H. C. Hodges
Week 13

h-hodges
8.1.23

http://www.txdigital.com/math311/8-1-23.dvi
http://www.txdigital.com/math311/8-1-23.pdf

Statement of Problem:

Find the eigenvalues and eigenvectors of the matrix. Remark upon any case where an eigenbasis of real eigenvectors does not exist.

$$A = \begin{pmatrix} 0 & 3 & 3 \\ -1 & 8 & 6 \\ 2 & -14 & -10 \end{pmatrix}$$

Begin by calculating $\det(A - \lambda)$, where λ is actually the scalar λ times the identity matrix:

$$\begin{aligned} \det(A-\lambda) &= \begin{vmatrix} -\lambda & 3 & 3 \\ -1 & 8-\lambda & 6 \\ 2 & -14 & -10-\lambda \end{vmatrix} \\ &= -\lambda\Big[(8-\lambda)(-10-\lambda)+(6)(14)\Big] + 3\Big[12-(10+\lambda)\Big] + 3\Big[14-2(8-\lambda)\Big] \\ &= -\lambda(\lambda+1)(\lambda+1) \end{aligned}$$

Therefore, $\lambda_1 = 0$ and $\lambda_2 = -1$. These are the two eigenvalues for the matrix. Placing these back into the matrix and solving for each variable yields the eigenvectors.

$$A - \lambda_1 = \begin{pmatrix} 0 & 3 & 3 \\ -1 & 8 & 6 \\ 2 & -14 & -10 \end{pmatrix}$$

$$\operatorname{rref}(A - \lambda_1) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore $x = -2z$, $y = -z$, and z is arbitrary. This indicates the eigenvector $\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$.

$$A - \lambda_2 = \begin{pmatrix} 1 & 3 & 3 \\ -1 & 9 & 6 \\ 2 & -14 & -9 \end{pmatrix}$$

$$\operatorname{rref}(A - \lambda_2) = \begin{pmatrix} 1 & 0 & 3/4 \\ 0 & 1 & 3/4 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore $x = -3z/4$, $y = -3z/4$, and z is arbitrary. This indicates the eigenvector $\begin{pmatrix} -3/4 \\ -3/4 \\ 1 \end{pmatrix}$.

Note that no eigenbasis exists since the eigenvectors do not span the entire vector space $\mathcal{D}$ (which in this case is $\mathbf{R}^3$).