

Patrick Barrett (p-barrett@calclab.math.tamu.edu)

Problem 5.5.5

http://www.aboynamedpat.com/math/assignment5_5_5.pdf

Problem:

Find the formulas for $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial x}$, $\frac{\partial w}{\partial x}$ in parabolic cylindrical coordinates,

$$x = 2uv, \quad y = u^2 - v^2, \quad z = w.$$

(The answers will be functions of (u, v, w)).

We'll set up the problem as a system of equations, take the partials with respect to x and solve for the unknown formulas.

$$\begin{aligned} \frac{\partial}{\partial x}(x) = \frac{\partial}{\partial x}(2uv) &\rightarrow 1 = 2\frac{\partial u}{\partial x}v + 2\frac{\partial v}{\partial x}u \\ \frac{\partial}{\partial x}(y) = \frac{\partial}{\partial x}(u^2) - \frac{\partial}{\partial x}(v^2) &\rightarrow 0 = 2u\frac{\partial u}{\partial x} - 2v\frac{\partial v}{\partial x} \\ \frac{\partial}{\partial x}(z) = \frac{\partial}{\partial x}w &\rightarrow 0 = \frac{\partial w}{\partial x} \end{aligned}$$

We can then solve the equations.

$$\begin{pmatrix} 2v & 2u & 0 & 1 \\ 2u & 2v & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & \frac{-v}{2(u^2-v^2)} \\ 0 & 1 & 0 & \frac{u}{2(u^2-v^2)} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Thus $\frac{\partial u}{\partial x} = \frac{-v}{2(u^2-v^2)}$, $\frac{\partial v}{\partial x} = \frac{u}{2(u^2-v^2)}$, $\frac{\partial w}{\partial x} = 0$