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Assignment #7 5.1.10
http://people.tamu.edu/~rsm4368/math311/5_1_10.pdf

5.1.10) Let *S* be the set of all twice-differentiable functions *y* on (-∞, ∞) such that either $y'' + 4y = 0$ or $y'' - 4y = 0$. Show that *S* is *not* a subspace of $C^{(2)}$(-∞, ∞).

Let $y_1, y_2 \in S$, such that y_1 is a solution to $y'' + 4y = 0$ and y_2 is a solution to $y'' - 4y = 0$.

Now, we must prove that a linear combination of the two functions, y_1 and y_2, is not an element of *S*.

Let $v = ry_1 + sy_2$.

$$v'' + 4v = (ry_1 + sy_2)'' + 4(ry_1 + sy_2) = ry_1'' + sy_2'' + 4ry_1 + 4sy_2 = r(y_1'' + 4y_1) + s(y_2'' + 4y_2)$$
$$v'' + 4v = 0 + s(y_2'' + 4y_2) \neq 0$$

And:

$$v'' - 4v = (ry_1 + sy_2)'' - 4(ry_1 + sy_2) = ry_1'' + sy_2'' - 4ry_1 - 4sy_2 = r(y_1'' - 4y_1) + s(y_2'' - 4y_2)$$
$$v'' - 4v = r(y_1'' - 4y_1) + 0 \neq 0$$

Therefore, *v*, which is an element of C^2, is not a solution to either of the differential equations, showing that it is not an element of *S*.