Initial assumptions

```
In[1]:= $Assumptions = {r > 0 , rp > 0, t > 0, θ1 > θ > 0, z > 0, u > 0, θp > 0, α > θ > 0};
```

Wedge Tbar pieces for general alpha, in polar coordinates

In[2]:= $\text{wedgeTbar1} = \left(\text{Csch}[u]\,\text{Sinh}\left[\frac{\pi u}{2\alpha}\right]\right) \Big/ \left(2\pi\, r\, rp \left(4\alpha\,\text{Cos}\left[\frac{\pi(\theta+\theta p)}{2\alpha}\right] + 4\alpha\,\text{Cosh}\left[\frac{\pi u}{2\alpha}\right]\right)\right);$

In[3]:= $\text{wedgeTbar2} = \left(\text{Csch}[u]\,\text{Sinh}\left[\frac{\pi u}{2\alpha}\right]\right) \Big/ \left(8\pi\, r\, rp\, \alpha \left(-\text{Cos}\left[\frac{\pi(\theta+\theta p)}{2\alpha}\right] + \text{Cosh}\left[\frac{\pi u}{2\alpha}\right]\right)\right);$

In[4]:= $\text{wedgeTbar3} = \left(-\text{Csch}[u]\,\text{Sinh}\left[\frac{\pi u}{2\alpha}\right]\right) \Big/ \left(2\pi\, r\, rp \left(4\alpha\,\text{Cos}\left[\frac{\pi(\theta-\theta p)}{2\alpha}\right] + 4\alpha\,\text{Cosh}\left[\frac{\pi u}{2\alpha}\right]\right)\right);$

In[5]:= $\text{wedgeTbar4} = \left(-\text{Csch}[u]\,\text{Sinh}\left[\frac{\pi u}{2\alpha}\right]\right) \Big/ \left(8\pi\, r\, rp\, \alpha \left(-\text{Cos}\left[\frac{\pi(\theta-\theta p)}{2\alpha}\right] + \text{Cosh}\left[\frac{\pi u}{2\alpha}\right]\right)\right);$

Definition of u

```
In[6]:= u = ArcCosh[(2 r * rp) ^ (-1) (r^2 + rp^2 + t^2 + (z - zp) ^2)];
```

For the time being, we specialize to alpha=pi/2.

```
In[7]:= zp := 0;
```

Wedge Tbar pieces for alpha=pi/2, in polar coordinates

```
In[8]:= wtbar1pi2 = Simplify[wedgeTbar1 /. α → Pi / 2]
```

Out[8]= $1 \Big/ \left(2\pi^2 \left(r^2 + rp^2 + t^2 + z^2 + 2\, r\, rp\, \text{Cos}[\theta + \theta p]\right)\right)$

```
In[9]:= wtbar2pi2 = Simplify[wedgeTbar2 /. α → Pi / 2]
```

Out[9]= $1 \Big/ \left(2\pi^2 \left(r^2 + rp^2 + t^2 + z^2 - 2\, r\, rp\, \text{Cos}[\theta + \theta p]\right)\right)$

```
In[10]:= wtbar3pi2 = Simplify[wedgeTbar3 /. α → Pi / 2]
```

Out[10]= $-1 \Big/ \left(2\pi^2 \left(r^2 + rp^2 + t^2 + z^2 + 2\, r\, rp\, \text{Cos}[\theta - \theta p]\right)\right)$

```
In[11]:= wtbar4pi2 = Simplify[wedgeTbar4 /. α → Pi / 2]
```

Out[11]= $-1 \Big/ \left(2\pi^2 \left(r^2 + rp^2 + t^2 + z^2 - 2\, r\, rp\, \text{Cos}[\theta - \theta p]\right)\right)$

Tbar pieces for planes at x=0 and y=0, in Cartesian coordinates

In[12]:= $\text{cartTbar1} = \frac{1/(2\,\text{Pi}^\wedge 2)}{t^2 + (x + xp)^2 + (y - yp)^2 + (z - zp)^2};$

In[13]:= $\text{cartTbar2} = \frac{1/(2\,\text{Pi}^\wedge 2)}{t^2 + (x - xp)^2 + (y + yp)^2 + (z - zp)^2};$

In[14]:= $\text{cartTbar3} = -\frac{1/(2\,\text{Pi}^\wedge 2)}{t^2 + (x + xp)^2 + (y + yp)^2 + (z - zp)^2};$

In[15]:= $\text{cartTbar4} = -\frac{1/(2\,\text{Pi}^\wedge 2)}{t^2 + (x - xp)^2 + (y - yp)^2 + (z - zp)^2};$

Tbar pieces for planes at x=0 and y=0, in polar coordinates

```
In[16]:= pctbar1 =
  Simplify[cartTbar1 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]
```

Out[16]= $1 \Big/ \left(2\pi^2 \left(r^2 + rp^2 + t^2 + z^2 + 2\, r\, rp\, \text{Cos}[\theta + \theta p]\right)\right)$

In[17]:=
```
pctbar2 =
 Simplify[cartTbar2 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]
```

Out[17]= $1 / \left(2 \pi^2 \left(r^2 + rp^2 + t^2 + z^2 - 2\, r\, rp \cos[\theta + \theta p]\right)\right)$

In[18]:=
```
pctbar3 =
 Simplify[cartTbar3 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]
```

Out[18]= $-1 / \left(2 \pi^2 \left(r^2 + rp^2 + t^2 + z^2 + 2\, r\, rp \cos[\theta - \theta p]\right)\right)$

In[19]:=
```
pctbar4 =
 Simplify[cartTbar4 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]
```

Out[19]= $-1 / \left(2 \pi^2 \left(r^2 + rp^2 + t^2 + z^2 - 2\, r\, rp \cos[\theta - \theta p]\right)\right)$

Do these agree with the wedge pieces at alpha=pi/2?

