Key for Quiz - Assignment 5

Instructions: In questions 1 to 4, define each space and describe the operations of vector addition (+) and scalar multiplication ($\cdot$) the corresponding to it. 10 points each.

1. $\mathcal{P}_n$ is the set of all polynomials of degree n or less; that is, $\mathcal{P}_n = \{a_0 + a_1x + \cdots + a_nx^n\}$. Here are the operations. If $p, q \in \mathcal{P}$, $p(x) = a_0 + a_1x + \cdots + a_nx^n$ $q(x) = b_0 + a_1x + \cdots + b_nx^n$, then
$(p + q)(x) = (a_0 + b_0) + (a_1 + b_1)x + \cdots + (a_n + b_n)x^n$.
If c is a scalar, then $c \cdot p$ is the polynomial
$(c \cdot p)(x) = ca_0 + ca_1x + \cdots + ca_nx^n$.

2. $\mathcal{S}^n$ is the set of all bi-infinite sequences with period n. A sequnce $\mathbf{x} = (\ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, x_3, \ldots)$ is in the space if $x_{k+n} = x_k$ for all k. Addition is defined by
$\mathbf{x} + \mathbf{y} = (\ldots, x_{-2} + y_{-2}, x_{-1} + y_{-1}, x_0 + y_0, x_1 + y_1, x_2 + y_2, x_3 + y_3, \ldots)$
and scalar multiplication by
$c \cdot \mathbf{x} = (\ldots, cx_{-2}, cx_{-1}, cx_0, cx_1, cx_2, cx_3, \ldots)$.

3. $C[0, 1]$ is the set of all functions f defined and continuous on the ineterval $[0, 1]$. If $f, g \in C[0, 1]$, then $f + g$ is defined by
$(f + g)(x) = f(x) + g(x)$
and $c \cdot f$ is defined by
$(c \cdot f)(x) = cf(x)$.

4. $C^{(2)}(-\infty, \infty)$ is the set of all functions defined and twice continuously differentiable on $(-\infty, \infty)$ – that is, $\mathbb{R}$. Addition and scalar multiplication are defined as in the previous question.

5. Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$ be a set of vectors in a vector space $\mathcal{V}$. Define $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$.

   Answer: The $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$ is the set of all linear combinations of the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$. Equivalently, $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\} = \{c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_m\mathbf{v}_m \,|\, c_1, \cdots, c_m \text{ are scalars}\}$.