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Assignment 8, Problem 5.2.4

http://calclab.tamu.edu/~j-macfarlane/math311/5_2_4.pdf

The matrix $A = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 1 & 1 \end{pmatrix}$ defines a linear function $L : \mathbb{R}^3 \rightarrow \mathbb{R}^2$.

- (a) Find the kernel of L .
 - (b) Is L “onto” (surjective)? Explain.
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Solution

- (a) The kernel of L can be found by reducing the augmented matrix of A . This gives us $\begin{pmatrix} 1 & 0 & \frac{-1}{5} & 0 \\ 0 & 1 & \frac{4}{5} & 0 \end{pmatrix}$.

From this we can see that the vectors that make up the kernel have the form $t \begin{pmatrix} \frac{1}{5} \\ \frac{-4}{5} \\ 1 \end{pmatrix}$ where t is an arbitrary real number.

- (b) The range of L is the span of the columns of A . The span of the columns of A is of dimension 2 since no column is a multiple of another. This means that the range of L spans \mathbb{R}^2 and therefore L is surjective.