In[20]:=
```
FullSimplify[wtbar1pi2 - pctbar1]
```

Out[20]= 0

In[21]:=
```
FullSimplify[wtbar2pi2 - pctbar2]
```

Out[21]= 0

In[22]:=
```
FullSimplify[wtbar3pi2 - pctbar3]
```

Out[22]= 0

In[23]:=
```
FullSimplify[wtbar4pi2 - pctbar4]
```

Out[23]= 0

Yes! Good.

Now, we will look at the energy density and perpendicular/radial pressure for each piece, in both cartesian and polar coordinates.

Cartesian T00 and pressures, in Cartesian coordinates

In[24]:=
```
cartrho1 = -1 / 2 (D[cartTbar1, {t, 2}]) // Simplify
```

Out[24]= $$-\frac{1}{4 \pi^2} \left(\frac{8 t^2}{\left(t^2 + (x + xp)^2 + (y - yp)^2 + z^2\right)^3} - \frac{2}{\left(t^2 + (x + xp)^2 + (y - yp)^2 + z^2\right)^2}\right)$$

In[25]:=
```
cartrho2 = -1 / 2 (D[cartTbar2, {t, 2}])
```

Out[25]= $$-\frac{1}{4 \pi^2} \left(\frac{8 t^2}{\left(t^2 + (x - xp)^2 + (y + yp)^2 + z^2\right)^3} - \frac{2}{\left(t^2 + (x - xp)^2 + (y + yp)^2 + z^2\right)^2}\right)$$

In[26]:=
```
cartrho3 = -1 / 2 (D[cartTbar3, {t, 2}])
```

Out[26]= $$\frac{1}{4 \pi^2} \left(\frac{8 t^2}{\left(t^2 + (x + xp)^2 + (y + yp)^2 + z^2\right)^3} - \frac{2}{\left(t^2 + (x + xp)^2 + (y + yp)^2 + z^2\right)^2}\right)$$

In[27]:=
```
cartrho4 = -1 / 2 (D[cartTbar4, {t, 2}])
```

Out[27]= $$\frac{1}{4 \pi^2} \left(\frac{8 t^2}{\left(t^2 + (x - xp)^2 + (y - yp)^2 + z^2\right)^3} - \frac{2}{\left(t^2 + (x - xp)^2 + (y - yp)^2 + z^2\right)^2}\right)$$

In[28]:=
```
px1 =
 1 / 8 (D[cartTbar1, {x, 2}] + D[cartTbar1, {xp, 2}] - 2 D[D[cartTbar1, x], xp]) // Simplify
```
Out[28]= 0

In[29]:=
```
py1 =
 1 / 8 (D[cartTbar1, {y, 2}] + D[cartTbar1, {yp, 2}] - 2 D[D[cartTbar1, y], yp])  // Simplify
```
Out[29]= $-\left(t^2+x^2+2\,x\,xp+xp^2-3\,y^2+6\,y\,yp-3\,yp^2+z^2\right)\Big/\left(2\,\pi^2\left(t^2+x^2+2\,x\,xp+xp^2+y^2-2\,y\,yp+yp^2+z^2\right)^3\right)$

In[30]:=
```
px2 =
 1 / 8 (D[cartTbar2, {x, 2}] + D[cartTbar2, {xp, 2}] - 2 D[D[cartTbar2, x], xp]) // Simplify
```
Out[30]= $-\left(t^2-3\,x^2+6\,x\,xp-3\,xp^2+y^2+2\,y\,yp+yp^2+z^2\right)\Big/\left(2\,\pi^2\left(t^2+x^2-2\,x\,xp+xp^2+y^2+2\,y\,yp+yp^2+z^2\right)^3\right)$

In[31]:=
```
py2 =
 1 / 8 (D[cartTbar2, {y, 2}] + D[cartTbar2, {yp, 2}] - 2 D[D[cartTbar2, y], yp]) // Simplify
```
Out[31]= 0

In[32]:=
```
px3 =
 1 / 8 (D[cartTbar3, {x, 2}] + D[cartTbar3, {xp, 2}] - 2 D[D[cartTbar3, x], xp])  // Simplify
```
Out[32]= 0

In[33]:=
```
py3 =
 1 / 8 (D[cartTbar3, {y, 2}] + D[cartTbar3, {yp, 2}] - 2 D[D[cartTbar3, y], yp])  //  Simplify
```
Out[33]= 0

In[34]:=
```
px4 =
 1 / 8 (D[cartTbar4, {x, 2}] + D[cartTbar4, {xp, 2}] - 2 D[D[cartTbar4, x], xp])  // Simplify
```
Out[34]= $\left(t^2-3\,x^2+6\,x\,xp-3\,xp^2+y^2-2\,y\,yp+yp^2+z^2\right)\Big/\left(2\,\pi^2\left(t^2+x^2-2\,x\,xp+xp^2+y^2-2\,y\,yp+yp^2+z^2\right)^3\right)$

In[35]:=
```
py4 =
 1 / 8 (D[cartTbar4, {y, 2}] + D[cartTbar4, {yp, 2}] - 2 D[D[cartTbar4, y], yp]) // Simplify
```
Out[35]= $\left(t^2+x^2-2\,x\,xp+xp^2-3\,y^2+6\,y\,yp-3\,yp^2+z^2\right)\Big/\left(2\,\pi^2\left(t^2+x^2-2\,x\,xp+xp^2+y^2-2\,y\,yp+yp^2+z^2\right)^3\right)$

In[36]:=
```
Txy1 = 1 / 8 (D[D[cartTbar1, x], y] + D[D[cartTbar1, xp], yp] -
      D[D[cartTbar1, x], yp] - D[D[cartTbar1, xp], y]) // FullSimplify
```
Out[36]= 0

In[37]:=
```
Txy2 = 1 / 8 (D[D[cartTbar2, x], y] + D[D[cartTbar2, xp], yp] -
    D[D[cartTbar2, x], yp] - D[D[cartTbar2, xp], y]) FullSimplify
```
Out[37]= 0

