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Assignment 4, Problem 3.3.9

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http://calclab.math.tamu.edu/˜r-waniska/339.pdf

Problem 3.3.9

In the remaining exercises, (a) identify n and p in the characterization

$$f : R^n \hookrightarrow R^p;$$

(b) state the largest natural domain D (a subset of R^n) of the function f; and (c) find all points (in D or on the boundary of D) where the function fails to be continuous.

$$f(x, y, z) = \begin{pmatrix} \frac{1}{x-y} \\ \frac{x^2-z^2}{x-z} \\ \frac{1}{\sqrt{y-z}} \end{pmatrix}$$

Solution:

(a) Since f is a function of three variables, $n = 3$, and since the output of f is three variable, $p = 3$. This can be wrttien

$$f : R^3 \hookrightarrow R^3.$$

(b) State the largest natural domain. To find the largest domain, we must look to see where the function is undefined.

$\frac{1}{x-y}$ is not defined along the line $x = y$, since the denominator of zero would cause the function to be undefined.

$\frac{x^2-z^2}{x-z}$ simplifies to $x + z$ after factoring but is still not defined along the line $x = z$. (This is a removable discontinuity and would be defined if the function were defined at its limit.)

In $\frac{1}{\sqrt{y-z}}$, the denominator cannot be equal to zero, so $y \neq z$. Also, since we are not allowing copmlex numbers for the value of the square root, the radicand must be nonnegative. Thus, $y \geq z$. These requirements can be combined to say $y > z$ must be true for the function to be defined.

So the domain of f is all triples of real numbers where $y > z$, $x \neq y$, and $x \neq z$.

(c) Find all points where the function fails to be continuous.

$\frac{1}{x-y}$ is continuous everywhere except where $x = y$.

$\frac{x^2-z^2}{x-z}$ is continuous everywhere except $x = z$. (It would be continuous everywhere if it were defined as its limit.)

$\frac{1}{\sqrt{y-z}}$ is not continuous along the plain $y = z$, but continuous everywhere else it is defined.