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Assignment #9 5.5.15
http://people.tamu.edu/~rsm4368/math311/5_5_15.pdf

5.5.15) Find the normal vectors to the coordinate surfaces for spherical coordinates in $\mathbf{R}^3$. Use $r = 2$, $\theta = \pi/4$, $\varphi = 0$.

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

We first need to find the Jacobian matrix for these equations, which is:

$$J = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{2}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-2}{\sqrt{2}} & 0 \end{pmatrix}$$

This will give us the tangent vectors to the coordinate surfaces and we want the normal vectors, so we must invert this matrix.

$$J^{-1} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} & \frac{\partial r}{\partial z} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} & \frac{\partial \theta}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{pmatrix} = \begin{pmatrix} \nabla r \\ \nabla \theta \\ \nabla \phi \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 1 & -2 & 0 & 0 & 0 & \sqrt{2} \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 1 & 0 & \frac{\sqrt{2}}{2} & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 & 0 & \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$$

$$J^{-1} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} & \frac{\partial r}{\partial z} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} & \frac{\partial \theta}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{pmatrix} = \begin{pmatrix} \nabla r \\ \nabla \theta \\ \nabla \phi \end{pmatrix} = \begin{pmatrix} (\frac{\sqrt{2}}{2},0,0) \\ (\frac{\sqrt{2}}{4},0,-\frac{\sqrt{2}}{2}) \\ (0,\frac{\sqrt{2}}{2},0) \end{pmatrix}$$