In[38]:=
```
Txy3 = 1 / 8 (D[D[cartTbar3, x], y] + D[D[cartTbar3, xp], yp] -
      D[D[cartTbar3, x], yp] - D[D[cartTbar3, xp], y]) // FullSimplify
```
Out[38]= 0

In[39]:=
```
Txy4 = 1 / 8 (D[D[cartTbar4, x], y] + D[D[cartTbar4, xp], yp] -
      D[D[cartTbar4, x], yp] - D[D[cartTbar4, xp], y]) // FullSimplify
```
Out[39]= $$-\frac{2\,(x-xp)\,(y-yp)}{\pi^2\left(t^2+(x-xp)^2+(y-yp)^2+z^2\right)^3}$$

In[40]:=
```
Txy4 /. {xp → x, yp → y}
```
Out[40]= 0

In[41]:= `py1 /. {xp → x, yp → y}`

Out[41]= $-\frac{1}{2\pi^2\left(t^2+4x^2+z^2\right)^2}$

In[42]:= `px2 /. {xp → x, yp → y}`

Out[42]= $-\frac{1}{2\pi^2\left(t^2+4y^2+z^2\right)^2}$

In[43]:= `py4 /. {xp → x, yp → y}`

Out[43]= $\frac{1}{2\pi^2\left(t^2+z^2\right)^2}$

In[44]:= `px4 /. {xp → x, yp → y}`

Out[44]= $\frac{1}{2\pi^2\left(t^2+z^2\right)^2}$

Cartesian T00 and pressures, in polar coordinates

In[45]:= `pcrho1 = Simplify[cartrho1 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]`

Out[45]= $\left(r^2+rp^2-3t^2+z^2+2\,r\,rp\,\mathrm{Cos}[\theta+\theta p]\right)\Big/\left(2\pi^2\left(r^2+rp^2+t^2+z^2+2\,r\,rp\,\mathrm{Cos}[\theta+\theta p]\right)^3\right)$

In[46]:= `pcrho1d = pcrho1 /. {rp → r, θp → θ}`

Out[46]= $\frac{2r^2-3t^2+z^2+2r^2\,\mathrm{Cos}[2\theta]}{2\pi^2\left(2r^2+t^2+z^2+2r^2\,\mathrm{Cos}[2\theta]\right)^3}$

In[47]:= `pcrho2 = Simplify[cartrho2 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}];`

In[48]:= `pcrho2d = pcrho2 /. {rp → r, θp → θ}`

Out[48]= $\frac{2r^2-3t^2+z^2-2r^2\,\mathrm{Cos}[2\theta]}{2\pi^2\left(2r^2+t^2+z^2-2r^2\,\mathrm{Cos}[2\theta]\right)^3}$

In[49]:= `pcrho3 = Simplify[cartrho3 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}];`

In[50]:= `pcrho3d = pcrho3 /. {rp → r, θp → θ}`

Out[50]= $-\frac{4r^2-3t^2+z^2}{2\pi^2\left(4r^2+t^2+z^2\right)^3}$

In[51]:= `pcrho4 = Simplify[cartrho4 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}];`

In[52]:= `pcrho4d = pcrho4 /. {rp → r, θp → θ}`

Out[52]= $-\frac{-3t^2+z^2}{2\pi^2\left(t^2+z^2\right)^3}$

In[53]:= `pcpx1 = Simplify[px1 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}];`

In[54]:= `pcpx1d = pcpx1 /. {rp → r, θp → θ}`

Out[54]= 0

```
In[55]:= pcpy1 = Simplify[py1 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[55]= -(-r^2 - rp^2 + t^2 + z^2 + 2 r^2 Cos[2 θ] + 4 r rp Cos[θ - θp] + 2 rp^2 Cos[2 θp] - 2 r rp Cos[θ + θp]) /
         (2 π^2 (r^2 + rp^2 + t^2 + z^2 + 2 r rp Cos[θ + θp])^3)

In[56]:= pcpy1d = pcpy1 /. {rp → r, θp → θ}

Out[56]= -1/(2 π^2 (2 r^2 + t^2 + z^2 + 2 r^2 Cos[2 θ])^2)

In[57]:= pcpx2 = Simplify[px2 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}];

In[58]:= pcpx2d = pcpx2 /. {rp → r, θp → θ} // Simplify

Out[58]= -1/(2 π^2 (2 r^2 + t^2 + z^2 - 2 r^2 Cos[2 θ])^2)

In[59]:= pcpy2 = Simplify[py2 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}];

In[60]:= pcpy2d = pcpy2 /. {rp → r, θp → θ}

Out[60]= 0

In[61]:= pcpx3 = Simplify[px3 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[61]= 0

In[62]:= pcpy3 = Simplify[py3 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[62]= 0

In[63]:= pcpx4 = Simplify[px4 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[63]= (-r^2 - rp^2 + t^2 + z^2 - 2 r^2 Cos[2 θ] + 2 r rp Cos[θ - θp] - 2 rp^2 Cos[2 θp] + 4 r rp Cos[θ + θp]) /
         (2 π^2 (r^2 + rp^2 + t^2 + z^2 - 2 r rp Cos[θ - θp])^3)

In[64]:= pcpx4d = pcpx4 /. {rp → r, θp → θ}

Out[64]= 1/(2 π^2 (t^2 + z^2)^2)

In[65]:= pcpy4 = Simplify[py4 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}];

In[66]:= pcpy4d = pcpy4 /. {rp → r, θp → θ}

Out[66]= 1/(2 π^2 (t^2 + z^2)^2)

In[67]:= pcTxy1 = Simplify[Txy1 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[67]= 0

In[68]:= pcTxy2 = Simplify[Txy2 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[68]= 0

In[69]:= pcTxy3 = Simplify[Txy3 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[69]= 0

In[70]:= pcTxy4 = Simplify[Txy4 /. {x → r Cos[θ], xp → rp Cos[θp], y → r Sin[θ], yp → rp Sin[θp]}]

Out[70]= -(2 (r Cos[θ] - rp Cos[θp]) (r Sin[θ] - rp Sin[θp])) / (π^2 (r^2 + rp^2 + t^2 + z^2 - 2 r rp Cos[θ - θp])^3)
```

