becomes steep enough so that no further support is required. However, for a hemispherical shell, the amount of support material is still relatively voluminous, and down-spikes in the upper, steeper parts of the bowl surface are still a problem.

The situation improves as we split the sphere surface into more, and correspondingly "flatter" parts. A natural next step is to split the sphere into *four* quadrants – or rather "*tetrants*" – since I plan to use a partitioning based on tetrahedral symmetry (Fig.22b). Now the total volume of support material is much smaller, and there are no longer any truly "down-pointing" spikes. Such a partitioning of the sphere also allows me to construct maquettes that are somewhat larger than the build volume of the available printer.

Gosper-Globe_276

As a first demonstration, I am constructing a spherical shell, which I call *Gosper-Globe_276* (Fig.22a), on an Ultimaker printer [7] with PLA build-material and also with PLA support-material. In order to make this approach work, I want all tubular elements appearing in each sphere-tetrant to be connected to each other, forming a reasonably rigid part that can be used as a building block for the whole sphere shell. Thus, rather than designing a single-loop Gosper path that winds its way serially through all four tetrant domains, I have introduced some branching points. The resulting tubular structure in the spherical shell now has the same connectivity as the wire-frame of a cube: Twelve zig-zag-y paths connect eight 3-way branch points. One of these valence-3 branching points can be found in the center of each of the four tetrant domains; the other four junctions appear at three symmetrically spread-out locations on the rims of the tetrant domains.

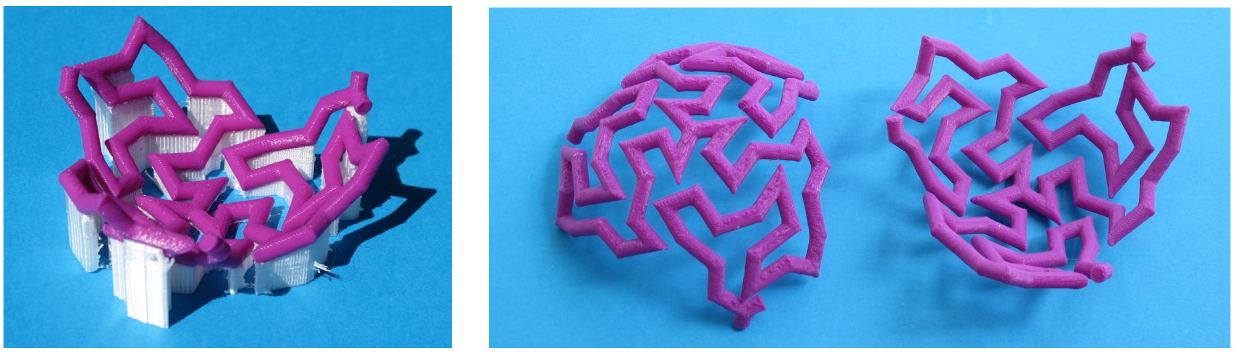
It was more difficult than anticipated to find a reasonable Gosper path to replace all twelve cube edges, so that three of them lie entirely in each tetrant part. To facilitate the assembly of the four tetrant parts, I combined the geometries of the three partial 3-way branch points lying on the periphery of the tetrant part into one fully modeled 3-way junction. I then placed just one of those onto the border of each tetrant part, – shown in contrasting colors in Figure 22b. Straight, cylindrical tube connections then accommodate the two incoming paths from the other two tetrant parts that connect to this branch point.



(a) (b) (c)

Figure 22: *(a) CAD model of Gosper-Globe_276; (b) one "tetrant" of it. (c) A first, unsuccessful print.*

