Comment on "Boundary conditions in the Unruh problem"

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We dispute the conclusions of Narozhny et al. [Phys. Rev. D **65**, 025004 (2002)] on numerous grounds.

I. INTRODUCTION AND NOTATION

The five authors of [1] have offered a variety of arguments that "principles of quantum field theory as of now do not give convincing arguments in favor of a universal thermal response of detectors uniformly accelerated in Minkowski space." We believe that when certain careful distinctions are made, the conclusions of [1] are seen to be unwarranted.

Following [1] we regard 2- or 4-dimensional Minkowski space (MS) as divided by the lines or hyperplanes (horizons) $t \pm z = 0$ into quadrants Past (P), Future (F), Left (L), and Right (R) (plus the horizons themselves); R is also called Rindler space (RS). Inside RS one uses the coordinates and line element

$$ds^2 = \rho^2 d\eta^2 - d\rho^2, \qquad -\infty < \eta < \infty, \quad 0 < \rho < \infty. \tag{1}$$

The abbreviations $x = \{t, z\}$, $\xi = \{\eta, \rho\}$ are useful. We sometimes refer to the locus $x = 0$ as "the origin", ignoring any transverse dimensions.

II. SOME DISTINCTIONS

A. Rindler and Minkowski representations

The first distinction that must be understood is between the physical focus of the paper of Unruh [2] and that of the paper of Fulling [3].

Ref. [3] is concerned with field quantization in RS regarded as a *self-contained*, globally hyperbolic, static space-time. (The statement in [1] that "Fulling ... considered the Fulling–Rindler vacuum as a state of quantum fields in MS" is historically incorrect, if "in" means "throughout", as the rest of the paragraph seems to imply.) Ref. [3] goes on to ask whether the notion of "particle" derived from a literal interpretation of the formalism of that "Fulling–Rindler" (FR) quantization is consistent with the standard particle concept in MS, when RS is embedded in MS as the region R; the answer is negative. (Furthermore, because of the singularity of the FR vacuum as the horizon is approached, to take any extension of that state seriously as a possible physical state of the field throughout MS is a dubious enterprise.)

In Ref. [2] the focus is on standard quantum field theory in MS and how it "looks" from the point of view of a uniformly accelerated observer. The principles of equivalence and general covariance imply that such an observer should be able to make sensible calculations in the coordinate system (1) [4]. *The Fock space of the FR quantization plays no direct role in [2].* The state considered is the ordinary Minkowski vacuum, which is defined throughout MS and therefore in particular for observables localized in the region R, or in the double wedge $R \cup L$. In the calculations the "boost modes" associated to the Rindler coordinates are used as a mathematical tool only. The basic conclusion of the thermal nature of the state can alternatively be derived without even mentioning these modes, by expressing the Minkowski vacuum two-point function in Rindler coordinates [5–8].

Conclusion: Any mathematical pathologies encountered in attempting to extend the FR representation of the field algebra right onto the horizon are not relevant, either to Unruh's work (because it does not deal with the FR representation) or to Fulling's (because the horizon and the regions beyond it are not part of the space-time considered there).

B. Boundary conditions and decay conditions

The term "boundary condition" ordinarily refers to (i) a constraint that is an essential part of the specification of the dynamics of a field, as at the surface of a perfect conductor. As [1] points out, field theory in RS leads to a self-adjoint operator of the *limit point* type, which means precisely that *no* boundary condition of type (i) needs to be imposed at $\rho = 0$ to complete the definition of the dynamical system under study. However, because there is a sense in which the point $\rho = 0$ lies "at infinity" from the RS point of view, there are some technical mathematical restrictions that arise within the FR construction: (ii) A test function, $f(\rho)$ or $f(\eta, \rho)$, must satisfy some de-

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cay condition as $\rho \to 0$ in order for the field operator $\phi_R(f) \equiv \int \phi_R(\xi) f$ to be well-defined in the FR representation. (iii) A one-particle state $|g\rangle$ with finite FR energy must have a spatial amplitude $\psi_g(\xi)$ with some decay as $\rho \to 0$. Ref. [1] lays great stress on (iii) (but in language that may suggest that (i) is involved).