In[71]:= `pcTxy4d = pcTxy4 /. {rp → r, θp → θ}`

Out[71]= 0

Wedge T00 and pressures, in polar coordinates, for alpha=pi/2

In[72]:= $\text{T001pi2} = -\frac{1}{2}\,\text{D[wtbar1pi2, \{t, 2\}]}$

Out[72]= $-\frac{1}{4\pi^2}\left(\left(8\,t^2\right)/\left(r^2+rp^2+t^2+z^2+2\,r\,rp\,\text{Cos}[\theta+\theta p]\right)^3-2/\left(r^2+rp^2+t^2+z^2+2\,r\,rp\,\text{Cos}[\theta+\theta p]\right)^2\right)$

In[73]:= `T001pi2d = T001pi2 /. {rp → r, θp → θ} // Simplify`

Out[73]= $\frac{2\,r^2-3\,t^2+z^2+2\,r^2\,\text{Cos}[2\,\theta]}{2\,\pi^2\left(2\,r^2+t^2+z^2+2\,r^2\,\text{Cos}[2\,\theta]\right)^3}$

In[74]:= $\text{T002pi2} = -\frac{1}{2}\,\text{D[wtbar2pi2, \{t, 2\}];}$

In[75]:= `T002pi2d = T002pi2 /. {rp → r, θp → θ} // Simplify`

Out[75]= $\frac{2\,r^2-3\,t^2+z^2-2\,r^2\,\text{Cos}[2\,\theta]}{2\,\pi^2\left(2\,r^2+t^2+z^2-2\,r^2\,\text{Cos}[2\,\theta]\right)^3}$

In[76]:= $\text{T003pi2} = -\frac{1}{2}\,\text{D[wtbar3pi2, \{t, 2\}];}$

In[77]:= `T003pi2d = T003pi2 /. {rp → r, θp → θ} // Simplify`

Out[77]= $-\frac{4\,r^2-3\,t^2+z^2}{2\,\pi^2\left(4\,r^2+t^2+z^2\right)^3}$

In[78]:= $\text{T004pi2} = -\frac{1}{2}\,\text{D[wtbar4pi2, \{t, 2\}];}$

In[79]:= `T004pi2d = T004pi2 /. {rp → r, θp → θ} // Simplify`

Out[79]= $-\frac{-3\,t^2+z^2}{2\,\pi^2\left(t^2+z^2\right)^3}$

In[80]:= Tpp1pi2 =

$\frac{1}{4\,r}\,\text{D[wtbar1pi2, r]}+\frac{1}{4\,r\text{\^{}}2}\,\text{(D[wtbar1pi2, \{}\theta\text{, 2\}] - D[D[wtbar1pi2, }\theta\text{], }\theta\text{p]) // Simplify}$

Out[80]= $-\left(r+rp\,\text{Cos}[\theta+\theta p]\right)/\left(4\,\pi^2\,r\left(r^2+rp^2+t^2+z^2+2\,r\,rp\,\text{Cos}[\theta+\theta p]\right)^2\right)$

In[81]:= `Tpp1pi2d = Tpp1pi2 /. {rp → r, θp → θ} // Simplify`

Out[81]= $-\frac{\text{Cos}[\theta]^2}{2\,\pi^2\left(2\,r^2+t^2+z^2+2\,r^2\,\text{Cos}[2\,\theta]\right)^2}$

In[82]:= $\text{Tpp2pi2} = \frac{1}{4\,r}\,\text{D[wtbar2pi2, r]}+\frac{1}{4\,r\text{\^{}}2}\,\text{(D[wtbar2pi2, \{}\theta\text{, 2\}] - D[D[wtbar2pi2, }\theta\text{], }\theta\text{p]);}$

In[83]:= `Tpp2pi2d = Tpp2pi2 /. {rp → r, θp → θ} // Simplify`

Out[83]= $-\frac{\text{Sin}[\theta]^2}{2\,\pi^2\left(2\,r^2+t^2+z^2-2\,r^2\,\text{Cos}[2\,\theta]\right)^2}$

In[84]:= **Tpp3pi2 =**

$$\frac{1}{4\,r}\,D[wtbar3pi2,\,r] + \frac{1}{4\,r^{\wedge}2}\,(D[wtbar3pi2,\,\{\theta,\,2\}] - D[D[wtbar3pi2,\,\theta],\,\theta p])\ //\ Simplify$$

Out[84]= $\left(r\left(r^2-4\,rp^2+t^2+z^2\right)+3\,r\,rp^2\,Cos[2\,\theta-2\,\theta p]-rp\left(-r^2+rp^2+t^2+z^2\right)Cos[\theta-\theta p]\right)\Big/\left(4\,\pi^2\,r\left(r^2+rp^2+t^2+z^2+2\,r\,rp\,Cos[\theta-\theta p]\right)^3\right)$

In[85]:= **Tpp3pi2d = Tpp3pi2 /. {rp → r, θp → θ} // Simplify**

Out[85]= 0

In[86]:= **Tpp4pi2 =**

$$\frac{1}{4\,r}\,D[wtbar4pi2,\,r] + \frac{1}{4\,r^{\wedge}2}\,(D[wtbar4pi2,\,\{\theta,\,2\}] - D[D[wtbar4pi2,\,\theta],\,\theta p])\ //\ Simplify$$

Out[86]= $\left(r\left(r^2-4\,rp^2+t^2+z^2\right)+3\,r\,rp^2\,Cos[2\,\theta-2\,\theta p]+rp\left(-r^2+rp^2+t^2+z^2\right)Cos[\theta-\theta p]\right)\Big/\left(4\,\pi^2\,r\left(r^2+rp^2+t^2+z^2-2\,r\,rp\,Cos[\theta-\theta p]\right)^3\right)$

