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Assignment 11, Problem 7.4.1
http://calclab.tamu.edu/~j-macfarlane/math311/7_4_1.pdf

A tin can has height 15 units and raduis 4 units. Write down an integral formula for the total mass of the material of the can, if the density (mass per unit area) is not uniform. HINTS: First choose a suitable coordinate system: A natural choice is to put the center of the bottom of the can at the origin, and the axis of the can along the positive z axis. Now write down an integral in cylindrical coordinates for the mass of each of the three pieces of metal making up the can, in terms of unspecified density functions.

Solution

If ρ is a function in terms of x, y, and z, then it is easy enough to put it in terms of the coordinate system suggested in the problem.

$$\rho(x, y, z) \to \rho(rcos\theta, rsin\theta, z)$$

The integral we are trying to find is

$$\int\int_S \rho(x, y, z)\, dS.$$

First we need to split this integral into three smaller integrals. This is one for each "piece" of the can.

$$\int\int_S \rho(x, y, z)\, dS = \int\int_{S_{side}} \rho(x, y, z)\, dS + \int\int_{S_{top}} \rho(x, y, z)\, dS + \int\int_{S_{bottom}} \rho(x, y, z)\, dS$$

Using the dimensions in the problem, this becomes

$$\int_0^{2\pi}\int_0^{15} 4\rho(4cos\theta, 4sin\theta, z)\, dzd\theta + \int_0^{2\pi}\int_0^{4} \rho(rcos\theta, rsin\theta, 15)r\, drd\theta + \int_0^{2\pi}\int_0^{4} \rho(rcos\theta, rsin\theta, 0)r\, drd\theta.$$