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Assignment 4 Problem 4.2.1 March 1, 2004
http://calclab.math.tamu.edu/~b-young/4_2_1.pdf

Problem Statement:

$x = \cosh u \cos v$ and $y = \sinh u \sin v$

define elliptical coordinates in the x-y plane.

a) Find the tangent vectors to the coordinate lines u = constant and v = constant.
b) Verify that these tangent vectors are orthogonal to each other.
c) Sketch two coordinate lines of each type, and sketch the two tangent vectors at the resulting intersection points.

A) The tangent vectors with u and v constant are the partial derivatives of x and y with respect to u and v. That is:

$$T_u = \begin{pmatrix} \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} -\cosh u \sin v \\ \sinh u \cos v \end{pmatrix}$$

$$T_v = \begin{pmatrix} \frac{\partial x}{\partial u} \\ \frac{\partial y}{\partial u} \end{pmatrix} = \begin{pmatrix} \sinh u \cos v \\ \cosh u \sin v \end{pmatrix}$$

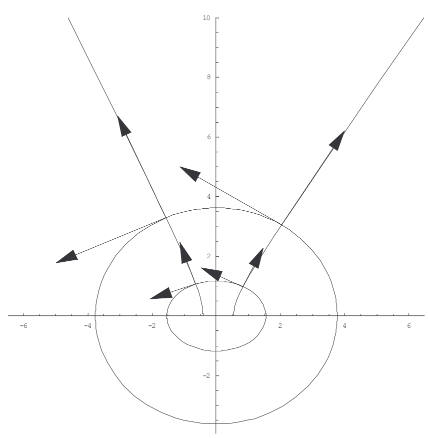
Where T_u is the tangent vector where u is constant and T_v is the tangent vector where v is constant.

B) To verify that the tangent vectors are orthogonal, the fact that perpendicular slopes multiply to equal negative one will be employed. That is:

$$\frac{\Delta y_{Tu}}{\Delta x_{Tu}} \frac{\Delta y_{Tv}}{\Delta x_{Tv}} = \frac{\sinh u \cos v}{-\cosh u \sin v} \frac{\cosh u \sin v}{\sinh u \cos v} = -1$$

Therefore the tangent vectors are orthogonal.

C) The following plot has the level curves *u=1, u=2, v=1, and v=2*.



From the plot it is easy to see that the tangent vectors are in fact orthogonal.