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Assignment #7	Problem 4.5.7	3/8/2004
URL: http://people.tamu.edu/~jrm7519/4.5.7.pdf		

4.5.7 A linear function $G:\Re^2 \to \Re^2$ has the matrix $B = \begin{pmatrix} -1 & 4 \\ 2 & 1 \end{pmatrix}$ with respect to the natural basis. Find the matrix of G with respect to the basis $\{\vec{b}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$. (Use the new basis for both domain and codomain.)

We are given matrix $B = \begin{pmatrix} -1 & 4 \\ 2 & 1 \end{pmatrix}$ with respect to the natural basis and need to find matrix M with respect to the new basis.

$$\begin{array}{ccc} g & \xleftarrow{C} & g' \\ \downarrow B & & \downarrow M \\ Lg & \xleftarrow{C} & (Lg)' \end{array}$$

From this figure outlining the placement of the matrices we can find that M is equal to:

$$M = C^{-1}BC$$

We are giving the matrix B and can define the matrix $C = \left(\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$ and calculate the inverse.

$$C = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$C^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Substituting these matrices in the equation above and solve for M.

$$M = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$M = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -5 & 3 \\ 1 & 3 \end{pmatrix}$$

$$M = \begin{pmatrix} -3 & 0 \\ -2 & 3 \end{pmatrix}$$