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Assignment 11, Problem 7.2.7

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http://calclab.math.tamu.edu/˜r-waniska/272.pdf

Problem 7.2.7

Show how the deformation of a deck of cards into a nonrectangular parallelepiped can be described by a 3-dimensional analogue of the shear transformation treated in Excersises 3.2.23 and 7.2.6.

The 2-D shear is given by the matrix $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + ay \\ y \end{pmatrix}$$

When $y = 0$, there is no shear, but as y increases, the x value is sheared more and more to the right.

To find a 3-D analouge, we need to note the essential qualities of the 2-D shear matrix.

1) In one dimension, the coordinate remains unchanged.

2) In the sheared dimension, the new coordinate is equal to the old coordinate plus a constant. This constant increases as the unchanging coordinate increases.

In the 3-D analouge, we need to shear two of the dimensions and leave the third alone. Note that the two sheared dimensions are not necessarily sheared at the same rate.

$$\begin{aligned} x' &= x + az \\ y' &= y + bz \\ z' &= z \end{aligned}$$

This can then be put into a matrix.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

In the 3-D analouge, the z coordinate remains constant, while the x and y coordinates are shifted a distance proportional to it's z value. This shearing describes the deformation of a deck of cards.