Raymond Waniska (r-waniska@calclab.math.tamu.edu)
Assignment 12, Problem 7.6.14
April 20, 2004
http://calclab.tamu.edu/~r-waniska/76E.pdf

Problem 7.6.14: Do example 3 of Sec. 7.4 by the parametric method.
Example 3: Evaluate $\int\int_S \vec{B} \cdot d\vec{S}$ when $\vec{B}(\vec{r}) = y\hat{i}$ and S is the triangle with corners at (0,0,1), (1,0,0), (1,1,0).

First, we will use the method described on page 383.

$$\int\int_S \vec{B} \cdot d\vec{S} = \int\int_S [B_x dy\, dz + B_y dz\, dx + B_z dx\, dy] = \int\int_S (y\, dy\, dz)$$

To integrate this, we need to project the triangle onto the y-z plane to find an equation for the surface. The points then become (0,0,1), (0,0,0), (0,1,0). This area is bounded above by $z = 1 - y$, below by $z = 0$, where $0 \leq y \leq 1$. The integral then bcomes:

$$\int_0^1 dy \int_0^{1-y} dz\,(y) = \int_0^1 y\,[z]_{z=0}^{z=1-y}\, dy = \int_0^1 (y - y^2)\, dy = \left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{y=0}^{y=1} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$