In[87]:= **Tpp4pi2d = Tpp4pi2 /. {rp → r, θp → θ} // Simplify**

Out[87]= $\dfrac{1}{2\,\pi^2\left(t^2+z^2\right)^2}$

In[88]:= **Trr1pi2** $= -\frac{1}{4}$ **(D[D[wtbar1pi2, rp], r] - D[wtbar1pi2, {r, 2}]) // Simplify**

Out[88]= $-\left((-1+Cos[\theta+\theta p])\left(3\,r^2-4\,r\,rp-rp^2-t^2-z^2+2\,(r-2\,rp)\,rp\,Cos[\theta+\theta p]\right)\right)\Big/\left(4\,\pi^2\left(r^2+rp^2+t^2+z^2+2\,r\,rp\,Cos[\theta+\theta p]\right)^3\right)$

In[89]:= **Trr1pi2d = Trr1pi2 /. {rp → r, θp → θ} // Simplify**

Out[89]= $-\dfrac{Sin[\theta]^2}{2\,\pi^2\left(2\,r^2+t^2+z^2+2\,r^2\,Cos[2\,\theta]\right)^2}$

In[90]:= **Trr2pi2** $= -\frac{1}{4}$ **(D[D[wtbar2pi2, rp], r] - D[wtbar2pi2, {r, 2}]);**

In[91]:= **Trr2pi2d = Trr2pi2 /. {rp → r, θp → θ} // Simplify**

Out[91]= $-\dfrac{Cos[\theta]^2}{2\,\pi^2\left(2\,r^2+t^2+z^2-2\,r^2\,Cos[2\,\theta]\right)^2}$

In[92]:= **Trr3pi2** $= -\frac{1}{4}$ **(D[D[wtbar3pi2, rp], r] - D[wtbar3pi2, {r, 2}]) // Simplify**

Out[92]= $\left((-1+Cos[\theta-\theta p])\left(3\,r^2-4\,r\,rp-rp^2-t^2-z^2+2\,(r-2\,rp)\,rp\,Cos[\theta-\theta p]\right)\right)\Big/\left(4\,\pi^2\left(r^2+rp^2+t^2+z^2+2\,r\,rp\,Cos[\theta-\theta p]\right)^3\right)$

In[93]:= **Trr3pi2d = Trr3pi2 /. {rp → r, θp → θ} // Simplify**

Out[93]= 0

In[94]:= **Trr4pi2** $= -\frac{1}{4}$ **(D[D[wtbar4pi2, rp], r] - D[wtbar4pi2, {r, 2}]) // Simplify**

Out[94]= $\left((1+Cos[\theta-\theta p])\left(-3\,r^2+4\,r\,rp+rp^2+t^2+z^2+2\,(r-2\,rp)\,rp\,Cos[\theta-\theta p]\right)\right)\Big/\left(4\,\pi^2\left(r^2+rp^2+t^2+z^2-2\,r\,rp\,Cos[\theta-\theta p]\right)^3\right)$

In[95]:=
```
Trr4pi2d = Trr4pi2 /. {rp → r, θp → θ} // Simplify
```

Out[95]= $\frac{1}{2\pi^2 (t^2+z^2)^2}$

In[193]:=
```
Tpr1pi2 =
  -1/4 (1/r D[D[wtbar1pi2, r], θp] - 1/r D[D[wtbar1pi2, θ], r]) - (4 r^2)^(-1) D[wtbar1pi2, θ]
```

Out[193]= $-(rp\,\mathrm{Sin}[\theta+\theta p]) \Big/ \left(4\pi^2 r \left(r^2+rp^2+t^2+z^2+2\,r\,rp\,\mathrm{Cos}[\theta+\theta p]\right)^2\right)$

In[194]:=
```
Tpr1pi2d = Tpr1pi2 /. {rp → r, θp → θ} // Simplify
```

Out[194]= $-\frac{\mathrm{Sin}[2\theta]}{4\pi^2 \left(2r^2+t^2+z^2+2r^2\mathrm{Cos}[2\theta]\right)^2}$

In[195]:=
```
Tpr2pi2 =
  -1/4 (1/r D[D[wtbar2pi2, r], θp] - 1/r D[D[wtbar2pi2, θ], r]) - (4 r^2)^(-1) D[wtbar2pi2, θ];
```

In[196]:=
```
Tpr2pi2d = Tpr2pi2 /. {rp → r, θp → θ} // Simplify
```

Out[196]= $\frac{\mathrm{Sin}[2\theta]}{4\pi^2 \left(2r^2+t^2+z^2-2r^2\mathrm{Cos}[2\theta]\right)^2}$

In[197]:=
```
Tpr3pi2 = -1/4 (1/r D[D[wtbar3pi2, r], θp] - 1/r D[D[wtbar3pi2, θ], r]) -
    (4 r^2)^(-1) D[wtbar3pi2, θ] // Simplify
```

Out[197]= $-\left(rp\left(-7r^2+rp^2+t^2+z^2-6\,r\,rp\,\mathrm{Cos}[\theta-\theta p]\right)\mathrm{Sin}[\theta-\theta p]\right) \Big/ \left(4\pi^2 r \left(r^2+rp^2+t^2+z^2+2\,r\,rp\,\mathrm{Cos}[\theta-\theta p]\right)^3\right)$

In[198]:=
```
Tpr3pi2d = Tpr3pi2 /. {rp → r, θp → θ} // Simplify
```

Out[198]= 0

In[199]:=
```
Tpr4pi2 =
  -1/4 (1/r D[D[wtbar4pi2, r], θp] - 1/r D[D[wtbar4pi2, θ], r]) - (4 r^2)^(-1) D[wtbar4pi2, θ];
```

In[200]:=
```
Tpr4pi2d = Tpr4pi2 /. {rp → r, θp → θ} // Simplify
```

Out[200]= 0

Do these energy densities agree with the Cartesian pieces?

In[104]:=
```
FullSimplify[pcrho1 - T001pi2]
```

Out[104]= 0

In[105]:=
```
FullSimplify[pcrho2 - T002pi2]
```

Out[105]= 0

In[106]:=
```
FullSimplify[pcrho3 - T003pi2]
```

Out[106]= 0

In[107]:=
```
FullSimplify[pcrho4 - T004pi2]
```

Out[107]= 0

Yes! Good.

