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Assignment 7
5.1.8 - **REVISED**
http://people.tamu.edu/~kmt4088/ch5sec1prob8.pdf

Show that the solution space of $x + 2y^2 = 0$ is *not* a subspace of $\mathbf{R}^2$.

First, the solution space is not empty, because $(x, y) = (0, 0)$ satisfies the condition, as $0 + 2(0) = 0$. So, the question becomes whether the solution space is closed to addition and multiplication. Checking either of these is sufficient.

Consider $(x_1 + x_2, y_1 + y_2)$:

$(x_1 + x_2) + 2(y_1 + y_2)^2 = x_1 + x_2 + 2(y_1^2 + 2y_1y_2 + y_2^2)$.

whereas for (x_1, y_1) and (x_2, y_2)

$x_1 + 2y_1^2 = 0$ and $x_2 + 2y_2^2 = 0$. However, this does not prove that $4y_1y_2 = 0$, and so the right hand side of the above equation does not necessarily equal zero. A numerical example is $(-8, 2)$ and $(-18, 3)$.

$$-8 + 2 \cdot 2^2 = 0 \text{ and } -18 + 2 \cdot 3^2 = 0, \text{ but } -26 + 2(4 + 12 + 9) = 24 \neq 0$$

This means that the space is not closed to addition, and therefore the solution space of $x + 2y^2 = 0$ is not a subspace of $\mathbf{R}^2$.