Scott McClelland r-mcclelland
Assignment #8 5.2.8
http://people.tamu.edu/~rsm4368/math311/5_2_8.pdf

5.2.8) Find the range and the kernel of each of these operators.

a.) $L : P_2 \rightarrow P_3$, $L(p)(t) = tp(t)$

Let $p(t) = at^2 + bt + c$

$L(p)(t) = at^3 + bt^2 + ct$

Therefore, L spans $\{t^3, t^2, t\}$, or:

ran L = span $\{t^3, t^2, t\}$

Now, we set $L(p)(t) = 0$, and solve for all a, b, and c. We see that:

a = b = c = 0 or:

ker $L = \vec{0}$

b.) $L : P_2 \rightarrow P_2$, $L(p)(t) = tp''(t)$

Let $p(t) = at^2 + bt + c$

$L(p)(t) = t(at^2 + bt + c)''$

$L(p)(t) = 2at$

ran L = span $\{t\}$

Set $L(p)(t) = 0$; which means that a = 0, b and c are arbitrary.

Therefore:

$$\ker L = \begin{pmatrix} 0 \\ b \\ c \end{pmatrix} = \text{span } \{t, 1\}$$