Do the pressures agree? We will find the components of the stress-energy tensor for the wedge in terms of the pressure components px, py, and Txy from the Cartesian case, and then compare the two.

Now, the on-diagonal terms, separated into pieces. The format is (Cartesian T$\mu\nu$ converted from Cartesian to polar components) - (wedge T$\mu\nu$ in polar components), so all of these should be zero if the two things are equal.

```
In[108]:= pcpx1d Cos[θ]^2 + pcpy1d Sin[θ]^2 + 2 pcTxy1 Sin[θ] Cos[θ] - Trr1pi2d // FullSimplify
Out[108]= 0
In[109]:= pcpx2d Cos[θ]^2 + pcpy2d Sin[θ]^2 + 2 pcTxy2 Sin[θ] Cos[θ] - Trr2pi2d // FullSimplify
Out[109]= 0
In[110]:= pcpx3 Cos[θ]^2 + pcpy3 Sin[θ]^2 + 2 pcTxy3 Sin[θ] Cos[θ] - Trr3pi2d // FullSimplify
Out[110]= 0
In[111]:= pcpx4d Cos[θ]^2 + pcpy4d Sin[θ]^2 + 2 pcTxy4d Sin[θ] Cos[θ] - Trr4pi2d // FullSimplify
Out[111]= 0
In[112]:= pcpx1d Sin[θ]^2 + pcpy1d Cos[θ]^2 - 2 pcTxy1 Sin[θ] Cos[θ] - Tpp1pi2d // FullSimplify
Out[112]= 0
In[113]:= pcpx2d Sin[θ]^2 + pcpy2d Cos[θ]^2 - 2 pcTxy2 Sin[θ] Cos[θ] - Tpp2pi2d // FullSimplify
Out[113]= 0
In[114]:= pcpx3 Sin[θ]^2 + pcpy3 Cos[θ]^2 - 2 pcTxy3 Sin[θ] Cos[θ] - Tpp3pi2d // FullSimplify
Out[114]= 0
In[115]:= pcpx4d Sin[θ]^2 + pcpy4d Cos[θ]^2 - 2 pcTxy4d Sin[θ] Cos[θ] - Tpp4pi2d // FullSimplify
Out[115]= 0
```

Off-diagonal terms

```
In[201]:= pcTxy1 (Cos[θ]^2 - Sin[θ]^2) + Sin[θ] Cos[θ] (-pcpx1d + pcpy1d) - Tpr1pi2d // FullSimplify
Out[201]= 0
In[202]:= pcTxy2 (Cos[θ]^2 - Sin[θ]^2) + Sin[θ] Cos[θ] (-pcpx2d + pcpy2d) - Tpr2pi2d // FullSimplify
Out[202]= 0
```

Tpr1 and 2 aren't canceling for some reason? Now fixed!

```
In[203]:= pcTxy3 (Cos[θ]^2 - Sin[θ]^2) + Sin[θ] Cos[θ] (-pcpx3 + pcpy3) - Tpr3pi2d // FullSimplify
Out[203]= 0
In[204]:= pcTxy4d (Cos[θ]^2 - Sin[θ]^2) + Sin[θ] Cos[θ] (-pcpx4d + pcpy4d) - Tpr4pi2d // FullSimplify
Out[204]= 0
```

This final section has to do with the torque problem in the wedge. Point-splitting is along z.

Wedge T00 and pressures, in polar coordinates, for general alpha

```
In[120]:= T001 = -1/2 D[wedgeTbar1, {t, 2}];
```

In[121]:=
```
T002 = -1/2 D[wedgeTbar2, {t, 2}];
```

In[122]:=
```
T003 = -1/2 D[wedgeTbar3, {t, 2}];
```

In[123]:=
```
T004 = -1/2 D[wedgeTbar4, {t, 2}];
```

In[124]:=
```
Tpp1 = 1/(4 r) D[wedgeTbar1, r] + 1/(4 r^2) (D[wedgeTbar1, {θ, 2}] - D[D[wedgeTbar1, θ], θp]);
```

In[125]:=
```
Tpp2 = 1/(4 r) D[wedgeTbar2, r] + 1/(4 r^2) (D[wedgeTbar2, {θ, 2}] - D[D[wedgeTbar2, θ], θp]);
```

In[126]:=
```
Tpp3 = 1/(4 r) D[wedgeTbar3, r] + 1/(4 r^2) (D[wedgeTbar3, {θ, 2}] - D[D[wedgeTbar3, θ], θp]);
```

In[127]:=
```
Tpp4 = 1/(4 r) D[wedgeTbar4, r] + 1/(4 r^2) (D[wedgeTbar4, {θ, 2}] - D[D[wedgeTbar4, θ], θp]);
```

Off-diagonal components

In[206]:=
```
Trp1 = -1/4 (1/r D[D[wedgeTbar1, r], θp] - 1/r D[D[wedgeTbar1, θ], r]) -
    (4 r^2) ^ (-1) D[wedgeTbar1, θ];
```

In[207]:=
```
Trp2 = -1/4 (1/r D[D[wedgeTbar2, r], θp] - 1/r D[D[wedgeTbar2, θ], r]) -
    (4 r^2) ^ (-1) D[wedgeTbar2, θ];
```

In[208]:=
```
Trp3 = -1/4 (1/r D[D[wedgeTbar3, r], θp] - 1/r D[D[wedgeTbar3, θ], r]) -
    (4 r^2) ^ (-1) D[wedgeTbar3, θ];
```

In[209]:=
```
Trp4 = -1/4 (1/r D[D[wedgeTbar4, r], θp] - 1/r D[D[wedgeTbar4, θ], r]) -
    (4 r^2) ^ (-1) D[wedgeTbar4, θ];
```

