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Assignment 6, Problem 4.4.13

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http://calclab.math.tamu.edu/˜r-waniska/44D.pdf

Problem 4.4.13:

Find the matrix (with respect to standard bases) of the linear operator $L : P_1 \to P_2$ defined by

$$L(p)(t) = \int_0^t p(\tau)d\tau + p'(t).$$

Since L is defined as going from P_1 to P_2, we need to find the values of $L(p)(t)$ at $p(t) = 1$ and $p(t) = t$.

$$L(t) = \int_0^t \tau d\tau + \frac{d}{dt}t = \frac{1}{2}\tau^2 \mid_{\tau=0}^{\tau=t} +1 = \frac{1}{2}t^2 + 1$$

$$L(1) = \int_0^t d\tau + \frac{d}{dt}1 = \tau \mid_{\tau=0}^{\tau=t} +0 = t$$

We then put these values into a matrix. Since the input of the linear operator is P_1, there are two columns ($\{L(t),\ L(1)\}$) for the input, there are two columns. Since the output of the linear operator is P_2, there are three rows ($\{t^2,\ t,\ 1\}$) .

$$\begin{matrix} & L(t) & L(1) \\ t^2 \to \\ t \to \\ 1 \to \end{matrix} \begin{pmatrix} .5 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$