As we have already indicated, these mathematical technicalities of the FR representation are simply not relevant to "the Unruh effect", which concerns the vacuum state, or other physically natural states, of the ordinary Minkowski representation of the field. Furthermore, these technicalities arise precisely because the origin of MS is effectively moved "to infinity" in FR quantization; far from indicating that the origin must be somehow "cut out" of the space for that theory to apply, or that a dynamical boundary condition must be imposed there, they reflect the fact that what happens at the origin (or on its light cone, the horizon) is totally irrelevant to what happens in R, identified with RS, which *dynamically* is a self-contained, globally hyperbolic universe, knowing and caring nothing about the topology of the rest of the space-time.

C. Causality; localized observables

Let us pursue the previous point further. The treatment of initial data at the origin is mathematically subtle, and data at that point may influence the solution of the field equation in the regions F and P. However, such data cannot influence the solution in the (open) regions R and L, and that is what is pertinent. Elementary relativistic causality implies that physics inside R is completely determined by initial data inside R (excluding the origin), and similarly that physics inside $R \cup L$ is completely determined by data on the spatial axis with the origin omitted. This statement does not contradict the Reeh–Schlieder theorem or the fact that the Minkowski vacuum contains field correlations between points in R and points outside R. A given physical state in MS may contain such correlations, or may even be generated from the vacuum by field operators with support entirely in L, but its internal dynamics is still causal in R.

Conclusion: The proper treatment of field excitations with initial data concentrated at the origin, however interesting a problem it may be, is not relevant to the analysis in [2] of physics inside R.

This same point must be made in the context of algebras of observables, discussed in Sec. VI of [1]. There the authors correctly rederive the equation

$$\omega_M = \tilde{\omega}_F^{(2\pi)} \quad \text{on } \tilde{\mathcal{U}}, \tag{2}$$

which states that the restriction of the Minkowski vacuum state (as a linear functional on self-adjoint operators) to observables localized in R or L coincides with the thermal state of temperature $(2\pi)^{-1}$ relative to the boost Hamiltonian (the generator of time translations in RS). They then state that (2) "holds only on the double wedge subalgebra $\tilde{\mathcal{U}} \subset \mathcal{U}, \ldots$ [and its RHS] does not admit continuation to the whole algebra $\mathcal{U}$ while the LHS admits such a continuation. Therefore the functionals ω_M and $\tilde{\omega}_F^{(2\pi)}$ describe *different* algebraic states over the algebra of observables of the free field in MS." This passage contradicts itself. The equation (2) states that $\tilde{\omega}_F^{(2\pi)}$ *does* have a continuation to $\mathcal{U}$, namely, ω_M. This extension has, of course, the property of Poincaré invariance. The objection in the next paragraph of [1] that $\tilde{U}$ itself is not Poincaré-invariant makes no more sense than to claim that the U. S. Postal Service is not nationwide because its Boston post office collects mail only from Boston.

D. The zero boost mode

Let us now address a technical argument introduced in [1] to indicate that the subtlety about the origin *does* affect the physics in $R \cup L$. The standard field theory in MS can be expressed in terms of normal modes, $\Psi_\kappa(x)$, that are eigenfunctions of the boost Hamiltonian. The variable κ ranges from $-\infty$ to ∞, and the modes have the normalization

$$(\Psi_\kappa, \Psi_{\kappa'})_M = \delta(\kappa - \kappa'), \tag{3}$$

$$[b_\kappa, b_{\kappa'}^\dagger] = \delta(\kappa - \kappa') \tag{4}$$

(equations (4.9) and (4.10) of [1]).