Calculations for T001 and Tpp1

In[132]:=
```
wedgerho1 = T001 /. {rp → r, θp → θ, t → 0, zp → 0};
```

In[133]:=
```
wedgepp1 = Tpp1 /. {rp → r, θp → θ, t → 0, zp → 0};
```

In[210]:=
```
Frp1 = Trp1 /. {rp → r, θp → θ, t → 0, zp → 0, α → Pi / 2} // Simplify
```

Out[210]=
$$-\frac{\mathrm{Sin}[2\,\theta]}{4\,\pi^2\,\left(2\,r^2+z^2+2\,r^2\,\mathrm{Cos}[2\,\theta]\right)^2}$$

Derivative of rho with respect to alpha

In[135]:=
```
drho1 = D[wedgerho1, α] /. α → Pi / 2;
```

Integral of that derivative over all theta

In[136]:=
```
int1 = Integrate[drho1, {θ, 0, Pi / 2}]
```

Out[136]=
$$-\frac{1}{2\,\pi^2\,z^4}$$

Integral of Frp over all theta

In[211]:=
```
Frp1int = Integrate[Frp1 * r, {θ, 0, Pi / 2}]
```
Out[211]=
$$-\frac{r}{4\pi^2\left(4 r^2 z^2 + z^4\right)}$$

Energy density and pressure at $\theta=\pi/2$

In[138]:=
```
wr1 = FullSimplify[wedgerho1 /. {θ → Pi / 2, α → Pi / 2}]
```
Out[138]=
$$\frac{1}{2\pi^2 z^4}$$

In[139]:=
```
wp1 = FullSimplify[wedgepp1 /. {θ → Pi / 2, α → Pi / 2}]
```
Out[139]= 0

In[140]:=
```
Integrate[wp1, {r, 0, Infinity}]
```
Out[140]= 0

Sum

In[141]:=
```
FullSimplify[int1 + wr1 + wp1]
```
Out[141]= 0

Energy density and pressure at $\theta=0$

In[142]:=
```
FullSimplify[wedgerho1 /. {θ → 0, α → Pi / 2}]
```
Out[142]=
$$\frac{1}{2\pi^2\left(4 r^2 + z^2\right)^2}$$

In[143]:=
```
FullSimplify[wedgepp1 /. {θ → 0, α → Pi / 2}]
```
Out[143]=
$$-\frac{1}{2\pi^2\left(4 r^2 + z^2\right)^2}$$

Calculations for T002 and Tpp2

In[144]:=
```
wedgerho2 = T002 /. {rp → r, θp → θ, t → 0, zp → 0};
```
In[145]:=
```
wedgepp2 = Tpp2 /. {rp → r, θp → θ, t → 0, zp → 0};
```
In[212]:=
```
Frp2 = Trp2 /. {rp → r, θp → θ, t → 0, zp → 0, α → Pi / 2} // Simplify
```
Out[212]=
$$\frac{\mathrm{Sin}[2\theta]}{4\pi^2\left(2 r^2 + z^2 - 2 r^2 \mathrm{Cos}[2\theta]\right)^2}$$

In[213]:=
```
Frp1 - Frp2 // Simplify
```
Out[213]=
$$-\left(\left(6 r^4 + 4 r^2 z^2 + z^4 + 2 r^4 \mathrm{Cos}[4\theta]\right) \mathrm{Sin}[2\theta]\right) \Big/ \left(2\pi^2 \left(2 r^4 + 4 r^2 z^2 + z^4 - 2 r^4 \mathrm{Cos}[4\theta]\right)^2\right)$$

So Frp is not the same for parts 1 and 2. But replace θ by $\pi/2$ - θ; it’s OK!

Derivative of rho with respect to alpha

In[148]:=
```
drho2 = D[wedgerho2, α] /. α → Pi / 2;
```

Integral of that derivative over all theta

In[149]:=
```
int2 = Integrate[drho2, {θ, 0, Pi / 2}]
```
Out[149]=
$$-\frac{1}{2\pi^2\left(4 r^2 + z^2\right)^2}$$

Integral of Frp over all theta

```
In[214]:= Frp2int = Integrate[Frp2 * r, {θ, 0, Pi / 2}]
```

Out[214]= $\frac{r}{4\pi^2\left(4r^2z^2+z^4\right)}$

Note that this is the negative of Frp1int. (Yes, still is.)

Energy density and pressure at $\theta=\pi/2$

```
In[151]:= wr2 = FullSimplify[wedgerho2 /. {θ → Pi / 2, α → Pi / 2}]
```

Out[151]= $\frac{1}{2\pi^2\left(4r^2+z^2\right)^2}$

```
In[152]:= wp2 = FullSimplify[wedgepp2 /. {θ → Pi / 2, α → Pi / 2}]
```

Out[152]= $-\frac{1}{2\pi^2\left(4r^2+z^2\right)^2}$

```
In[153]:= Integrate[wp2, {r, 0, Infinity}]
```

Out[153]= $-\frac{1}{16\pi z^3}$

Sum

```
In[154]:= FullSimplify[int2 + wr2 + wp2]
```

Out[154]= $-\frac{1}{2\pi^2\left(4r^2+z^2\right)^2}$

Energy density and pressure at $\theta=0$

```
In[155]:= FullSimplify[wedgerho2 /. {θ → 0, α → Pi / 2}]
```

Out[155]= $\frac{1}{2\pi^2z^4}$

```
In[156]:= FullSimplify[wedgepp2 /. {θ → 0, α → Pi / 2}]
```

Out[156]= 0

Calculations for T003 and Tpp3

```
In[157]:= wedgerho3 = T003 /. {rp → r, θp → θ, t → 0, zp → 0};
In[158]:= wedgepp3 = Tpp3 /. {rp → r, θp → θ, t → 0, zp → 0};
In[215]:= Frp3 = Trp3 /. {rp → r, θp → θ, t → 0, zp → 0, α → Pi / 2} // Simplify
```

Out[215]= 0

Derivative of rho with respect to alpha

```
In[160]:= drho3 = D[wedgerho3, α] /. α → Pi / 2;
```

