MATH 172 Homework 5

$\boxed{7.2}$

3. $\displaystyle\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta = \int_0^{\pi/2} \sin^7\theta \cos^4\theta \cos\theta \, d\theta$

$\displaystyle = \int_0^{\pi/2} \sin^7\theta (1-\sin^2\theta)^2 \cos\theta \, d\theta$

Let $u = \sin\theta \Rightarrow du = \cos\theta \, d\theta$

@ $x = \pi/2$, $u = 1$; @ $x = 0$, $u = 0$

Substitute:

$\displaystyle \int_0^1 u^7 (1-u^2)^2 \, du = \int_0^1 u^7 (1 - 2u^2 + u^4) \, du$

$\displaystyle = \int_0^1 (u^7 - 2u^9 + u^{11}) \, du = \left[\frac{1}{8} u^8 - \frac{2}{10} u^{10} + \frac{1}{12} u^{12} \right]_0^1$

$\displaystyle = \left(\frac{1}{8} - \frac{1}{5} + \frac{1}{12} \right) - 0 = \frac{15-24+10}{120} = \boxed{\frac{1}{120}}$

11. $\displaystyle\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$

We will use the trig identity

$2 \sin x \cos x = \sin 2x$.

$\Rightarrow \sin x \cos x = \frac{1}{2} \sin 2x$

$\displaystyle \int_0^{\pi/2} (\sin x \cos x)^2 \, dx = \int_0^{\pi/2} \left(\frac{1}{2} \sin 2x\right)^2 dx$

$\displaystyle = \frac{1}{4} \int_0^{\pi/2} \sin^2 2x \, dx$

We can then use

$\displaystyle \sin^2 x = \frac{1 - \cos(2x)}{2}$

$\displaystyle = \frac{1}{4} \int_0^{\pi/2} \frac{1}{2} (1 - \cos(4x)) \, dx = \frac{1}{8} \int_0^{\pi/2} (1 - \cos(4x)) \, dx$

Using substitution $(u=4x)$

$$= \frac{1}{8}\left[x - \frac{1}{4}\sin(4x)\right]_0^{\pi/2}$$

$$= \frac{1}{8}\left[\pi/2 - \frac{1}{4}(0) - \left(0 - \frac{1}{4}(0)\right)\right] = \boxed{\frac{\pi}{16}}$$

41. $\int \sin 8x \cos 5x \, dx$

Use 2(a)

$\sin A \sin B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

$$= \int \frac{1}{2}[\sin(8x-5x) + \sin(8x+5x)]\,dx$$

$$= \frac{1}{2}\int(\sin 3x + \sin 13x)\,dx$$

By substitution:

$$= \frac{1}{2}\left[-\frac{1}{3}\cos 3x - \frac{1}{13}\cos 13x\right] + C$$

$$\boxed{-\frac{1}{6}\cos 3x - \frac{1}{26}\cos 13x + C}$$

$\boxed{7.3}$

4. $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

$a^2 = 9 \Rightarrow a = 3$

Let $x = 3\sin\theta$, where $-\pi/2 \le \theta \le \pi/2$

$dx = 3\cos\theta\, d\theta$

Then,

A $\sqrt{9-x^2} = \sqrt{9 - 9\sin^2\theta} = \sqrt{9(1-\sin^2\theta)}$

$= 3\sqrt{\cos^2\theta} = 3\cos\theta$

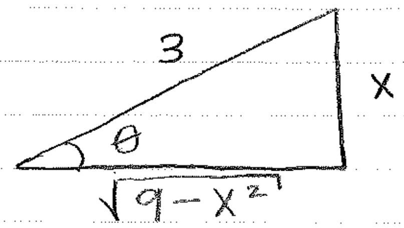
Substitute:

$$\int \frac{9\sin^2\theta}{\cancel{3\cos\theta}}\,\cancel{3\cos\theta}\,d\theta$$

$$= 9\int \sin^2\theta\,d\theta = 9\int \frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C = \frac{9}{2}\theta - \frac{9}{4}(2\cos\theta\sin\theta) + C$$

We must find θ in terms of x



$x = 3\sin\theta$

$\Rightarrow \sin\theta = \frac{x}{3}$

From ⊛ $\sqrt{9-x^2} = 3\cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{9-x^2}}{3}$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{2}\cdot\frac{\sqrt{9-x^2}}{3}\cdot\frac{x}{3} + C$

$= \boxed{\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C}$

10. $\int_0^{2/3}\sqrt{4-9x^2}\,dx$ $\quad a = 4 \Rightarrow a = 2$

$= \int_0^{2/3}\sqrt{(2)^2-(3x)^2}\,dx$

Let $3x = 2\sin\theta$

$\Rightarrow x = \frac{2}{3}\sin\theta \Rightarrow dx = \frac{2}{3}\cos\theta\,d\theta$

@ $x = 0$, $0 = 2\sin\theta \Rightarrow \theta = 0$

@ $x = 2/3$, $3(2/3) = 2\sin\theta \Rightarrow \sin\theta = 1 \Rightarrow \theta = \pi/2$

Substitute:

$\int_0^{\pi/2}\sqrt{4-9\left(\frac{2}{3}\sin\theta\right)^2}\cdot\frac{2}{3}\cos\theta\,d\theta$

$= \frac{2}{3}\int_0^{\pi/2}\sqrt{4(1-\sin^2\theta)}\,\cos\theta\,d\theta$

$= \frac{2}{3}\int_0^{\pi/2}2\cos\theta\cdot\cos\theta\,d\theta = \frac{4}{3}\int_0^{\pi/2}\cos^2\theta\,d\theta$

$= \frac{4}{3}\int_0^{\pi/2}\frac{1}{2}(1+\cos 2\theta)\,d\theta$

$= \frac{2}{3}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = \frac{2}{3}\left[\frac{\pi}{2} + \frac{1}{2}(0) - 0\right] = \boxed{\frac{\pi}{3}}$