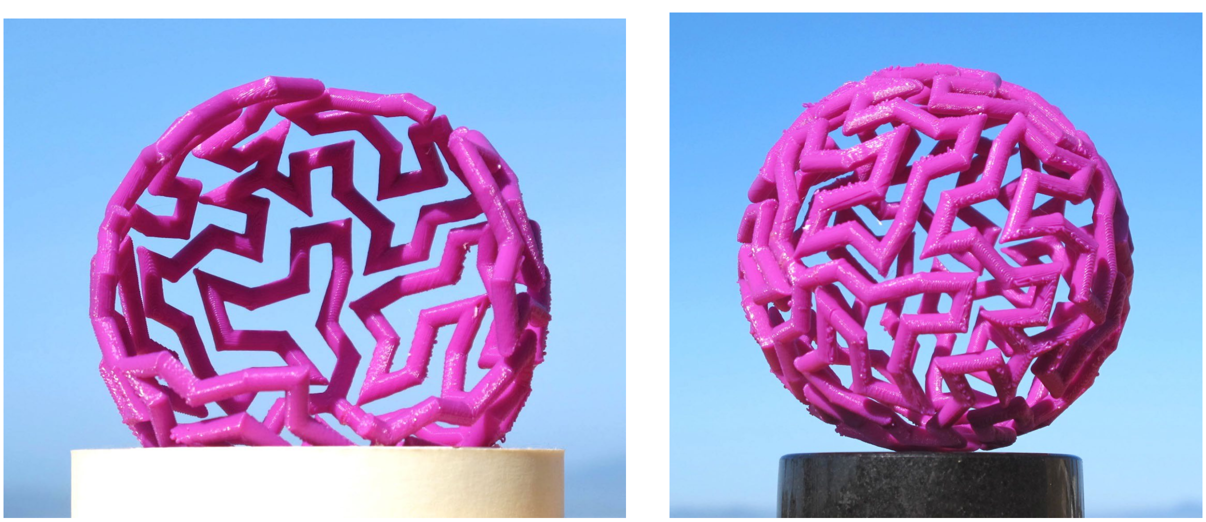
In my first attempt, I used the default settings for the Ultimaker printer as supplied by the Cloud 3DPrinterOS service [14] offered through the Jacobs Institute for Design Innovation at Berkeley. The resulting print had an incomplete support structure that caused some breaks in the 3D tubular pathways (Fig.22c). In my second attempt, I used the *expert* mode in the 3DPrinterOS. I lowered the support overhang angle from 60° to 40°, and I increased the support density from 15% to 20%. This resulted in a good support structure and nice, sturdy parts. Figure 23a shows one tetrant part as it came off the Ultimaker printer. The reduced overhang angle was sufficient to firmly support the many tubular segments that then serve as anchors for additional tubular branches, which may take off at steeper angles and which need no further support. The support was easy to break away, and the "spidery" parts (Figs.23b,c) were rigid enough to keep their shapes when handled for clean-up and for assembling the complete spherical shell.



(a) (b) (c)

Figure 23: *(a) Tetrant part as it came off the Ultimaker printer. (b, c) Outside and inside view of the tetrant part after its (white) support structure had been removed.*

The first two tetrants connect to each other with just two tubular joins. This leaves the angle of rotation around the line through these two connection points rather poorly defined. To make sure that the two parts join at the right angle so that they nicely adhere to the sphere of the final sculpture, I formed a bowl-shaped supporting jig that kept the parts tilted at the right angles. Joining a third tetrant part is less ambiguous, since it joins the previous two parts at *three* tubular join-points (Fig.24a). Adding the fourth tetrant by fusing another three joints then completes *Gosper-Globe_276* (Fig.24b).



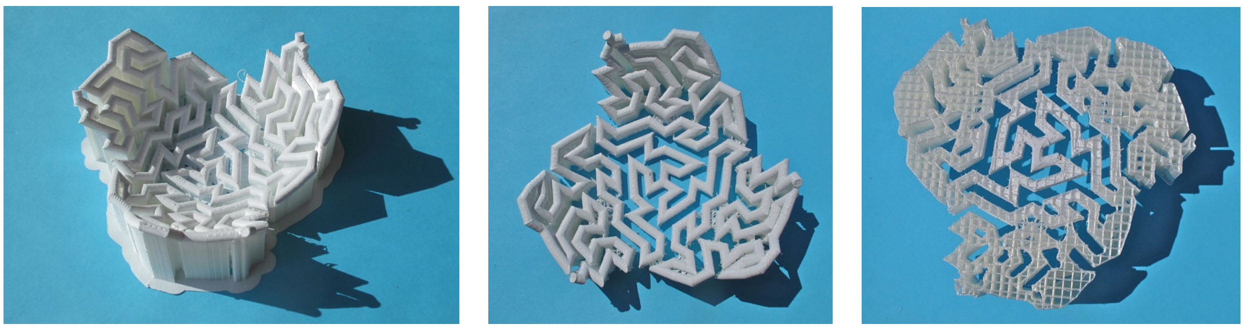
(a) (b)

Figure 24: *(a) Three tetrant parts assembled into a partial spherical shell. (b) The complete spherical shell of Gosper-Globe_276.*

Gosper-Globe_756

The construction of the purple *Gosper-Globe_276* gave me the confidence that this approach is indeed practical. So, I embarked on building a globe of much higher complexity and of a size that exceeds the build volume of the Ultimaker printers to which I had access through the Cloud 3DprintOS.

The 3D-printed parts from which the *Gosper-Globe_276* was assembled have three zig-zag paths running from a central 3-way branch point to the three branch points on the periphery of the tetrant in 23 steps on a set of vertices that correspond to a level-3 subdivision of the 60 triangles of the pentakis-dodecahedron [11]. In the new *Gosper-Globe_756*, I used a level-5 subdivision of these triangles, which results in the three zig-zag paths in each tetrant having 63 steps. Figure 25 shows one of those parts as it comes off the printer, and later, after about half the support material has been removed (Figs.25b,c).