Integral of that derivative over all theta

```
In[161]:= int3 = Integrate[drho3, {θ, 0, Pi / 2}] // FullSimplify
```

Out[161]= $\left(-8r^4z+2r^2z^3+z^5+4r^2\left(r^2+z^2\right)\sqrt{4r^2+z^2}\,\text{ArcCosh}\left[1+\frac{z^2}{2r^2}\right]\right)\Big/\left(2\pi^2\left(4r^2z+z^3\right)^3\right)$

Integral of Frp over all theta

In[216]:= `Frp3int = Integrate[Frp3 * r, {θ, 0, Pi / 2}]`

Out[216]= 0

Energy density and pressure at $\theta=\pi/2$

In[163]:= `wr3 = FullSimplify[wedgerho3 /. {θ → Pi / 2, α → Pi / 2}]`

Out[163]= $-\frac{1}{2\,\pi^2\,(4\,r^2+z^2)^2}$

In[164]:= `wp3 = FullSimplify[wedgepp3 /. {θ → Pi / 2, α → Pi / 2}]`

Out[164]= 0

In[165]:= `Integrate[wp3, {r, 0, Infinity}]`

Out[165]= 0

Sum

In[166]:= `FullSimplify[int3 + wr3 + wp3]`

Out[166]= $-\left(r^2\left(4\,r^2\,z+z^3-2\,(r^2+z^2)\,\sqrt{4\,r^2+z^2}\,\text{ArcCosh}\left[1+\frac{z^2}{2\,r^2}\right]\right)\right)\Big/\left(\pi^2\,(4\,r^2\,z+z^3)^3\right)$

Energy density and pressure at $\theta=0$

In[167]:= `FullSimplify[wedgerho3 /. {θ → 0, α → Pi / 2}]`

Out[167]= $-\frac{1}{2\,\pi^2\,(4\,r^2+z^2)^2}$

In[168]:= `FullSimplify[wedgepp3 /. {θ → 0, α → Pi / 2}]`

Out[168]= 0

Calculations for T004 and Tpp4

In[169]:= `wedgerho4 = T004 /. {rp → r, θp → θ, t → 0, zp → 0};`

In[170]:= `wedgepp4 = Tpp4 /. {rp → r, θp → θ, t → 0, zp → 0};`

In[217]:= `Frp4 = Trp4 /. {rp → r, θp → θ, t → 0, zp → 0, α → Pi / 2} // Simplify`

Out[217]= 0

In[172]:= `drho4 = D[wedgerho4, α] /. α → Pi / 2;`

In[173]:= `int4 = Integrate[drho4, {θ, 0, Pi / 2}] // FullSimplify`

Out[173]= $\left(24\,r^4\,z+10\,r^2\,z^3+z^5-4\,r^2\,(3\,r^2+z^2)\,\sqrt{4\,r^2+z^2}\,\text{ArcCosh}\left[1+\frac{z^2}{2\,r^2}\right]\right)\Big/\left(2\,\pi^2\,z^5\,(4\,r^2+z^2)^2\right)$

Integral of Frp over all theta

In[218]:= `Frp4int = Integrate[Frp4 * r, {θ, 0, Pi / 2}]`

Out[218]= 0

Energy density and pressure at $\theta=\pi/2$

In[175]:= `wr4 = FullSimplify[wedgerho4 /. {θ → Pi / 2, α → Pi / 2}]`

Out[175]= $-\frac{1}{2\pi^2 z^4}$

In[176]:= `wp4 = FullSimplify[wedgepp4 /. {θ → Pi / 2, α → Pi / 2}]`

Out[176]= $\frac{1}{2\pi^2 z^4}$

In[177]:= `Integrate[wp4, {r, 0, Infinity}]`

Out[177]= $\frac{\infty}{\text{Sign}[z]^4}$

Sum

In[178]:= `FullSimplify[int4 + wr4 + wp4]`

Out[178]= $\left(24 r^4 z + 10 r^2 z^3 + z^5 - 4 r^2 (3 r^2 + z^2) \sqrt{4 r^2 + z^2}\ \text{ArcCosh}\left[1 + \frac{z^2}{2 r^2}\right]\right) \Big/ \left(2 \pi^2 z^5 (4 r^2 + z^2)^2\right)$

Energy density and pressure at θ=0

In[179]:= `FullSimplify[wedgerho4 /. {θ → 0, α → Pi / 2}]`

Out[179]= $-\frac{1}{2\pi^2 z^4}$

In[180]:= `FullSimplify[wedgepp4 /. {θ → 0, α → Pi / 2}]`

Out[180]= $\frac{1}{2\pi^2 z^4}$

Sum of all parts from 1 + 2 + 3 + 4

In[181]:= `FullSimplify[int1 + int2 + int3 + int4 + wr1 + wr2 + wr3 + wr4 + wp1 + wp2 + wp3 + wp4]`

Out[181]= $\frac{1}{\pi^2 z^5 (4 r^2 + z^2)^3} 4 r^2 \left(12 r^4 z + 7 r^2 z^3 + z^5 - 3 r^2 (2 r^2 + z^2) \sqrt{4 r^2 + z^2}\ \text{ArcCosh}\left[1 + \frac{z^2}{2 r^2}\right]\right)$

Sum of all parts from 1 + 2

In[182]:= `FullSimplify[int1 + int2 + wr1 + wr2 + wp1 + wp2]`

Out[182]= $-\frac{1}{2\pi^2 (4 r^2 + z^2)^2}$

Sum of all parts from 3 + 4

In[183]:= `FullSimplify[int3 + int4 + wr3 + wr4 + wp3 + wp4]`

Out[183]= $\left(96 r^6 z + 56 r^4 z^3 + 12 r^2 z^5 + z^7 - 24 r^4 (2 r^2 + z^2) \sqrt{4 r^2 + z^2}\ \text{ArcCosh}\left[1 + \frac{z^2}{2 r^2}\right]\right) \Big/ \left(2 \pi^2 z^5 (4 r^2 + z^2)^3\right)$