Test I

Instructions: *Show all work in your bluebook. Calculators that do linear algebra or calculus are* not *allowed.*

1. **(10 pts.)** Find both the parametric equation for the plane passing through the three points $P(1,0,1)$, $Q(1,2,2)$, $R(0,1,1)$ and the area of the triangle $\triangle PQR$.

2. **(10 pts.)** Let $\mathbf{x} = (3,2,-3,-1)$ and $\mathbf{v} = (\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$. Find the component of $\mathbf{x}$ in the direction of $\mathbf{v}$ (i.e., the projection $\mathbf{p}$ of $\mathbf{x}$ on $\mathbf{v}$), and the component $\mathbf{q}$ of $\mathbf{x}$ perpendicular to $\mathbf{v}$.

3. A linear system $A\mathbf{x} = \mathbf{b}$ has the augmented matrix $[A|\mathbf{b}]$ given below.

$$[A|b] = \left(\begin{array}{cccc|c} 1 & 1 & -3 & -2 & 2 \\ -1 & -1 & 4 & 3 & -1 \\ 1 & 1 & -2 & -1 & 3 \end{array}\right)$$

   (a) **(10 pts.)** The corresponding homogeneous system is $A\mathbf{x} = \mathbf{0}$. Is the system overdetermined or underdetermined? Does it have any non-trivial (i.e., non-zero) solutions? Are the columns of A linearly dependent or linearly independent?

   (b) **(15 pts.)** Put $[A|\mathbf{b}]$ in reduced row echelon form. Write the solution to $A\mathbf{x} = \mathbf{b}$ in parametric form.

4. **(20 pts.)** Use row reduction either to find C^{-1} or to show that it does not exist, given that the matrix C is

$$C = \left(\begin{array}{ccc} 1 & -1 & 3 \\ 2 & 3 & 4 \\ -1 & 3 & -4 \end{array}\right).$$

5. **(15 pts.)** Evaluate $\det(B)$, where B is given below, using any method. Using your answer, determine whether B is singular or invertible.

$$B = \left(\begin{array}{cccc} 1 & 2 & 0 & 0 \\ -2 & -3 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & -1 & 3 \end{array}\right).$$

6. **(10 pts.)** Find $\det(FG)$, given that F and G are the matrices below.

$$F = \begin{pmatrix} 2 & 0 & 0 \\ -2 & -3 & 0 \\ 9 & 1 & 2 \end{pmatrix}, \qquad G = \begin{pmatrix} -5 & 9 & 7 \\ 0 & 2 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

7. **(10 pts.)** Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear function for which $f([1\ -1]^T) = [1\ 2]^T$ and $f([1\ 1]^T) = [-2\ 3]^T$. Find the matrix A that represents the function f.