Extra Problems - Math 311-503

1. Consider the following operators, spaces, and inner products. In each case, show that the operator is self adjoint.

   (a) $L = \dfrac{d^2}{dx^2}$
   $V = \{f \in C^{(2)}[2,4] \mid f(2) = 0, f(4) = 0\}$
   $\langle f, g \rangle = \int_2^4 f(x)g(x)dx$

   (b) $L = \dfrac{d^2}{dx^2} - \dfrac{d}{dx}$
   $V = \{f \in C^{(2)}[0,\infty) \mid f(0) = 0, f \text{ is "nice" at } \infty\}$
   $\langle f, g \rangle = \int_0^\infty f(x)g(x)e^{-x}dx.$

   (c) $L = \dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2\dfrac{d}{dr}\right)$
   $V = \{f \in C^{(2)}[0,1] \mid f \text{ is "nice" at } 0,\ f'(1) = 0\}$
   $\langle f, g \rangle = \int_0^1 f(r)g(r)r^2dr$

2. Use the method of Frobenius in the following differential equations to find the indicial equation, the recurrence relation, and the the first few terms of the series solution.

   (a) $x^2y'' + xy' + (x^2 - 1)y = 0.$

   (b) $9x^2y'' + (9x^2 + 2)y = 0.$

   (c) $25x^2y'' + 25xy' + (x^4 - 1)y = 0$