James Macfarlane (j-macfarlane)
Assignment 6, Problem 4.3.1
http://calclab.tamu.edu/~j-macfarlane/math311/4_3_1.pdf

Write out a proof of Proposition 1.

Proposition 1:

1. If a set S is independent, then any subset of S is also independent.
2. If a set S spans V, then any superset of S also spans V.

Solution

1. If vectors $\{\vec{v}_1, \ldots, \vec{v}_n\}$ make up a set S and S is independent, then there is no nontrivial conbination of these vectors that cancel. This means that when

$$r_1\vec{v}_1 + \ldots + r_n\vec{v}_n = 0$$

then $r_1 = 0$, $\ldots$, $r_n = 0$. A subset of S would not introduce any new vectors that would allow the cancellation of any combination of vectors from the original set and therefore also not allow the cancellation of any combination of vectors from the subset. This means that the subset of S is also independent.

2. The superset of S still contains the vectors from the set S. This means that any combination of vectors from the set S can also be written from the same vectors in the superset of S that are also in the set S.