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Assignment 7 Problem 4.5.11
http://calclab.math.tamu.edu/~b-young/4_5_11.pdf

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Problem Statement: How would the change-of-basis matrix change if you decided to write the vectors of both bases in reverse order?

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Solution: First consider the matrix of the change of basis in $\mathcal{R}^n$ as:

$$M = \begin{pmatrix} m_{11} & \dots & m_{1(n-1)} & m_{1n} \\ m_{21} & \dots & m_{2(n-1)} & m_{2n} \\ \vdots & & \vdots & \vdots \\ m_{n1} & \dots & m_{n(n-1)} & m_{nn} \end{pmatrix}$$

Then consider the reversal of the basis as the matrix:

$$A = \begin{pmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{pmatrix}$$

Then the change of basis with the basis reversed on either side could be thought of as applying the change of basis matrix $A$ to each side. That is:

$$R = AMA^{-1}$$

Thus

$$R = \begin{pmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{pmatrix} \begin{pmatrix} m_{11} & \dots & m_{1(n-1)} & m_{1n} \\ m_{21} & \dots & m_{2(n-1)} & m_{2n} \\ \vdots & & \vdots & \vdots \\ m_{n1} & \dots & m_{n(n-1)} & m_{nn} \end{pmatrix} \begin{pmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} m_{nn} & \dots & m_{n2} & m_{n1} \\ m_{(n-1)n} & \dots & m_{(n-1)2} & m_{(n-1)1} \\ \vdots & & \vdots & \vdots \\ m_{1n} & \dots & m_{12} & m_{11} \end{pmatrix}$$