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Assignment 7 Problem 5.1.21 Revised
http://people.tamu.edu/˜mzr8136/mathhomework.html

5.1.21

Prove that each of these subsets of $R^{\infty}$is a subspace of R^{∞}.

U_2 =the set of sequences that converge: $\lim_{n\to\infty} x_n \equiv L$ exists (and is not ∞). Hint: use the theorem from calculus that states:
$\lim_{n\to\infty} x_n + y_n \equiv \lim_{n\to\infty} x_n + \lim_{n\to\infty} y_n$
if the two limits on the right exist.

Solution:

Since we know that $U_2$is a subset of R^{∞}, to prove that it is a subspace we need to prove that it is closed under addition and scalar multiplication. The set of sequences that converge are closed under multiplication. Multiplying by a scalar constant still allows the sequence to converge, albeit to a different number.
$\lim_{n\to\infty} x_n \equiv M$,
$\lim_{n\to\infty} y_n \equiv N$,
$\lim_{n\to\infty} r \cdot x_n = r \cdot M$.
Therefore this set is closed under multiplication.

Also, the sequences that converge satisfy a closed additive property as well. Since the two sequences converge, the limits are finite numbers which can be added.
So $\lim_{n\to\infty} x_n + y_n = \lim_{n\to\infty} x_n + \lim_{n\to\infty} y_n = M + N$, this proves that the set is also closed under addition and is therefore a subspace of R^{∞}.