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Problem 6.3.6
http://www.aboynamedpat.com/math/assignment6_3_6.pdf

Problem:

Find $\hat{T}$, $\hat{N}$, $\hat{B}$, κ, τ for the curve $\vec{f}(t) = \begin{pmatrix} a \\ bt \\ t^2 \end{pmatrix}$.

$\hat{T}$ =the unit tangent vector; so we'll take the derivative with respect to t and divide by the length of the derivative.

$$\vec{f}'(t) = \begin{pmatrix} 0 \\ b \\ 2t \end{pmatrix}, \qquad \left\|\vec{f}'(t)\right\| = \sqrt{0 + b^2 + (2t)^2} = \sqrt{b^2 + 4t^2}$$

$$\hat{T} = \frac{1}{\sqrt{b^2 + 4t^2}} \begin{pmatrix} 0 \\ b \\ 2t \end{pmatrix}.$$

$\hat{N}$ =the unit normal vector; which is the derivative of $\hat{T}$ wrt t over the length of that.

$$\hat{T}'(t) = \begin{pmatrix} 0 \\ \frac{-4bt}{(4t^2+b^2)^{\frac{3}{2}}} \\ \frac{2b^2}{(4t^2+b^2)^{\frac{3}{2}}} \end{pmatrix}, \quad \left\|\hat{T}'(t)\right\| = \frac{2b}{4t^2 + b^2}$$

$$\hat{N} = \left(0, \frac{-2t}{\sqrt{4t^2 + b^2}}, \frac{b}{\sqrt{4t^2 + b^2}}\right)$$

$\hat{B}$ =the unit binormal vector which is found by crossing $\hat{T}$ with $\hat{N}$.

$$\hat{T} \times \hat{N} = \begin{bmatrix} i & j & k \\ 0 & \frac{b}{\sqrt{b^2+4t^2}} & \frac{2t}{\sqrt{b^2+4t^2}} \\ 0 & \frac{-2t}{\sqrt{4t^2+b^2}} & \frac{b}{\sqrt{4t^2+b^2}} \end{bmatrix} \to (1, 0, 0) = \hat{B}$$

κ =curvature which is equal to $\left\|d\hat{T}/ds\right\| = \left\|\hat{T}'(t)\,\frac{dt}{ds}\right\| = \left\|\frac{\hat{T}'(t)}{\vec{f}'(t)}\right\|$

$$\left\|\hat{T}'(t)\right\| = \frac{2b}{4t^2 + b^2}, \quad \left\|\vec{f}'(t)\right\| = \sqrt{b^2 + 4t^2}$$

$$\left\|\frac{\hat{T}'(t)}{\vec{f}'(t)}\right\| = \frac{2b\sqrt{b^2 + 4t^2}}{4t^2 + b^2} = \frac{2b}{(b^2 + 4t^2)^{\frac{3}{2}}} = \kappa$$

τ =the torsion which is equal to $\left\|\hat{B}'(t)/\vec{f}'(t)\right\|$

$$\|\hat{B}'(t)\| = 0 \text{ and therefore it can be seen that } \tau \text{ is } 0.$$