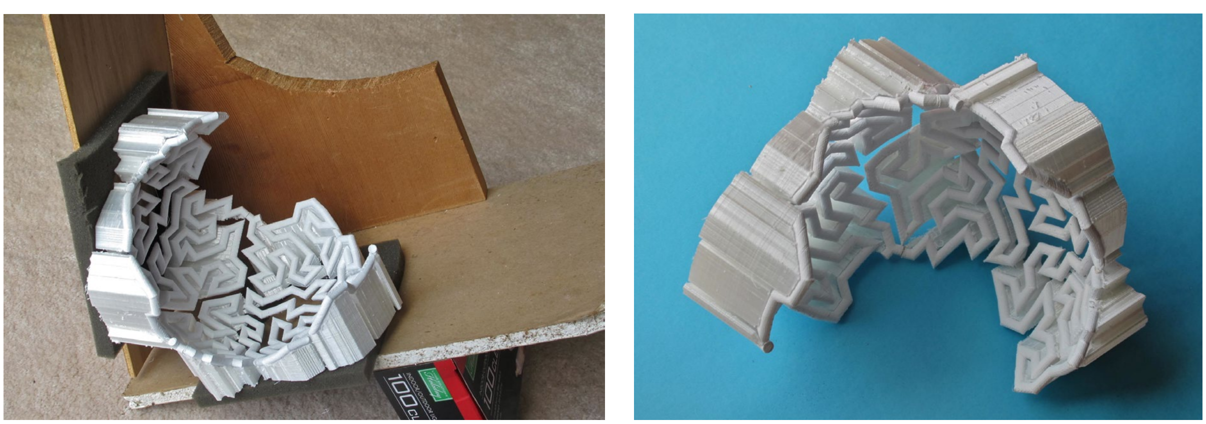


(a) (b) (c)

Figure 25: *One tetrant of the Gosper-Globe_756: (a) as it comes off the Ultimaker 3D printer, (b) after some support removal, (c) seen from the support side.*

At first, I was worried that each tetrant part would be rather "flimsy" and would deform seriously under gravity once it was freed from its support structure. Therefore, I kept much of the support in place until I had properly joined two such tetrants, and I planned to remove the remaining support structure after having joined all four parts.

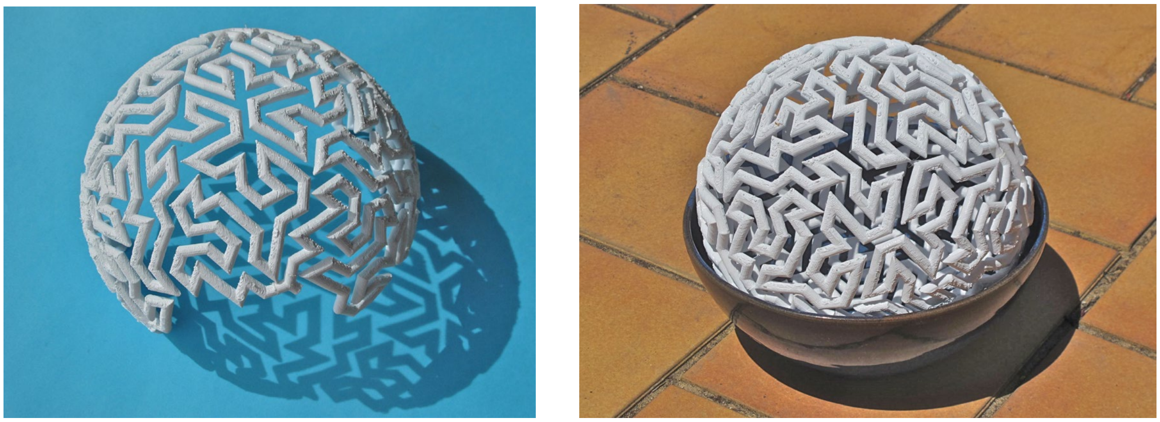
The first two tetrant parts connect in only two of the 3-way branch points. To make sure that they get joined at the right angle, I constructed a jig that provides two mounting planes with the dihedral angle of the tetrahedron (70.5 degrees) between them (Fig.26a). Figure 26b shows the resulting assembly of the first two tetrants joined in this jig.