Now, the authors of [1] make the interesting observation that $\Psi_0(x)$ is (up to a finite numerical factor) equal to the standard Wightman two-point function of the field with the second point fixed at the origin. Since $\Psi_0(x)$ does not vanish for $x \in R \cup L$, it may seem that omitting or including the mode $\kappa = 0$ changes the dynamics of the field in $R \cup L$ (or in R), by turning this mode on or off. However, it follows from (3) and (4) that the natural measure for integration over κ is ordinary Lebesgue measure, as indeed appears on the left side of (5.8) and several other (unnumbered) equations in [1]. When a test function or wave function is expanded in these modes, the omission of the single point $\kappa = 0$ will not change the value of the integral. That is, $\kappa = 0$ is a point of *zero measure* in the spectral resolution of the boost Hamiltonian. If this were not so, then (3) and (4) would necessarily contain Kronecker delta terms, $\delta_{\kappa 0}$, for $\kappa' = 0$. (For example, since Ψ_0 is not normalizable, changing the amplitude corresponding to it by a discrete amount would convert a normalized test function to a nonnormalizable function. That can't be correct.)

It is precisely this mathematical fact about $\kappa = 0$ that [1] denies; the paper repeatedly states that $\kappa = 0$ is a point of *nonzero measure*. What does this mean? What [1] has shown is that $\Psi_\kappa(x)$ itself is singular as a function of κ at $\kappa = 0$ when x is fixed at the origin or any point on the horizon, and also that $\Psi_\kappa(0)$ vanishes when $\kappa \neq 0$.

But these things are not relevant to the behavior of an eigenfunction expansion at points inside the wedges R and L, where all the modes are smooth functions of both κ and x, and they are not inconsistent with the claim that $\kappa = 0$ does not affect those regions except as a Lebesgue-negligible point.

It may seem strange that a mode of zero measure can be entirely responsible for the value of a function at the origin, so we digress to show how it can happen. By hypothesis we are dealing with the eigenfunction expansion of a *continuous* function, say $g(x)$. Thus $g(0)$ is recoverable from the values of g elsewhere, which in turn are obtainable without any contribution from the $\kappa = 0$ mode. For this to work out, it is important that the vanishing of $\Psi_\kappa(x)$ as $x \to 0$ is nonuniform in κ. An elementary partial analog of the phenomenon is manifested by the equation

$$\delta(x - x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} e^{-ikx'}\, dk,$$

which means that

$$\begin{aligned} g(x) &= \int_{-\infty}^{\infty} \delta(x - x') g(x')\, dx' \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\, e^{ikx} \int_{-\infty}^{\infty} dx'\, g(x') e^{-ikx'} \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\, e^{ikx} \lim_{\epsilon \downarrow 0} \int_{|x'|>\epsilon} dx'\, g(x') e^{-ikx'} \\ &= \lim_{\epsilon \downarrow 0} \int_{|x'|>\epsilon} \delta(x - x') g(x')\, dx'. \end{aligned}$$

When $x = 0$ we see here the value of g at 0 being built up (distributionally) out of objects $\{\delta(x - x') \colon x' \neq 0\}$ that vanish near $x = 0$! (The interchanges of integrals are valid here only because we are considering distributional limits.)

E. Superselection rules; correlations; state preparation

When an observable A, such as electric charge, commutes with all other observables, it is said to determine a *superselection rule* [9–11]. (Here "observable" is meant in the strict, literal sense: something that, at least in principle, can be observed. An arbitrary self-adjoint operator is not an observable in this sense.) Then a coherent linear superposition of eigenstates of A cannot be distinguished from a statistical mixture, or from another superposition with different relative phases. Contrary to a common misconception, this does not mean that the physical system can never be in a state where A does not have a definite value; it merely says that there is no operational difference between pure states and mixed states of that type. In a completely classical world, all observables would be superselection rules; a macroscopic observable, such as the top side of a die, defines a superselection rule for all practical purposes. But classical statistical mechanics and the classical probability theory of dice are not thereby invalidated.

