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Assignment #4 | Problem 3.5.7 | 3/1/2004
URL: http://people.tamu.edu/~jrm7519/3.5.7.pdf

**3.5.7** The temperature in a square plate is given by the equation $T = x + 3y$. An ant crawled on the plate along the parabolic path $y = x^2$ until it reached the point $(2,4)$, whereupon it found itself fried. What is wrong with the following argument?

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial x}$$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial x}$$

We can cancel the two equal terms, concluding that

$$0 = \frac{\partial T}{\partial y}\frac{\partial y}{\partial x}$$

Substituting the given functions and numbers, we learn that

$$12 = 0$$

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To accurately evaluate this problem either (1) a parameterization of $x = t$ needs to be used for the chain rule or (2) a simple substitution of $y = x^2$.

(1) Evaluation by chain rule:

Let $x = t$; therefore, $y = t^2$ and $T = x + 3y$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{\partial T}{\partial t} = (1)(1) + (3)(2t)$$

$$\frac{\partial T}{\partial t} = 1 + 6t$$

Substituting $x$ back into the equation gives:

$$\frac{\partial T}{\partial x} = 1 + 6x$$

(2) Substitution method:

$y = x^2$ and $T = x + 3y$

$$T = x + 3(x^2)$$

$$\frac{\partial T}{\partial x} = (1) + (3)(2x)$$

$$\frac{\partial T}{\partial x} = 1 + 6x$$

The problem with the initial argument in the problem was in the setup of the chain rule. The problem set the partial derivate of 'T' with respect to 'x', $\frac{\partial T}{\partial x}$, equal to the derivate of 'T' with respect to 'x', $\frac{dT}{dx}$. These to terms are not equal; therefore, in the 3rd step the two $\frac{\partial T}{\partial x}$ terms cannot be canceled out.