(a) (b)

Figure 26: *(a) Jig to hold two tetrants at the proper angle between them. (b) The resulting fused tetrant pair.*

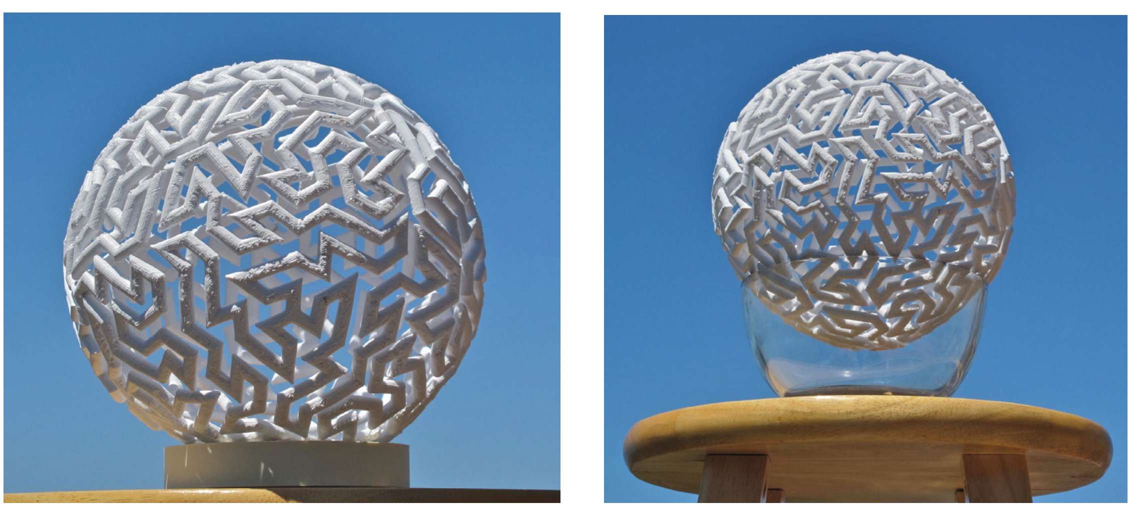
At this point I started the tricky task of removing some support material from this joined pair without stressing or breaking the actual Gosper path. It tuned out that handling this assembly was awkward, and soon one of the glued joints broke. This encouraged me to remove all the support from one of the tetrants to see how "flimsy" the unsupported tetrant part really was. Fortunately, it turned out that it was stable enough, so that I no longer had to worry whether I could properly join unsupported tetrant parts (Fig.27a).



(a) (b)

Figure 27: *(a) Two tetrant parts fused together. (b) Combining the two pairs in a suitable bowl.*

The tetrant parts #3 and #4 were individually freed of all support material and then joined into a pair-constellation like Figure 27a, using again the jig shown in Figure 26a. Joining the two pairs into the complete sphere turned out to more difficult. The two pairs were indeed rather "flimsy" or "squishy" components! I joined them by placing both parts with the appropriate rough alignment between them into a small salad bowl of the right size (Fig.27b) and then lining up and gluing together a first joint between the two assemblies (using Duco Cement). After that joint had hardened sufficiently, I then lined up and glued a second joint. I repeated this process for joints #3 and #4. With all joints properly fused, the completed globe was firm enough to hold its shape under the influence of gravity, even when just placed on a flat surface or onto a small supporting ring (Fig.28a). A nicer way to show off the assembled *Gosper-Globe_756* is to support it with a small transparent cylinder or a small glass bowl (Fig.28b).



(a) (b)

Figure 28: *The final Gosper-Globe_756: (a) Side view, (b) View from below.*

Conclusions

There are many different kinds of Gosper-Sculptures, and there is no standard procedure to design and fabricate such sculptures. Every type of geometry has different design challenges, and the size and type of realization of such a sculpture presents additional difficulties. But in all cases where the complete model cannot be built as a single coherent object on a suitable 3D-printer, judiciously partitioning the overall sculpture into a few sub-assemblies is an important step. Smaller sub-assemblies may be easier to build, but then the composition of these smaller parts into the final sculpture becomes a bigger problem. Complicated scaffolding may be required for the assembly of many small parts into an accurately aligned and balanced sculpture.

For the simple *Gosper-Onion_38* metal sculpture, the described, promising approach was driven by looking for sub-assemblies that have large co-planar regions, for which it was easy to accurately implement the proper torsion angles; these sub-assemblies can then be assembled with the help of a few simple jigs. For the complex *Gosper-Globes*, I was aiming to have as few partitions as possible and to make the largest parts that could be 3D-printed without a lot of difficult-to-remove support material. Different Gosper-Sculpture types and different construction materials will require a re-evaluation of these tradeoffs.

Acknowledgements

I would like to thank the reviewers for their constructive comments as well as good suggestions for future work. One such suggestion was to break *Gosper-Onion_38* into 38 individual parts with individually keyed junctions between the tubular segments to turn this into a 3D snap-together puzzle.

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