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Assignment #6 4.4.9
http://people.tamu.edu/~rsm4368/math311/4_4_9.pdf

4.4.9) Consider the two bases for P_2:

$$\varepsilon \equiv \{1, t, t^2\} \equiv \{e_1, e_2, e_3\}$$

$$\beta \equiv \{(1-t), (1+t), 2t^2\} \equiv \{b_1, b_2, b_3\}$$

a.) Find the change-of-basis matrix that takes coordinates relative to β into coordinates relative to ε.

$$b_1 = e_1 - e_2$$
$$b_2 = e_1 + e_2$$
$$b_3 = 2e_3$$

By definition, you can use these bases to describe a certain function f(t):

$$f(t) = a_1e_1 + a_2e_2 + a_3e_3 = c_1b_1 + c_2b_2 + c_3b_3$$
$$a_1e_1 + a_2e_2 + a_3e_3 = c_1(e_1 - e_2) + c_2(e_1 + e_2) + c_3(2e_3)$$
$$= (c_1 + c_2)e_1 + (c_1 - c_2)e_2 + 2c_3e_3$$
$$a_1 = c_1 + c_2$$
$$a_2 = -c_1 + c_2$$
$$a_3 = 2c_3$$

Now, you can put these equations into matrix form:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

b.) Find the change-of-basis matrix that takes coordinates relative to ε into coordinates relative to β.

By definition, the answer to this is M^{-1}.

$$M^{-1} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right)$$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} 1/2 & -1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$