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Assignment #8	Problem 6.1.6	3/29/2004
URL: http://people.tamu.edu/~jrm7519/6.1.6.pdf		

6.1.6 Tell whether each of these formulas defines an inner product on the subspace of bounded functions in the space $C(0,\infty)$. (Explain your answers briefly.) The scalars in this problem are real, not complex. The boundedness condition should help you decide whether the integrals converge.

(a) $\langle f, g\rangle = \int_0^\infty f(t)g(t)\frac{1}{t^2+1}dt$

(b) $\langle f, g\rangle = \int_0^\infty f(t)g(t)t^2+1dt$

(c) $\langle f, g\rangle = \int_0^2 f(t)g(t)dt$

We can note that all inner products must converge and meet the following conditions:

RI1. $\vec{x}\cdot\vec{x} > 0$ – except that $\vec{0}\cdot\vec{0} = 0$

RI2. $\vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{x}$

RI3. $(r\vec{x}+\vec{y})\cdot\vec{z} = r(\vec{x}\cdot\vec{z})+\vec{y}\cdot\vec{z}$

We can see that all three functions meet RI1, RI2, and RI3, therefore in order for these formulas to be inner products they must converge.

(a) $\langle f, g\rangle = \int_0^\infty f(t)g(t)\frac{1}{t^2+1}dt$

We can note that this interval goes from zero to infinity and f and g are bounded functions therefore we can say

$$\int_0^\infty f(t)g(t)\frac{1}{t^2+1}dt < \int_0^\infty \frac{1}{t^2+1}dt$$

Where $\int_0^\infty \frac{1}{t^2+1}dt = \tan^{-1}(t)$ which converges; therefore the left integral converges and (a) is an inner product.

(b) $\langle f, g\rangle = \int_0^\infty f(t)g(t)t^2+1dt$

Similarly to part (a)

$$\int_0^\infty f(t)g(t)t^2+1dt < \int_0^\infty t^2+1dt$$

However, in this case $\int_0^{\infty} t^2 + 1dt$ does not converge; therefore, (b) is not an inner product.

(c) $\langle f, g \rangle = \int_0^2 f(t)g(t)dt$

Since (c) is bounded to a region of $C(0,2)$ we can see that this function will converge; therefore (c) is an inner product.