Ref. [1] asserts that a superselection rule relating observables in R and L makes it impossible to represent the Minkowski vacuum as a superposition of states of different Rindler particle numbers; this superselection rule, it is said, prohibits correlations between particles in R and particles in L, and this prohibition is intimately related to the cutting out of the origin, or the imposition of a boundary condition there, which alters the topology of space-time. This claim is faulty for several reasons. First, as just pointed out, a superselection rule does not forbid superpositions. Second, what is involved here is not really a superselection rule, but just the commutativity of observables belonging to spacelike-separated regions, which is a general feature of relativistic theories. The argument proves far too much: Consider two bounded regions of space-time, far spatially separated. The respective subalgebras of observables commute with each other. This is a statement about standard quantum field theory in full MS; it does not hinge on the excision of any point lying between the two regions. (Which point would it be?) Finally, causality (which this commutativity represents) in no way prevents observables in one region from being correlated with observables in the other; the correlations can be created by some common cause in the far past, and are encoded in the quantum state of the system.

A simpler example is provided by the operators of the electron and proton in a hydrogen atom. They commute with each other, but correlations between the electron and proton wave functions still exist and have observational consequences (e.g., hyperfine levels).

Similar remarks apply to [1]'s complaint that a uniformly accelerated observer cannot "prepare the Minkowski vacuum state as the initial state of the field." *No* observer can *prepare* an absolute vacuum throughout all space. The question being studied is what an observer detects in whatever initial state is *assumed*. *Which* state to investigate must be decided on the basis of some statistical and cosmological assumptions, but the MS vacuum is certainly a natural place to start. Once the vacuum is understood, calculations for other, related states (e.g., a 3 K thermal state, or a beam at CERN) are relatively easy.

III. FINAL REMARKS

The reality of acceleration temperature has been confirmed by various theoretical analyses [7, 12–14] of concrete systems, involving only local physics. The general pattern is that any phenomenon that appears "thermal" in an accelerated frame is also present in an inertial frame, but with a different physical interpretation.

We have already mentioned that thermality can be deduced by expressing the MS vacuum Green function in terms of the (Fermi normal) coordinates comoving with the accelerating observer or test system [5–8]. *This elementary algebraic calculation is an entirely local matter.* No appeal is necessary to topology, horizons, periodic coordinates, Bogolubov transformations, etc., however interesting and instructive those concepts may be. In [7] and [8] one sees the topic entering in this way the literature of standard quantum electrodynamics and atomic physics.

The effects in the atomic system of [7] and the nucleonic one of [14] should be observable in principle. The residual polarization of electrons undergoing circular acceleration in storage rings [13] *is* observed; in that case, it is the theoretical part of the analysis that is complicated, and there have been some controversies over the years, but recent, sufficiently inclusive analysis [15] demonstrates consistency. One of the causes of controversy has been that there is no analog of the FR vacuum in the case of circular acceleration; in the present context, that fact simply strengthens our argument that acceleration temperature does not depend on the FR quantization in a central way.

In response to the authors' Reply [16]: We acknowledge that the statement on p. 879 of [2] is technically incorrect when applied to distributions. However, the conclusions of [2] and the arguments of this Comment do not depend on that claim. The Cauchy problem for the wave equation is well-posed within certain function spaces, such as smooth functions of compact support (in the roles of initial and final data). Such a function can be expanded in the boost eigenfuctions; the expansion is an integral, not a sum, so it is unaffected by the omission of one point of the spectrum. In this sense, the paragraph of [2] in question is correct in concluding "The expansion of Eq. (2.12) is then valid in the full Minkowski space-time." In principle one need never use (2.12) in Minkowski space except with smooth test functions, and hence the omission of the zero boost mode will not affect the calculations.

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